# CONCISE BUSINESS ARITHMETIC

MOORE
AND
MINER

GINN AND COMPANY

Area of simple shapes etc

Rectangle Length by width
L × W.

Triangle
× Base × alt — ÷ 2
.001 for 6 da
.01 " 60 "
.1 " 600 "
8 1 " 6000 "
Hyp. — R. ang. Tri = Summ of sq
of other 2 sides
× B × alt. — sq-root

.6 6
.6 6 5
.0 0 0$\frac{1}{6}$

11,00 - 10% per year
6
66
33

Digitized by the Internet Archive
in 2022 with funding from
Kahle/Austin Foundation

https://archive.org/details/concisebusinessa0000unse

MOORE AND MINER SERIES

# CONCISE BUSINESS ARITHMETIC

BY

JOHN H. MOORE

AND

GEORGE W. MINER

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON
ATLANTA · DALLAS · COLUMBUS · SAN FRANCISCO

COPYRIGHT, 1915, BY GEORGE W. MINER

PRACTICAL BUSINESS ARITHMETIC

COPYRIGHT, 1906, BY JOHN H. MOORE AND GEORGE W. MINER
COPYRIGHT, 1915, BY GEORGE W. MINER

ALL RIGHTS RESERVED

515.4

The Athenæum Press
GINN AND COMPANY · PRO-
PRIETORS · BOSTON · U.S.A.

# PREFACE

The "Concise Business Arithmetic" has been prepared to meet the needs of those schools that have short courses in business arithmetic. It emphasizes fundamental processes and those subjects that are commonly used in the business world. The average student needs all the drill he can get to acquire accuracy and facility in the handling of numbers.

This book is made up of twenty chapters selected from the "Practical Business Arithmetic" (Revised) and retains the features of that volume that have been so highly commended by teachers, namely: the presentation of each subject in a logical order; the selection of problems that appeal to the needs and the interest of the student and of the community as well; the omission of complex and useless problems and the inclusion of such material as will meet the needs of persons who have time for only a brief study of business arithmetic; a plan of grading and grouping problems which aids the student in acquiring facility and in advancing his educational equipment; the inclusion of an amount of work that will contribute to real efficiency; the development of subjects inductively and the omission of set rules; the unusual amount of work in the different chapters.

The text includes the computation of loss and gain on the selling price, tests on a time limit, additional work on graphs, statistical matter based on the latest census, and an appendix on the varied uses of the adding machine.

As this work is based on the "Practical Business Arithmetic," of which the late John H. Moore was the senior author, it seems proper to retain his name in the first place on the title-page of this volume, the actual preparation of which has necessarily been entirely in the hands of the junior author of the Moore and Miner textbooks.

Grateful acknowledgment for helpful service is due to Dr. David Eugene Smith, Professor of Mathematics, Teachers College, Columbia University, New York; to Mr. George Abbot of Brown Bros. & Co., Boston; to Mr. H. T. Smith, assistant cashier of the Shawmut National Bank, Boston, for valuable assistance on the chapters on interest and banking; to Mr. Wm. B. Medlicott, Lecturer on Property Insurance at Harvard University, for his work on the chapter on property insurance; to Mr. Montgomery Rollins of Boston, author of "Money and Investments"; to Mr. Harold T. Sibley of Chicago for suggestions on the chapter on stocks and bonds; to Mr. Alexander H. Sproul of the State Normal School, Salem, Massachusetts, and to Mr. C. D. McGregor of Des Moines, Iowa.

GEORGE W. MINER

# CONTENTS

## FUNDAMENTAL PROCESSES

CHAPTER PAGE
I. Introduction . . . 1
II. Addition . . . 2
III. Subtraction . . . 23
IV. Multiplication . . . 36
V. Division . . . 52
VI. Checking Results . . . 71

## FRACTIONS

VII. Decimal Fractions . . . 75
VIII. Factors, Divisors, and Multiples . . . 95
IX. Common Fractions . . . 101
X. Aliquot Parts . . . 138
XI. Bills and Accounts . . . 150

## DENOMINATE NUMBERS

XII. Denominate Quantities . . . 169

## PERCENTAGE AND ITS APPLICATIONS

XIII. Percentage . . . 175
XIV. Commercial Discounts . . . 188
XV. Gain and Loss . . . 196
XVI. Marking Goods . . . 204
XVII. Property Insurance . . . 209

## INTEREST AND BANKING

XVIII. Interest . . . 216
XIX. Bank Discount . . . 230
XX. Domestic Exchange . . . 242

DIVIDENDS AND INVESTMENTS

CHAPTER PAGE
XXI. Stocks and Bonds . . . . . . . . . . . . . . 256

APPENDIX A
Adding Machines . . . . . . . . . . . . . . . 273

APPENDIX B
Tables of Measures . . . . . . . . . . . . . . 275
Business Abbreviations . . . . . . . . . . . . . 278
Business Symbols . . . . . . . . . . . . . . . 279

INDEX . . . . . . . . . . . . . . . . . . . 281

# CONCISE BUSINESS ARITHMETIC

## FUNDAMENTAL PROCESSES

## CHAPTER I

### INTRODUCTION

**1.** The student who is prepared to study business arithmetic must be familiar with the ordinary symbols used in the statement or the solution of problems; he must have the ability to read and to write numbers with facility; he must know the fundamentals, and he must be able to perform ordinary operations in United States money, and in both common and decimal fractions.

**2.** In this course in business arithmetic one learns many simple methods for handling numbers and solving problems, and the adaptation of arithmetic to important business operations; he also acquires skill, rapidity, and accuracy, and he learns how to prove his own work, thus developing self-reliance. Because arithmetic deals with the problems of the home as well as the business office, the study of its practical and everyday features increases one's knowledge of the usages, the phraseology, and the literature of business and commerce.

**3.** Much attention is given, in the text, to the fundamental processes, for these are at the foundation of all arithmetic. One must acquire a high degree of accuracy and speed in the handling of these fundamentals if he is to achieve any marked degree of success in his subsequent work.

The text contains an unusual amount of material for the student's work, and portions of it may be omitted, at the discretion of the instructor, if the advancement of the class warrants it.

# CHAPTER II

## ADDITION

### ORAL EXERCISE

**1.** Find the sum of 1, 2, 3, 7, 5, 9, 4, 8, and 6.

**2.** Read each of the numbers in problem 1 increased by 2; by 5; by 3; by 7; by 8; by 9; by 17; by 23.

**3.** Find the sum of 8, 7, 9, 5, 6, 11, and 12.

**4.** Read each of the numbers in problem 3 increased by 12; by 15; by 18; by 24; by 42; by 19; by 16.

**5.** Illustrate what is meant by like numbers.

**4.** *Only like numbers can be added.*

**5.** To secure speed and accuracy in addition name *results only* and express these in the fewest words possible.

Thus, in adding 2, 4, 7, 8, 3, 2, and 8 say *6, 13, 21, 4, 6, 34*; do *not* say *2 and 4 are 6 and 7 are 13 and 8 are 21 and 3 are 24 and 2 are 26 and 8 are 34.*

### ORAL EXERCISE

*Name the sum in each of the following problems:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** | **11.** | **12.** | **13.** | **14.** | **15.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 2 | 8 | 1 | 5 | 8 | 1 | 3 | 5 | 5 | 1 | 3 | 4 | 2 |
| 2 | 1 | 4 | 2 | 3 | 2 | 2 | 3 | 3 | 1 | 4 | 7 | 2 | 5 | 7 |
| 1 | 6 | 3 | 1 | 6 | 1 | 3 | 6 | 4 | 6 | 4 | 2 | 1 | 2 | 3 |
| 2 | 8 | 2 | 2 | 4 | 3 | 7 | 4 | 2 | 2 | 3 | 7 | 5 | 8 | 5 |
| 8 | 4 | 1 | 3 | 4 | 4 | 4 | 9 | 8 | 7 | 2 | 3 | 2 | 6 | 4 |
| 4 | 8 | 4 | 4 | 3 | 7 | 7 | 5 | 3 | 3 | 1 | 4 | 8 | 4 | 2 |
| 5 | 6 | 3 | 5 | 2 | 2 | 3 | 8 | 6 | 2 | 0 | 5 | 2 | 5 | 1 |
| 6 | 0 | 6 | 2 | 3 | 1 | 4 | 2 | 2 | 5 | 7 | 2 | 6 | 3 | 4 |
| 3 | 8 | 1 | 7 | 7 | 6 | 1 | 1 | 1 | 1 | 7 | 7 | 1 | 2 | 3 |
| 3 | 6 | 2 | 2 | 4 | 2 | 2 | 4 | 3 | 4 | 2 | 1 | 1 | 1 | 2 |
| 2 | 2 | 3 | 5 | 1 | 8 | 3 | 2 | 2 | 3 | 1 | 3 | 8 | 6 | 2 |
| 4 | 1 | 5 | 1 | 2 | 3 | 2 | 4 | 1 | 2 | 4 | 4 | 9 | 8 | 7 |

**6.** Addition is the basis of all mathematical processes. It constitutes a large part of all the computations of business life and concerns, to some extent, every citizen of to-day. Ability to add rapidly and accurately is therefore a valuable accomplishment.

**7. Rapid addition** depends mainly upon the ability to group; that is, to instantly combine two or more figures into a single number. In reading it is never necessary to stop to name the individual letters in the words. All the letters of a word are taken in at a glance; hence the whole word is known at sight. Words are then grouped in rapid succession and a whole line is practically read at a glance. This is just the principle upon which rapid addition depends. From two to four figures should be read at sight as a single number, and the group so formed should be rapidly combined with other groups until the result of any given column is determined. This can be done only by intelligent, persistent practice.

**8.** The following list contains **all possible groups of two figures each.**

### ORAL EXERCISE

*Pronounce at sight the sum of each of the following groups:*

| | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 1 | 1 | 2 | 2 | 4 | 1 | 3 | 3 | 4 | 3 | 1 | 4 | 2 | 4 | 7 |
| | 1 | 3 | 1 | 2 | 1 | 5 | 2 | 3 | 2 | 6 | 7 | 3 | 5 | 6 | 7 |
| **2.** | 8 | 9 | 8 | 5 | 6 | 4 | 5 | 5 | 7 | 1 | 5 | 6 | 6 | 8 | 9 |
| | 9 | 9 | 8 | 5 | 1 | 4 | 3 | 4 | 2 | 8 | 6 | 6 | 9 | 6 | 1 |
| **3.** | 8 | 7 | 7 | 4 | 9 | 7 | 6 | 7 | 5 | 3 | 2 | 4 | 5 | 7 | 6 |
| | 2 | 3 | 5 | 8 | 3 | 8 | 7 | 9 | 9 | 8 | 9 | 9 | 8 | 4 | 2 |

The above exercise may be copied on the board and each student in turn required to name the results from left to right, from right to left, from top to bottom, and from bottom to top. The drill should be continued until the sums can be named at the rate of 150 per minute. This is the first and most important step in grouping.

### ORAL EXERCISE

*Name the sum in each of the following problems:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** | **11.** | **12.** | **13.** | **14.** | **15.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 3 | 5 | 6 | 7 | 9 | 9 | 9 | 1 | 2 | 5 | 8 | 2 | 8 |
| 3 | 1 | 4 | 2 | 4 | 7 | 9 | 8 | 8 | 4 | 7 | 2 | 3 | 7 | 2 |
| 8 | 7 | 7 | 5 | 8 | 2 | 5 | 9 | 4 | 5 | 8 | 3 | 5 | 4 | 1 |
| 7 | 9 | 6 | 9 | 3 | 9 | 8 | 4 | 7 | 1 | 1 | 7 | 9 | 5 | 9 |
| 3 | 8 | 5 | 3 | 8 | 1 | 6 | 4 | 9 | 2 | 6 | 5 | 7 | 3 | 4 |
| 9 | 4 | 7 | 7 | 2 | 9 | 8 | 5 | 1 | 3 | 5 | 7 | 6 | 5 | 5 |
| 5 | 6 | 6 | 8 | 2 | 4 | 4 | 3 | 6 | 3 | 6 | 8 | 7 | 4 | 6 |
| 5 | 6 | 5 | 5 | 7 | 5 | 4 | 2 | 1 | 3 | 6 | 4 | 9 | 4 | 8 |
| 2 | 3 | 2 | 1 | 1 | 2 | 3 | 1 | 1 | 2 | 5 | 3 | 8 | 1 | 9 |
| 4 | 3 | 3 | 1 | 4 | 2 | 1 | 5 | 6 | 4 | 5 | 9 | 7 | 6 | 6 |

Name the results only and make groups of two figures each. Thus, in problem 1, beginning at the bottom and adding up, say *6, 16, 28, 43, 52.*

**16–45.** Add the numbers in the exercise on page 2 by groups of two figures each.

**9.** It is practically as easy to add 54 and 9, 59 and 6, etc., as it is 4 and 9, 9 and 6, etc. 4 and 9 are always equal to 1 ten and 3 units, and 9 and 6 to 1 ten and 5 units. Hence in adding 54 and 9 think of the tens as increased by 1, call the units 3, and the result is 63; in adding 59 and 6 think of the tens as 6, the units as 5, and the result as 65.

### ORAL EXERCISE

*Pronounce at sight the sum of each of the following groups:*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 27 | 48 | 59 | 77 | 58 | 52 | 59 | 75 | 95 | 84 | 39 | 59 | 84 | 76 | 91 |
| | 7 | 8 | 6 | 8 | 7 | 8 | 8 | 6 | 9 | 7 | 6 | 5 | 9 | 8 | 8 |
| **2.** | 75 | 59 | 77 | 88 | 74 | 23 | 24 | 44 | 89 | 78 | 67 | 37 | 56 | 58 | 68 |
| | 8 | 9 | 9 | 5 | 6 | 8 | 9 | 9 | 9 | 9 | 9 | 7 | 7 | 4 | 5 |
| **3.** | 37 | 49 | 38 | 37 | 45 | 95 | 98 | 87 | 54 | 72 | 63 | 42 | 73 | 97 | 88 |
| | 5 | 8 | 7 | 6 | 9 | 8 | 7 | 7 | 9 | 9 | 8 | 9 | 8 | 5 | 9 |

**10.** In combining numbers between 10 and 20 think of them as one ten and a certain number of units and not as a certain number of units and 1 ten.

Thus, in combining 17 and 18 think of 28 and 7, or 35; in combining 19 and 15 think of 29 and 5, or 34; and so on.

### ORAL EXERCISE

*Pronounce at sight the sum of each of the following groups:*

| | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 12 | 17 | 12 | 16 | 11 | 12 | 18 | 16 | 17 | 11 | 19 | 13 | 18 | 12 | 17 |
| | 15 | 17 | 12 | 13 | 14 | 11 | 18 | 12 | 18 | 19 | 15 | 13 | 12 | 14 | 19 |
| **2.** | 13 | 11 | 15 | 19 | 14 | 19 | 17 | 15 | 13 | 19 | 16 | 14 | 18 | 18 | 12 |
| | 18 | 16 | 16 | 14 | 15 | 16 | 16 | 13 | 11 | 18 | 14 | 14 | 11 | 15 | 19 |
| **3.** | 11 | 17 | 12 | 17 | 15 | 15 | 12 | 18 | 16 | 14 | 19 | 14 | 19 | 17 | 11 |
| | 11 | 14 | 13 | 13 | 17 | 15 | 17 | 16 | 16 | 13 | 19 | 18 | 13 | 11 | 15 |

The above exercise contains all combinations possible with the numbers from 11 to 19 inclusive. Drill on the exercise should be continued until results can be named at the rate of 120 per minute.

**11.** Numbers between 10 and 20 may be combined with numbers above 20 in practically the same manner as in § 10

Thus, in adding 62 and 12 think of 72 and 2, or 74; in adding 79 and 17 think of 89 and 7, or 96.

### ORAL EXERCISE

*Pronounce at sight the sum of each of the following groups:*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 25 | 48 | 59 | 87 | 91 | 75 | 86 | 75 | 48 | 78 | 57 | 89 | 37 | 56 | 75 |
| | 17 | 17 | 16 | 14 | 18 | 18 | 19 | 12 | 16 | 13 | 16 | 14 | 17 | 18 | 14 |
| **2.** | 29 | 47 | 83 | 92 | 36 | 54 | 59 | 78 | 67 | 92 | 77 | 86 | 53 | 78 | 85 |
| | 13 | 14 | 19 | 14 | 19 | 13 | 18 | 15 | 13 | 13 | 19 | 19 | 17 | 14 | 14 |
| **3.** | 31 | 32 | 45 | 69 | 74 | 95 | 98 | 92 | 96 | 87 | 86 | 34 | 43 | 64 | 38 |
| | 19 | 17 | 19 | 15 | 8 | 18 | 14 | 19 | 15 | 17 | 19 | 18 | 18 | 19 | 17 |

### ORAL EXERCISE

**1.** Count by 7's from 1 to 85.

Solution. 8, 15, 22, 9, 36, 43, 50, 7, 64, 71, 8, 85.

*Count by:*

**2.** 2's from 39 to 55.
**3.** 5's from 11 to 86.
**4.** 6's from 15 to 63.
**5.** 5's from 2 to 107.
**6.** 7's from 11 to 60.
**7.** 8's from 25 to 89.
**8.** 9's from 31 to 112.
**9.** 8's from 32 to 192.
**10.** 7's from 18 to 102.
**11.** 6's from 72 to 126.
**12.** 9's from 10 to 136.
**13.** 9's from 17 to 152.
**14.** 8's from 10 to 138.
**15.** 7's from 19 to 152.
**16.** 6's from 20 to 128.
**17.** 6's from 15 to 111.
**18.** 9's from 12 to 102.
**19.** 8's from 17 to 113.
**20.** 7's from 24 to 108.
**21.** 6's from 27 to 117.
**22.** 4's from 19 to 183.
**23.** 11's from 14 to 102.
**24.** 12's from 17 to 161.
**25.** 13's from 17 to 121.

**26.** Beginning at 1 count by 4's to 17; going on from 17 count by 7's to 52; from 52 count by 9's to 133; from 133 count by 5's to 158; from 158 count by 12's to 206; from 206 count by 13's to 271.

This exercise furnishes one of the best possible drills in addition, and it should be continued until the successive results can be named at the rate of 150 per minute.

**12.** If the student is accurate and rapid in making groups of two figures each, he is ready for practice in groups of three figures each. In the following exercise are **all the possible groups of three figures each.**

### ORAL EXERCISE

*Name at sight the sum of each of the following groups:*

4, 2, and 3 should be thought of as 9 just as p-e-n is thought of as pen.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 4 | 1 | 9 | 8 | 1 | 1 | 3 | 1 | 8 | 1 | 4 | 5 | 1 | 7 | 8 |
| | 1 | 3 | 1 | 2 | 2 | 3 | 1 | 7 | 3 | 3 | 1 | 4 | 4 | 1 | 4 |
| | 3 | 3 | 2 | 1 | 7 | 5 | 6 | 3 | 1 | 9 | 4 | 1 | 6 | 4 | 1 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2.** | 1 | 6 | 1 | 4 | 1 | 2 | 1 | 1 | 1 | 1 | 7 | 6 | 9 | 8 | 1 |
| | 4 | 1 | 2 | 1 | 2 | 2 | 9 | 1 | 1 | 6 | 6 | 6 | 5 | 5 | 5 |
| | 9 | 2 | 5 | 2 | 3 | 1 | 1 | 8 | 7 | 8 | 1 | 1 | 1 | 1 | 7 |
| **3.** | 6 | 5 | 2 | 5 | 2 | 3 | 9 | 2 | 2 | 2 | 2 | 6 | 1 | 1 | 2 |
| | 1 | 1 | 3 | 3 | 3 | 2 | 2 | 8 | 7 | 6 | 5 | 1 | 1 | 1 | 2 |
| | 5 | 5 | 6 | 2 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 5 | 4 | 4 |
| **4.** | 3 | 2 | 1 | 2 | 2 | 6 | 2 | 6 | 5 | 5 | 7 | 1 | 1 | 1 | 1 |
| | 2 | 2 | 1 | 7 | 6 | 8 | 6 | 2 | 2 | 2 | 2 | 1 | 1 | 6 | 9 |
| | 2 | 2 | 3 | 7 | 9 | 2 | 7 | 6 | 9 | 8 | 5 | 2 | 1 | 9 | 9 |
| **5.** | 9 | 8 | 9 | 8 | 7 | 3 | 4 | 5 | 6 | 6 | 5 | 4 | 3 | 3 | 4 |
| | 1 | 1 | 1 | 1 | 1 | 5 | 8 | 7 | 7 | 7 | 5 | 4 | 4 | 4 | 4 |
| | 8 | 8 | 7 | 7 | 7 | 5 | 4 | 5 | 9 | 8 | 6 | 7 | 9 | 8 | 6 |
| **6.** | 5 | 6 | 6 | 9 | 5 | 7 | 3 | 4 | 9 | 6 | 6 | 8 | 3 | 3 | 3 |
| | 5 | 7 | 6 | 4 | 4 | 3 | 4 | 4 | 4 | 8 | 7 | 4 | 9 | 4 | 4 |
| | 5 | 7 | 9 | 9 | 4 | 4 | 6 | 4 | 8 | 6 | 6 | 8 | 9 | 5 | 4 |
| **7.** | 3 | 4 | 6 | 9 | 8 | 5 | 4 | 3 | 3 | 2 | 3 | 3 | 4 | 5 | 8 |
| | 8 | 7 | 6 | 9 | 9 | 9 | 7 | 8 | 3 | 5 | 3 | 7 | 7 | 8 | 8 |
| | 9 | 9 | 6 | 9 | 9 | 9 | 8 | 8 | 9 | 6 | 8 | 9 | 7 | 9 | 8 |
| **8.** | 8 | 5 | 4 | 3 | 3 | 5 | 2 | 3 | 3 | 4 | 5 | 7 | 7 | 5 | 4 |
| | 8 | 8 | 9 | 8 | 7 | 2 | 4 | 3 | 7 | 6 | 7 | 9 | 8 | 7 | 6 |
| | 9 | 5 | 6 | 7 | 3 | 5 | 9 | 6 | 7 | 8 | 9 | 9 | 9 | 8 | 7 |
| **9.** | 3 | 3 | 2 | 2 | 3 | 3 | 4 | 5 | 7 | 9 | 9 | 9 | 7 | 3 | 6 |
| | 6 | 3 | 4 | 4 | 3 | 6 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 3 | 4 |
| | 9 | 5 | 8 | 7 | 4 | 8 | 6 | 7 | 8 | 7 | 5 | 4 | 3 | 3 | 2 |
| **10.** | 2 | 2 | 3 | 4 | 5 | 7 | 2 | 2 | 3 | 4 | 5 | 7 | 9 | 6 | 6 |
| | 4 | 9 | 6 | 5 | 6 | 7 | 4 | 8 | 5 | 5 | 6 | 7 | 9 | 6 | 5 |
| | 5 | 9 | 6 | 8 | 8 | 8 | 4 | 9 | 9 | 7 | 7 | 7 | 6 | 5 | 4 |
| **11.** | 8 | 8 | 9 | 2 | 2 | 3 | 4 | 5 | 6 | 8 | 8 | 9 | 6 | 8 | 7 |
| | 5 | 8 | 3 | 3 | 7 | 5 | 5 | 5 | 8 | 8 | 5 | 4 | 5 | 7 | 3 |
| | 3 | 2 | 2 | 8 | 9 | 7 | 5 | 9 | 9 | 6 | 5 | 4 | 3 | 2 | 2 |

This exercise should be drilled upon until the sums of the groups, in any order, can be named at the rate of 120 per minute.

### ORAL EXERCISE

**1–15.** Turn to the exercise on page 2 and find the sum of the numbers given.

Name results only, and make groups of three figures each. Thus, in problem 1, say *9, 23, 37, 43.*

Add from the bottom up and check the work by adding from the top down.

*Find the sum in each of the following problems:*

| **16.** | **17.** | **18.** | **19.** | **20.** | **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 4 | 2 | 2 | 2 | 4 | 4 | 5 | 1 | 2 | 9 | 5 | 4 |
| 1 | 1 | 3 | 3 | 3 | 1 | 6 | 3 | 9 | 5 | 7 | 4 | 0 | 7 | 3 |
| 1 | 1 | 4 | 1 | 5 | 2 | 2 | 4 | 5 | 0 | 2 | 4 | 1 | 2 | 1 |
| 2 | 1 | 3 | 1 | 3 | 1 | 4 | 1 | 8 | 8 | 9 | 2 | 8 | 0 | 1 |
| 2 | 4 | 1 | 4 | 6 | 4 | 5 | 8 | 3 | 2 | 0 | 3 | 0 | 0 | 6 |
| 2 | 2 | 3 | 8 | 1 | 1 | 2 | 1 | 7 | 1 | 1 | 5 | 2 | 5 | 8 |
| 2 | 4 | 2 | 2 | 2 | 2 | 2 | 3 | 8 | 3 | 5 | 7 | 2 | 6 | 1 |
| 5 | 2 | 1 | 4 | 5 | 3 | 7 | 6 | 2 | 7 | 3 | 7 | 2 | 6 | 6 |
| 1 | 2 | 9 | 4 | 3 | 2 | 3 | 1 | 8 | 2 | 2 | 1 | 6 | 0 | 7 |
| 5 | 1 | 8 | 3 | 4 | 2 | 1 | 2 | 9 | 9 | 6 | 7 | 2 | 3 | 3 |
| 3 | 5 | 2 | 3 | 3 | 6 | 9 | 3 | 3 | 1 | 2 | 8 | 2 | 6 | 3 |
| 1 | 3 | 1 | 3 | 3 | 1 | 0 | 5 | 6 | 3 | 7 | 0 | 4 | 1 | 1 |
| 3 | 2 | 7 | 2 | 4 | 3 | 0 | 2 | 8 | 8 | 4 | 7 | 2 | 5 | 9 |
| 5 | 4 | 2 | 5 | 2 | 4 | 8 | 5 | 1 | 2 | 3 | 3 | 2 | 3 | 2 |
| 2 | 4 | 1 | 4 | 4 | 3 | 2 | 2 | 0 | 4 | 3 | 0 | 5 | 2 | 1 |
| 1 | 2 | 1 | 2 | 6 | 6 | 4 | 4 | 6 | 6 | 3 | 6 | 2 | 5 | 8 |
| 8 | 6 | 2 | 3 | 3 | 3 | 5 | 2 | 4 | 4 | 3 | 3 | 2 | 8 | 2 |
| 1 | 2 | 6 | 5 | 1 | 1 | 1 | 3 | 0 | 5 | 6 | 1 | 6 | 2 | 1 |
| 4 | 4 | 1 | 3 | 7 | 2 | 9 | 3 | 7 | 9 | 1 | 5 | 7 | 5 | 7 |
| 3 | 5 | 2 | 2 | 2 | 6 | 2 | 2 | 3 | 1 | 7 | 3 | 3 | 7 | 2 |
| 4 | 2 | 5 | 6 | 1 | 3 | 1 | 3 | 0 | 3 | 2 | 2 | 1 | 3 | 1 |
| 4 | 2 | 1 | 2 | 1 | 2 | 2 | 7 | 7 | 7 | 1 | 1 | 9 | 2 | 2 |
| 9 | 7 | 2 | 2 | 3 | 8 | 3 | 1 | 2 | 3 | 9 | 1 | 2 | 5 | 2 |
| 1 | 3 | 4 | 4 | 4 | 1 | 7 | 7 | 1 | 0 | 0 | 8 | 4 | 8 | 4 |
| 2 | 1 | 3 | 7 | 3 | 2 | 5 | 7 | 6 | 5 | 5 | 2 | 4 | 4 | 3 |
| 1 | 6 | 2 | 1 | 5 | 5 | 3 | 2 | 3 | 2 | 8 | 1 | 3 | 6 | 3 |
| 2 | 3 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 | 5 | 7 | 1 | 1 |

**13.** It is always an advantage to find **groups of figures aggregating 10 and 20** in the body of a column.

These groups should be added immediately to the sum already obtained by simply combining the tens of the two numbers. It is not a good plan, however, to take the digits in irregular order in order to form groups of 10 and 20.

### ORAL EXERCISE

*Find the sum in each of the following problems, taking advantage of groups of 10 and 20 wherever possible:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** | **11.** | **12.** | **13.** | **14.** | **15.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 7 | 6 | 5 | 2 | 5 | 3 | 4 | 3 | 7 | 8 | 2 | 5 | 9 |
| 9 | 8 | 3 | 4 | 5 | 5 | 4 | 3 | 2 | 5 | 5 | 4 | 7 | 8 | 9 |
| 7 | 4 | 5 | 1 | 8 | 5 | 6 | 7 | 8 | 5 | 5 | 6 | 3 | 2 | 1 |
| 3 | 6 | 5 | 9 | 2 | 7 | 9 | 8 | 7 | 4 | 0 | 2 | 5 | 8 | 1 |
| 2 | 7 | 7 | 2 | 4 | 3 | 1 | 2 | 3 | 6 | 9 | 7 | 5 | 2 | 5 |

| **16.** | **17.** | **18.** | **19.** | **20.** | **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 9 | 1 | 4 | 6 | 5 | 7 | 1 | 2 | 4 | 4 | 1 | 6 | 1 |
| 3 | 1 | 2 | 6 | 1 | 2 | 2 | 2 | 4 | 2 | 2 | 3 | 9 | 3 | 1 |
| 3 | 8 | 9 | 3 | 5 | 2 | 3 | 1 | 5 | 6 | 4 | 3 | 2 | 1 | 8 |
| 7 | 1 | 4 | 6 | 5 | 4 | 4 | 7 | 5 | 1 | 8 | 7 | 8 | 7 | 0 |
| 2 | 6 | 1 | 2 | 2 | 4 | 3 | 3 | 5 | 7 | 6 | 2 | 3 | 4 | 9 |
| 1 | 3 | 5 | 2 | 3 | 2 | 3 | 4 | 4 | 2 | 4 | 5 | 7 | 5 | 0 |
| 6 | 5 | 9 | 2 | 4 | 4 | 8 | 6 | 6 | 5 | 3 | 1 | 8 | 1 | 1 |

| **31.** | **32.** | **33.** | **34.** | **35.** | **36.** | **37.** | **38.** | **39.** | **40.** | **41.** | **42.** | **43.** | **44.** | **45.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 9 | 8 | 6 | 6 | 6 | 6 | 5 | 8 | 6 | 6 | 2 | 7 | 6 |
| 2 | 8 | 4 | 5 | 6 | 7 | 7 | 5 | 7 | 4 | 3 | 9 | 8 | 9 | 7 |
| 9 | 9 | 7 | 6 | 8 | 7 | 8 | 6 | 8 | 9 | 8 | 5 | 9 | 9 | 4 |
| 9 | 7 | 9 | 9 | 7 | 9 | 7 | 8 | 7 | 7 | 9 | 6 | 9 | 2 | 9 |

| **46.** | **47.** | **48.** | **49.** | **50.** | **51.** | **52.** | **53.** | **54.** | **55.** | **56.** | **57.** | **58.** | **59.** | **60.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 38 | 42 | 25 | 35 | 46 | 14 | 21 | 12 | 18 | 29 | 57 | 17 | 13 | 14 | 15 |
| 8 | 1 | 4 | 2 | 4 | 5 | 0 | 3 | 2 | 5 | 5 | 4 | 6 | 2 | 7 |
| 1 | 2 | 1 | 9 | 7 | 6 | 6 | 3 | 4 | 7 | 6 | 8 | 6 | 7 | 2 |
| 1 | 7 | 3 | 9 | 9 | 0 | 6 | 8 | 4 | 8 | 9 | 8 | 8 | 5 | 8 |
| 2 | 4 | 6 | 5 | 2 | 9 | 8 | 6 | 7 | 4 | 5 | 5 | 2 | 3 | 6 |

**14.** When three figures are in consecutive order the sum may be found by multiplying the middle figure by 3; when five figures are in consecutive order the sum may be found by multiplying the middle figure by 5; etc.; or the sum of any number of consecutive numbers may be found by taking one half the sum of the first and last numbers and multiplying it by the number of terms.

### ORAL EXERCISE

*By inspection find the sum of:*

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. | 15. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 10 | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 |
| 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 50 |
| 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 |

| 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. | 29. | 30. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
| 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 | 61 | 66 | 71 | 76 | 81 |
| 12 | 17 | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 | 67 | 72 | 77 | 82 |
| 13 | 18 | 23 | 28 | 33 | 38 | 43 | 48 | 53 | 58 | 63 | 68 | 73 | 78 | 83 |
| 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 69 | 74 | 79 | 84 |

| 31. | 32. | 33. | 34. | 35. | 36. | 37. | 38. | 39. | 40. | 41. | 42. | 43. | 44. | 45. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 10 | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 |
| 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 50 |
| 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 |
| 10 | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 | 52 |
| 11 | 14 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 50 | 53 |
| 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 |
| 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 55 |
| 14 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 50 | 53 | 56 |
| 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 |
| 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 55 | 58 |
| 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 50 | 53 | 56 | 59 |

**15.** When a figure is repeated several times the sum may be found by multiplication.

### ORAL EXERCISE

*By inspection find the sum of:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** | **11.** | **12.** | **13.** | **14.** | **15.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 4 | 5 | 3 | 7 | 8 | 8 | 15 | 6 | 7 | 8 | 15 | 13 | 9 |
| 9 | 7 | 4 | 5 | 3 | 7 | 5 | 7 | 15 | 6 | 8 | 7 | 14 | 13 | 8 |
| 9 | 8 | 4 | 5 | 9 | 7 | 5 | 9 | 15 | 12 | 7 | 8 | 15 | 13 | 8 |
| 9 | 8 | 9 | 5 | 9 | 8 | 6 | 9 | 8 | 12 | 7 | 7 | 14 | 7 | 9 |
| 9 | 8 | 9 | 9 | 8 | 8 | 6 | 9 | 8 | 12 | 7 | 8 | 15 | 7 | 8 |

| **16.** | **17.** | **18.** | **19.** | **20.** | **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 7 | 4 | 2 | 7 | 5 | 12 | 2 | 4 | 6 | 8 | 9 | 8 | 5 | 16 |
| 3 | 7 | 4 | 2 | 7 | 5 | 5 | 2 | 4 | 6 | 8 | 9 | 8 | 5 | 16 |
| 3 | 7 | 4 | 2 | 4 | 5 | 5 | 2 | 4 | 6 | 8 | 9 | 8 | 5 | 16 |
| 2 | 2 | 7 | 8 | 4 | 4 | 5 | 3 | 5 | 4 | 3 | 5 | 8 | 5 | 16 |
| 2 | 2 | 7 | 8 | 2 | 4 | 5 | 3 | 5 | 4 | 3 | 5 | 8 | 5 | 20 |
| 2 | 2 | 7 | 8 | 2 | 4 | 5 | 3 | 5 | 4 | 3 | 5 | 9 | 8 | 1 |

**16.** In all written work make plain, legible figures of a uniform size, write them equal distances from each other, and be sure that the units of the same order stand in the same vertical column.

*1 2 3 4 5 6 7 8 9 0*

**17.** Many of the errors that occur in business are in simple addition. Errors in addition result from two main causes: irregularity in the placing of figures; poor figures.

**18.** In business it is important that figures be made rapidly; but rapidity should never be secured at the expense of legibility.

### WRITTEN EXERCISE

*Copy and find the sum of:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** |
|---|---|---|---|---|---|
| 1745 | 1842 | 1249 | 4271 | 6229 | 1481 |
| 1862 | 1695 | 1810 | 8614 | 4813 | 1862 |
| 7529 | 4716 | 6241 | 9217 | 7142 | 4129 |
| 8721 | 8412 | 1728 | 8214 | 6212 | 2412 |

| **7.** | **8.** | **9.** | **10.** | **11.** | **12.** |
|---|---|---|---|---|---|
| 4216 | 2110 | 4142 | 1061 | 4113 | 4112 |
| 8912 | 8420 | 4347 | 1875 | 8217 | 1012 |
| 4729 | 1641 | 1012 | 6214 | 8614 | 1862 |
| 8624 | 1722 | 1816 | 1931 | 1692 | 1721 |
| 4829 | 1837 | 4112 | 1648 | 1591 | 1692 |
| 6212 | 4216 | 4210 | 1721 | 1686 | 1486 |
| 4110 | 4117 | 1618 | 1728 | 2172 | 4112 |
| 4210 | 1832 | 4060 | 1421 | 1754 | 1010 |

**19.** The simplest way to **check addition** is to add the columns in reverse order. If the results obtained by both processes agree, the work may be assumed to be correct.

**20.** In adding long columns of figures it is generally advisable to record the entire sum of each column separately; then if interruptions occur, it will not be necessary to re-add any portions already completed. After the total of each column has been found the entire total may be determined by combining the separate totals of the columns.

**21.** The best way to test the accuracy of columns added in this manner is to begin at the left and repeat the addition in reverse order. The entire total of each column should again be written and the complete total of the problem found by adding the separate totals of the several columns. If the results obtained by the two processes agree, the work may be assumed to be correct.

**22. Example.** Find the sum of 54,669, 15,218, 36,425, 45,325, and 68,619. Check the result.

Solution. Beginning at the bottom of the right-hand column, add each column in regular order and write the entire totals as shown in (*a*). Beginning at the top of the left-hand column again add each column and write the entire totals as shown in (*b*). Next add the totals obtained by the first and second additions and compare the results. Since the total shown by (*a*) is equal to the total shown by (*b*), the result, 220,256, is assumed to be correct. All work in addition should be carefully checked.

| (*b*) |  | (*a*) |
|---|---|---|
|  | 54669 |  |
| 19 | 15218 | 36 |
| 28 | 36425 | 12 |
| 21 | 45325 | 21 |
| 12 | 68619 | 28 |
| 36 | 220256 | 19 |
| 220256 |  | 220256 |

### WRITTEN EXERCISE

*See how many times the following numbers can be written in one minute. Write each number in form for vertical addition.*

**1.** 426579.
**2.** 123987.
**3.** $7983.21.
**4.** $4080.91.
**5.** $70812.34.
**6.** $41182.50.

Thus, in repeating the number in problem 1 write it as follows:

4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
Etc.

Be sure that the spacing between the lines and between the columns is uniform. Increase the speed gradually until from 150 to 200 figures can be written per minute.

**23.** Skill in writing figures from dictation should be cultivated. The dictation should be slow at first, but it should be gradually increased until the requisite speed is acquired.

**24.** In calling off numbers to another great care should be taken in order that no errors may be made. In reading United States money the word *dollars* should be called with each amount. The word *cents* may be omitted in all cases except where there are no dollars.

Thus, in calling $400.37 say *four hundred dollars, thirty-seven;* in calling $25.11 say *twenty-five dollars, eleven;* in calling $1573.86 say *fifteen hundred seventy-three dollars, eighty-six;* in calling $5.31 say *five dollars, thirty-one.*

### WRITTEN EXERCISE

*Write from dictation and find the sum of:*

**1.** $75.18, $123.95, $147.25, $9.50, $181.45, $172.16, $84.98, $314.95, $49.10, $69.90, $312.60, $415.90.

**2.** $3140.19, $310.92, $3164.96, $3162.19, $18.62, $410.95, $690.18, $10.75, $3100.40, $300.40, $200.50, $100.90, $410.80, $100.85, $310.60, $80.90, $399.80, $412.60.

### WRITTEN EXERCISE

*Copy, find the sum, and check:*

**1.**

$1692564.21
1345678.50
3790269.17
6721234.23
4212612.45
1249212.54
1282964.16
2179216.54
6923469.48
2468627.16
5492165.18

**2.**

$6242654.92
1259216.05
2179203.04
9131426.04
1259212.46
1321652.17
2141621.46
3151259.06
2114201.58
3174257.69
4269237.50

**3.**

$2151621.54
1262141.67
2131472.03
1462141.07
9648968.03
1242472.42
9868638.68
2141262.49
8969849.61
1262479.14
3416218.49

**4.**

$126724.45
437.26
12.91
7.69
647686.42
54164.95
712.78
92.14
569.86
11.46
7269.42
6924.75
12798.69
124.60
1.59

**5.**

$1275246.48
7512047.72
126056.42
216414.79
16147.92
72146.54
12.75
126.92
2146.01
32645.91
4260.01
3001.98
75.06
1.92
4120.78

**6.**

$1251624.64
721266.44
411200.54
1500000.98
1969214.54
2401206.25
129863.64
1264.92
1512692.07
1692162.04
1721.90
416.75
6721.90
14.95
4261.74

| 7. | 8. | 9. |
|---|---|---|
| $126412.80 | $214621.64 | $1620177.25 |
| 98536.41 | 92684.75 | 2516214.29 |
| 6812.00 | 35789.44 | 1261426.41 |
| 232714.18 | 16929.75 | 1019027.60 |
| 982.75 | 18927.95 | 991833.40 |
| 12568.30 | 374261.42 | 721164.74 |
| 229942.78 | 1026406.47 | 389946.36 |
| 26054.90 | 621457.92 | 194172.45 |
| 226491.37 | 742467.48 | 916938.55 |
| 8362.40 | 692416.24 | 12012.40 |
| 426834.75 | 521207.84 | 6412.75 |
| 82964.35 | 37292.20 | 1616912.62 |
| 359482.20 | 12040.00 | 7518.95 |
| 7960.00 | 7245.69 | 16219.75 |
| 86521.46 | 36047.46 | 1417521.64 |
| 754376.50 | 694116.52 | 1121490.75 |
| 1016823.25 | 721205.11 | 69876.62 |
| 62943.12 | 621476.01 | 724138.75 |
| 4836.90 | 741627.03 | 126421.26 |
| 927859.46 | 621264.75 | 217221.64 |
| 262582.74 | 69128.60 | 69214.75 |
| 126129.45 | 1012016.59 | 2111694.41 |
| 96852.17 | 992017.29 | 999940.69 |
| 14912.18 | 121202.60 | 72126.73 |
| 251926.86 | 101427.65 | 251327.14 |
| 46532.45 | 241659.64 | 759783.96 |
| 22614.92 | 69398.95 | 16126.47 |
| 462284.20 | 70120.92 | 7212.45 |
| 97052.04 | 6954.87 | 277269.18 |
| 68052.65 | 1267026.54 | 189247.90 |
| 126125.65 | 321614.70 | 69246.78 |
| 37580.54 | 162367.90 | 24126.25 |

**25.** Some accountants practice adding two columns at once when the columns are short. The method generally employed is similar to the method explained for combining groups in regular addition.

**26. Example.** Find the sum of 83, 72, 89.

Solution. Beginning at the bottom and adding up, think of 89 and 72 as 159 and 2, or 161; of 161 and 83 as 241 and 3, or 244.

In adding name results only. Thus say *159, 161, 241, 244.*

83
72
89
244

### ORAL EXERCISE

*By inspection give the sum of each of the following groups:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** | **11.** | **12.** | **13.** | **14.** | **15.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 64 | 52 | 37 | 65 | 38 | 52 | 85 | 93 | 68 | 58 | 76 | 83 | 57 | 62 |
| 25 | 18 | 29 | 56 | 27 | 43 | 67 | 34 | 72 | 75 | 46 | 39 | 47 | 25 | 39 |

| **16.** | **17.** | **18.** | **19.** | **20.** | **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 53 | 52 | 61 | 34 | 91 | 68 | 48 | 24 | 78 | 54 | 94 | 57 | 92 | 76 | 43 |
| 46 | 43 | 37 | 76 | 13 | 47 | 69 | 96 | 76 | 35 | 36 | 44 | 37 | 31 | 56 |

| **31.** | **32.** | **33.** | **34.** | **35.** | **36.** | **37.** | **38.** | **39.** | **40.** | **41.** | **42.** | **43.** | **44.** | **45.** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | 44 | 46 | 48 | 67 | 44 | 53 | 25 | 54 | 46 | 33 | 16 | 67 | 83 | 88 |
| 86 | 57 | 65 | 25 | 48 | 57 | 45 | 31 | 65 | 39 | 64 | 34 | 43 | 82 | 25 |
| 75 | 21 | 34 | 31 | 39 | 21 | 67 | 69 | 87 | 87 | 77 | 25 | 41 | 98 | 31 |

## HORIZONTAL ADDITION

**27.** In some kinds of invoicing and in short-extending the items of an account numbers to be added are written in horizontal lines. Much time may be saved by adding these numbers as they stand. After careful practice it will be found possible to add numbers written in horizontal lines with as much facility as numbers written in vertical columns.

**28.** In adding numbers written horizontally care should be exercised to combine only units of the same order. It is generally best to add from left to right and to verify the work from right to left. Grouping may be employed to advantage in horizontal addition.

### WRITTEN EXERCISE

*Copy and add the following numbers horizontally. Verify the work.*

Thus, in problem 1, beginning at the left, say *10, 20, 32, 52*. In verifying the work from the right say *20, 32, 42, 52*.

**1.** 8, 2, 1, 1, 7, 1, 4, 6, 2, 3, 8, 9.

**2.** 7, 9, 6, 5, 4, 8, 7, 4, 3, 7, 3, 1, 3.

**3.** 6, 2, 4, 8, 3, 1, 7, 6, 4, 2, 8, 9, 4, 2.

**4.** 15, 23, 46, 83, 29, 35, 42, 15, 21, 26.

**5.** 64, 48, 56, 35, 47, 87, 32, 45, 67, 91.

**6.** 52, 64, 86, 28, 76, 41, 15, 32, 12, 87.

**7.** 32, 48, 24, 62, 85, 14, 63, 54, 78, 94, 23, 45.

**8.** 42, 76, 49, 81, 17, 42, 17, 19, 21, 43, 64, 17.

**9.** 45, 48, 34, 46, 48, 53, 25, 42, 35, 56, 70, 10.

**10.** 291, 196, 855, 578, 210, 354, 102, 232, 241, 162.

**11.** 469, 388, 962, 764, 351, 899, 111, 232, 190, 175.

**12.** $15^{25}$, $50^{25}$, $16^{84}$, $31^{42}$, $86^{38}$, $19^{10}$, $23^{12}$, $10^{13}$, $64^{80}$, $40^{10}$.

It is frequently desirable to express dollars and cents without the dollar sign and the decimal point. This may be done by slightly raising the cents of the amount. Thus, \$17.17 may be written $17^{17}$; \$2.08 may be written $2^{08}$.

**13.** $15^{25}$, $8^{93}$, $4^{88}$, $21^{84}$, $16^{35}$, $18^{46}$, $29^{14}$, $44^{60}$, $62^{90}$, $84^{60}$, $40^{50}$.

**14.** $76^{75}$, $84^{97}$, $67^{05}$, $95^{74}$, $68^{63}$, $52^{21}$, $13^{25}$, $42^{18}$, $60^{95}$, $80^{13}$, $90^{62}$.

**29.** It is important that the student acquire the ability to carry a series of numbers in mind. The following exercises are suggestive of what may be done to cultivate ability in this direction.

The dictation suggested should not be slower than at the rate of one hundred twenty words per minute. Nothing should be written by the students until all of the numbers of a problem have been called by the instructor; then one student may be sent to the blackboard and required to write the numbers from memory. If the numbers are correctly written, the instructor may require another student to give the sum of them without using pen or pencil. The numbers may be written on the board in either vertical or horizontal order, as the instructor may direct.

### ORAL EXERCISE

*From the instructor's dictation find mentally the sum of each of the following problems:*

1. 6, 9, 8, 4, 8, 6.
2. 14, 17, 20, 5, 9.
3. 24, 17, 16, 9, 5.
4. 5, 6, 7, 1, 3, 8.
5. 6, 2, 8, 1, 7, 4.
6. 364, 436, 657, 25.
7. 438, 212, 750, 64.
8. 859, 441, 769, 71.
9. 2140, 3160, 4000.
10. 200, 415, 600, 95.
11. 857, 643, 237, 500.
12. $4150, $4050, $850.
13. $5.15, $2.15, $6.70.
14. $167.14, $232.86, $9.
15. 147, 253, 179, 121.
16. 423, 517, 81, 49.
17. 255, 45, 89, 121.
18. 25, 65, 27, 133.
19. $48, $32, $138.
20. $135, $275, $418.
21. $23, $67, $281.
22. $284, $36, $245.
23. 133, 167, 29, 61.
24. 2319, 1681, 2335.
25. 3310, 2790, 1565.
26. 2740, 1365, 2135.
27. 2273, 1237, 1145.
28. 1432, 1058, 1210.

### WRITTEN REVIEW EXERCISE

**1.** Find the sum of all the integers from 2165 to 2260 inclusive.

**2.** Find the sum of all the integers from 1137 to 1200 inclusive.

**3.** Complete the following sales sheet. Add by columns and by lines and check the work by adding the vertical and horizontal totals.

SUMMARY OF SALES FOR WEEK ENDING AUG. 25

| | PINE | | OAK | | MAPLE | | SPRUCE | | WALNUT | | CHERRY | | TOTAL | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Monday | 1216 | 18 | 16161 | 47 | 649 | 58 | 860 | 40 | 315 | 64 | 186 | 50 | | |
| Tuesday | 5160 | 40 | 3214 | 90 | 316 | 40 | 160 | 50 | 513 | 80 | 216 | 54 | | |
| Wednesday | 6152 | 18 | 2150 | 18 | 163 | 59 | 430 | 17 | 968 | 52 | 756 | 14 | | |
| Thursday | 1216 | 18 | 2160 | 50 | 130 | 98 | 115 | 67 | 413 | 60 | 314 | 75 | | |
| Friday | 4160 | 80 | 1215 | 40 | 315 | 16 | 218 | 90 | 411 | 50 | 132 | 75 | | |
| Saturday | 3165 | 80 | 2115 | 72 | 218 | 50 | 165 | 37 | 118 | 50 | 17 | 05 | | |
| Total | | | | | | | | | | | | | | |

**4.** Add the following by columns and by lines, and check the work by adding the vertical and horizontal totals :

| | | | | | |
|---:|---:|---:|---:|---:|---|
| 21162 49 | 962 18 | 1245 76 | 54168 97 | 52 19 | |
| 176 19 | 1278 95 | 52698 13 | 7529 87 | 95162 87 | |
| 2164 89 | 7524 16 | 47612 87 | 6842 23 | 5948 23 | |
| 76 95 | 87 14 | 2150 49 | 172 93 | 1745 86 | |
| 51276 92 | 18187 95 | 75 19 | 162 14 | 5290 18 | |
| 9834 18 | 92923 15 | 25 91 | 162 18 | 14 95 | |
| 754 95 | 2167 92 | 2584 16 | 9176 92 | 3164 82 | |
| 1356 05 | 1314 93 | 7125 95 | 2167 18 | 2645 97 | |
| 756 92 | 142 18 | 167 42 | 926 44 | 3167 18 | |
| 75162 19 | 82195 78 | 72162 18 | 9165 97 | 168 44 | |
| 7162 95 | 4167 18 | 7156 95 | 172 18 | 1 56 | |
| 2 15 | 6843 82 | 3954 05 | 60 65 | 9 18 | |
| 8 85 | 9162 19 | 5144 65 | 8162 18 | 91684 57 | |
| 2416 45 | 1829 32 | 4217 64 | 1492 95 | 8647 64 | |
| 168 94 | 257 16 | 417 86 | 952 17 | 347 18 | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | |

**5.** Complete the following sales sheet. Add by columns and by lines and then check the work by adding the vertical and horizontal totals.

SUMMARY OF CLERKS' DAILY SALES

| NAMES OF CLERKS | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | TOTAL FOR WEEK |
|---|---:|---:|---:|---:|---:|---:|---|
| J. E. Snow | 167 18 | 194 67 | 98 46 | 241 80 | 175 66 | 314 90 | |
| W. B. Moore | 78 20 | 65 14 | 50 42 | 60 93 | 51 19 | 64 86 | |
| T. B. Welch | 112 40 | 118 64 | 192 40 | 146 18 | 110 50 | 140 12 | |
| E. H. Ross | 164 90 | 143 18 | 192 64 | 214 10 | 110 60 | 190 18 | |
| Minnie Davis | 165 19 | 214 78 | 120 42 | 167 18 | 164 27 | 140 51 | |
| Ada Benton | 68 49 | 90 81 | 64 75 | 120 14 | 142 16 | 60 90 | |
| Elmer S. Frey | 240 18 | 920 41 | 718 52 | 167 59 | 840 72 | 143 86 | |
| Joseph White | 22 49 | 72 86 | 51 47 | 62 14 | 91 26 | 72 15 | |
| Margaret Dix | 47 26 | 91 18 | 21 64 | 18 42 | 61 19 | 64 86 | |
| F. O. Beck | 127 16 | 95 27 | 114 82 | 162 15 | 102 15 | 112 61 | |
| L. O. Avery | 214 91 | 218 46 | 920 41 | 172 14 | 152 86 | 142 71 | |
| B. W. Snyder | 162 14 | 153 46 | 118 64 | 162 14 | 182 15 | 69 58 | |
| Ella Harding | 21 27 | 18 92 | 17 65 | 28 64 | 59 18 | 72 41 | |
| Carrie Simpson | 21 18 | 45 30 | 16 98 | 42 41 | 20 68 | 75 98 | |
| W. F. Baldwin | 162 10 | 114 80 | 115 90 | 116 84 | 117 41 | 200 60 | |
| E. O. Burrill | 84 90 | 90 10 | 116 80 | 114 30 | 65 20 | 300 75 | |
| Total | | | | | | | |

**6.** On the page following are a number of inventory extensions; find the footing of each.

Each column should be added in approximately three minutes.

| *a.* | *b.* | *c.* | *d.* |
|---:|---:|---:|---:|
| $1628.45 | $1743.19 | $2065.32 | $1156.78 |
| 176.22 | 78.91 | 145.55 | 11.69 |
| 453.26 | 1011.45 | 28.49 | 189.24 |
| 1102.65 | 125.60 | 217.86 | 338.54 |
| 45.22 | 101.25 | 207.41 | 24.75 |
| 143.51 | 74.24 | 1078.44 | 8.95 |
| 55.68 | 212.35 | 45.60 | 1250.05 |
| 425.70 | 338.90 | 256.85 | 74.33 |
| 119.45 | 227.83 | 194.61 | 258.97 |
| 7.15 | 46.21 | 112.45 | 364.20 |
| 89.52 | 117.25 | 34.65 | 126.70 |
| 48.75 | 57.81 | 133.27 | 82.56 |
| 170.25 | 9.11 | 24.13 | 219.63 |
| 613.81 | 764.35 | 495.34 | 360.42 |
| 1203.05 | 88.75 | 6.17 | 24.45 |
| 327.16 | 401.19 | 98.75 | 175.26 |
| 654.32 | 1145.22 | 856.21 | 1364.12 |
| 108.17 | 366.18 | 1408.95 | 374.11 |
| 75.45 | 201.13 | 376.23 | 211.06 |
| 9.58 | 64.33 | 112.50 | 21.08 |
| 142.53 | 8.78 | 56.40 | 312.74 |
| 189.70 | 89.56 | 76.50 | 118.44 |
| 20.65 | 192.35 | 244.56 | 94.65 |
| 673.13 | 98.75 | 339.54 | 951.15 |
| 120.06 | 320.16 | 87.40 | 73.07 |
| 65.34 | 135.20 | 467.80 | 239.15 |
| 753.25 | 846.18 | 515.05 | 1856.22 |
| 371.10 | 1904.76 | 287.01 | 475.02 |
| 69.16 | 128.44 | 301.04 | 602.13 |
| 231.06 | 473.08 | 1654.23 | 342.50 |
| 503.44 | 95.16 | 270.15 | 89.65 |
| 78.42 | 143.65 | 69.40 | 439.20 |
| 250.21 | 54.30 | 355.11 | 175.06 |
| 2859.89 | 1756.84 | 742.13 | 1914.56 |

### WRITTEN EXERCISE

Each of these groups should be added in approximately one minute.

| 1. | 2. | 3. |
|---|---|---|
| 6,354,276,742 | 6,917,408,583 | 8,632,714,509 |
| 5,116,433,854 | 5,880,734,112 | 6,875,352,272 |
| 8,446,750,284 | 6,538,922,553 | 8,539,168,753 |
| 2,141,991,648 | 7,543,276,445 | 8,335,674,912 |
| 4,653,752,816 | 6,441,819,263 | 5,321,841,174 |
| 3,256,851,539 | 2,574,438,911 | 2,435,627,819 |
| 5,462,175,255 | 4,156,717,549 | 1,546,721,973 |
| 6,435,621,953 | 1,817,908,667 | 5,167,534,217 |
| 4,717,880,945 | 6,745,243,517 | 4,576,819,054 |
| 5,601,523,764 | 3,546,798,212 | 4,151,762,492 |

| 4. | 5. | 6. |
|---|---|---|
| 2,334,515,637 | 5,432,317,892 | 6,453,615,809 |
| 5,734,516,772 | 6,534,317,865 | 4,576,879,253 |
| 7,354,618,227 | 4,531,841,962 | 2,432,653,274 |
| 3,542,618,906 | 3,132,341,264 | 7,819,457,836 |
| 5,115,616,874 | 4,651,272,335 | 1,918,776,425 |
| 7,175,437,256 | 3,256,728,143 | 5,327,731,224 |
| 1,735,273,540 | 1,830,925,165 | 4,532,551,243 |
| 8,134,556,773 | 2,543,175,213 | 5,231,605,228 |
| 3,115,617,221 | 3,413,214,605 | 7,615,257,818 |
| 3,255,617,711 | 5,443,216,448 | 5,663,338,119 |

| 7. | 8. | 9. |
|---|---|---|
| 4,334,561,228 | 3,653,212,716 | 4,176,234,562 |
| 1,765,371,442 | 7,661,823,394 | 5,934,736,548 |
| 9,717,632,461 | 5,749,287,716 | 6,254,817,259 |
| 5,616,727,640 | 3,264,723,445 | 1,218,735,143 |
| 2,615,431,817 | 2,845,367,621 | 7,342,235,907 |
| 4,256,713,332 | 4,256,443,486 | 6,543,217,194 |
| 3,078,395,668 | 7,254,563,823 | 2,546,718,911 |
| 7,845,446,772 | 9,117,224,383 | 3,664,293,189 |
| 8,250,114,337 | 1,493,786,332 | 9,226,847,821 |

### A WRITTEN REVIEW TEST

*Write the following problems from dictation, and then complete the work. Time, approximately, thirty minutes, including the dictation. Add the following:*

**1.**

| | |
|---|---|
| 23,418 | 17,546 |
| 43,621 | 23,811 |
| 71,892 | 32,264 |
| 22,457 | 31,542 |
| 34,256 | 45,623 |
| 41,243 | 54,819 |
| 44,172 | 22,716 |
| 34,523 | 45,613 |
| 31,113 | 41,414 |
| 20,125 | 13,064 |
| 62,157 | 93,845 |
| ? | ? |

**2.**

| | |
|---|---|
| 28,356 | 43,572 |
| 32,718 | 21,471 |
| 10,918 | 37,509 |
| 27,354 | 42,311 |
| 45,612 | 20,175 |
| 33,452 | 75,604 |
| 31,174 | 70,219 |
| 44,530 | 28,332 |
| 45,517 | 37,743 |
| 50,197 | 22,508 |
| 29,875 | 45,612 |
| ? | ? |

In problems 3 and 4 add by columns and then by lines, and check the work by adding the vertical and horizontal totals.

**3.**

| | | |
|---|---|---|
| 542 | 236 | ? |
| 437 | 653 | ? |
| 572 | 246 | ? |
| 445 | 354 | ? |
| 236 | 716 | ? |
| 423 | 394 | ? |
| 347 | 616 | ? |
| 455 | 494 | ? |
| 337 | 651 | ? |
| 453 | 283 | ? |
| 624 | 912 | ? |
| 538 | 496 | ? |
| 712 | 324 | ? |
| ? | ? | ? |

**4.**

| | | |
|---|---|---|
| 123 | 235 | ? |
| 947 | 834 | ? |
| 174 | 215 | ? |
| 313 | 208 | ? |
| 364 | 312 | ? |
| 252 | 733 | ? |
| 243 | 317 | ? |
| 203 | 406 | ? |
| 543 | 392 | ? |
| 527 | 618 | ? |
| 624 | 483 | ? |
| 713 | 626 | ? |
| 235 | 439 | ? |
| ? | ? | ? |

# CHAPTER III

## SUBTRACTION

### ORAL EXERCISE

*State the number that, added to the smaller number, makes the larger one in each of the following:*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 9 | 9 | 9 | 8 | 8 | 7 |
| | 1 | 2 | 1 | 3 | 2 | 3 | 3 | 2 | 3 | 1 | 6 | 4 | 4 | 1 | 2 |
| **2.** | 12 | 11 | 12 | 11 | 12 | 11 | 12 | 11 | 10 | 11 | 10 | 11 | 10 | 12 | 10 |
| | 9 | 2 | 3 | 9 | 8 | 3 | 4 | 8 | 4 | 7 | 6 | 4 | 7 | 5 | 3 |
| **3.** | 18 | 17 | 16 | 17 | 16 | 15 | 14 | 15 | 14 | 13 | 13 | 16 | 15 | 14 | 13 |
| | 9 | 8 | 7 | 9 | 8 | 6 | 9 | 7 | 8 | 4 | 7 | 9 | 8 | 5 | 9 |
| **4.** | 13 | 14 | 14 | 15 | 16 | 17 | 18 | 18 | 19 | 19 | 19 | 19 | 18 | 18 | 17 |
| | 11 | 12 | 11 | 13 | 12 | 13 | 13 | 12 | 13 | 11 | 16 | 14 | 14 | 11 | 12 |
| **5.** | 22 | 21 | 22 | 21 | 22 | 21 | 22 | 21 | 20 | 21 | 20 | 21 | 20 | 22 | 20 |
| | 19 | 12 | 13 | 19 | 18 | 13 | 14 | 18 | 14 | 17 | 16 | 14 | 17 | 15 | 13 |
| **6.** | 38 | 27 | 26 | 37 | 26 | 35 | 44 | 25 | 34 | 53 | 43 | 36 | 45 | 54 | 73 |
| | 29 | 18 | 17 | 29 | 18 | 26 | 39 | 17 | 28 | 44 | 37 | 29 | 38 | 45 | 69 |
| **7.** | 42 | 51 | 72 | 81 | 92 | 71 | 32 | 41 | 70 | 61 | 90 | 81 | 30 | 62 | 50 |
| | 39 | 42 | 63 | 79 | 88 | 63 | 24 | 38 | 64 | 57 | 86 | 74 | 27 | 55 | 47 |

**30.** A **parenthesis** ( ) signifies that the numbers included within it are to be considered together. A **vinculum** $\overline{\qquad}$ has the same signification as a parenthesis.

Thus, $15 - (4 + 2)$, or $15 - \overline{4 + 2}$ signifies that the sum of 4 and 2 is to be subtracted from 15.

**31. Examples.** 1. Find the difference between 849 and 162.

Solution. 2 from 9 leaves **7**. 6 cannot be subtracted from 4, but 6 from 14 leaves **8**. Since 1 of the 8 hundreds has been taken, there are but 7 hundreds remaining. 1 from 7 leaves **6**.

849
162
687

Check. 687 + 162 = 849.

The above is a common method of subtraction. For practical computation, however, the "making change" method is best. It is easily understood and is much more rapid when once learned. The "making change" method is illustrated in the following example and solution.

2. Find the difference between 7246 and 4824.

Solution. Think "4 + **2** = 6," and write 2; "2 + **2** = 4," and write 2; "8 + **4** = 12," and write 4; "1 and 4 + **2** = 7," and write 2.

7246
4824
2422

Check. 2422 + 4824 = 7246.

### ORAL EXERCISE

1. 16 + 23 + ? = 54?
2. 27 + 14 + ? = 72?
3. 17 + 36 + ? = 62?
4. 19 + 17 + 12 + ? = 57?
5. 25 + 14 + 11 + ? = 75?
6. 18 + 17 + 16 + ? = 70?
7. 16 + 18 + 16 = 25 + ?
8. 72 + 17 + 11 = 37 + ?
9. 14 + 18 + 38 = 42 + ?
10. 12 + 16 + 12 + 14 + ? = 75?
11. 16 + 15 + 19 + 15 + ? = 93?
12. 18 + 17 + 15 + 29 + ? = 98?

### WRITTEN EXERCISE

1. Without copying the individual problems, find quickly the sum of the twenty differences in the following:

| | | | | |
|---|---|---|---|---|
| $2140.50<br>714.23 | $4157.50<br>1236.80 | $5000.24<br>249.17 | $9000.72<br>1246.18 | $3145.62<br>2000.79 |
| $5500.89<br>2799.14 | $1624.14<br>957.80 | $1985.72<br>645.92 | $1379.54<br>923.18 | $1742.18<br>842.16 |
| $9275.17<br>842.99 | $2446.80<br>1321.44 | $3169.14<br>874.36 | $3156.19<br>1400.72 | $4756.83<br>2738.44 |
| $7514.85<br>721.92 | $7291.80<br>1642.95 | $1756.92<br>921.74 | $8721.13<br>2049.79 | $1872.14<br>742.12 |

**2.** Copy the following table and show (*a*) the total exports for each year given; (*b*) the excess of exports for each year given; (*c*) the total exports and imports for the ten years; (*d*) the total excess of exports for the ten years. Check.

IMPORTS AND EXPORTS IN THE UNITED STATES FOR TEN YEARS

| Year Ending June 30 | Exports | | Total Exports | Imports | Excess of Exports |
|---|---|---|---|---|---|
| | Domestic | Foreign | | | |
| 1904 | $1 435 179 017 | $25 648 254 | | $991 090 978 | |
| 1905 | 1 491 744 641 | 26 817 025 | | 1 117 513 071 | |
| 1906 | 1 717 953 382 | 25 911 118 | | 1 226 562 446 | |
| 1907 | 1 853 718 034 | 27 133 044 | | 1 434 421 425 | |
| 1908 | 1 834 786 357 | 25 986 357 | | 1 194 341 792 | |
| 1909 | 1 638 355 593 | 24 655 511 | | 1 311 920 224 | |
| 1910 | 1 710 083 998 | 34 900 722 | | 1 556 947 430 | |
| 1911 | 2 013 549 025 | 35 771 174 | | 1 527 226 105 | |
| 1912 | 2 170 319 828 | 34 002 581 | | 1 653 264 934 | |
| 1913 | 2 428 506 358 | 37 377 791 | | 1 812 978 234 | |
| Total | | | | | |

Under the term *domestic exports* are included exports of merchandise, the growth, produce, or manufacture of the United States; under *foreign exports* are included articles of merchandise previously imported into the United States and subsequently reëxported. Under the term *imports* are included imports of all merchandise of whatever origin received into the United States.

**32.** The common method of **making change** is to add to the price of the goods purchased a sum that will equal the amount offered in payment.

Thus, if a person buys groceries amounting to 74¢ and tenders $1 in payment, the mental process of the clerk in making the change is as follows: "74¢ + 1¢ + 25¢ = $1"; the customer should receive as change a 1-cent piece and a quarter of a dollar.

Change may be made in a number of ways. In the above example two dimes and a 5-cent piece might be given instead of the quarter of a dollar. In the following exercise name the largest coins and bills that could be used.

### ORAL EXERCISE

**1.** Name the coins and the amount of change to be given from $1 for each of the following purchases: 17¢; 24¢; 31¢; 38¢; 45¢; 52¢; 59¢; 66¢; 73¢; 80¢; 87¢; 18¢; 25¢.

**2.** Name the coins and the amount of change to be given from $2 for each of the following purchases: $1.19; $1.26; $1.33; $1.40; $1.47; $1.54; $1.61; $1.68; $1.75; $1.82; $1.89; $1.20; $1.27; $1.34; $1.41; $1.48; $1.55; $1.62; $1.69; $1.76; $1.83; $1.90.

**3.** Name the bills and coins and the amount of change to be given from $5 for each of the following purchases: $1.21; $1.28; $1.35; $1.42; $2.22; $2.29; $2.36; $4.43; $3.49; $4.50; $3.51; $3.56; $4.57; $2.58; $1.63; $2.64; $1.65; $1.70; $2.71; $3.72; $2.77; $3.84; $1.91; $2.85; $2.92.

**4.** Name the bills and coins and the amount of change to be given from $10 for each of the following purchases: $4.93; $3.86; $7.70; $2.44; $8.37; $5.30; $3.23; $5.17; $4.24; $3.31; $8.38; $2.45; $6.52; $4.59; $3.66; $5.73; $4.80; $3.87; $2.88; $7.81; $9.74; $5.67; $3.60; $4.53; $2.46; $3.29; $8.32; $7.25; $2.18; $7.49; $9.42; $3.67; $1.93.

**33.** It is frequently necessary to find the difference between a minuend and several subtrahends. If the "making change" method of subtraction is employed, the operation is a simple one.

**34. Example.** From a farm of 578 A. I sold at one time 162 A., at another 98 A., and at another 121 A. How many acres remained unsold?

SOLUTION. Arrange the numbers as shown in the margin. Eleven (1 + 8 + 2) and **seven** are 18; write 7. Three (1 carried + 2), eighteen (3 + 9 + 6) and **nine** are twenty-seven; write 9. Four (2 carried + 1 + 1) and **one** are 5; write 1.

CHECK. 197 + 121 + 98 + 162 = 578.

$$\begin{array}{r} \underline{578\text{ A.}} \\ 162\text{ A.} \\ 98 \\ \underline{121\phantom{\text{ A.}}} \\ 197\text{ A.} \end{array}$$

### WRITTEN EXERCISE

*Find the amount each person has remaining on deposit:*

**1.** A. Deposit, $900; checks, $210, $175, $198.

**2.** B. Deposit, $875; checks, $157, $218, $157.

**3.** C. Deposit, $750; checks, $120, $117, $121, $118.

**4.** D. Deposit, $960; checks, $128, $109, $118, $117.

**5.** E. Deposit, $967; checks, $192, $102, $117, $128, $146.

**6.** F. Deposit, $998; checks, $119, $117, $105, $123, $173.

Do not neglect to check all work. The bank clerk who makes an error a day in work like the above, and who fails to discover and correct this error, will not long retain his position.

**7.** Copy the following, supplying the missing terms and checking the results:

$148.90 + $149.75 + $421.77 = $???.??
118.60 + 172.12 + ???.?? = ???.??
242.30 + ???.?? + 210.96 = ???.??
???.?? + 168.72 + 130.41 = ???.??
$718.95 + $698.75 + $978.00 = $????.??

The following problem shows a portion of a bank discount register. In the first column are recorded the amounts of several notes that have been discounted; in the second, the discount charges; and in the third, the collection and exchange charges. The proceeds of any note is the difference between the amount (face) of the note and the total charges upon it.

**8.** Copy and complete the following bank record. Check the work. ($j + i + h$ should equal $g$.)

| FACE OF PAPER | | DISCOUNT | | COLL. & EXCH. | | PROCEEDS | |
|---|---|---|---|---|---|---|---|
| 729 | 14 | 7 | 29 | | 73 | *a* | |
| 862 | 29 | 4 | 31 | | 86 | *b* | |
| 725 | 74 | 7 | 26 | | 73 | *c* | |
| 832 | 16 | 12 | 48 | 1 | 26 | *d* | |
| 426 | 19 | 6 | 39 | | 43 | *e* | |
| 378 | 36 | 3 | 78 | | 38 | *f* | |
| *g* | | *h* | | *i* | | *j* | |

**35.** The **complement** of a number is the difference between the number and a unit of the next higher order.

Thus, 2 is the complement of 8, 23 is the complement of 77, and 152 is the complement of 848. 3 and 7, 24 and 76, 250 and 750, are complementary numbers. Observe that *when two numbers of more than one figure each are complementary, the sum of the units' figure is 10 and the sum of the figures in each corresponding higher order is 9.*

**36.** Since numbers are read from left to right, in finding the complement of a number, begin at the left to subtract.

**37.** In beginning at the left to subtract take 1 from the highest order in the minuend and regard the other orders as 9's, except the last, which regard as 10.

**38. Example.** A man gave a 100-dollar bill in payment for an account of \$77.52. How much change should he receive?

SOLUTIONS. (*a*) Begin at the left. 7 from 9 leaves **2**; 7 from 9 leaves **2**; 5 from 9 leaves **4**; 2 from 10 leaves **8**. Or

(*b*) 7 and **2** are 9; 7 and **2** are 9; 5 and **4** are 9; 2 and **8** are 10. \$22.48.

$$\begin{array}{r} \$100.00 \\ 77.52 \\ \hline \$22.48 \end{array}$$

This method of finding the *amount of change* is used by many clerks and cashiers. The work is in all cases proved by counting out to the customer the bills and coins necessary to make the amount of the purchase equal to the amount offered in payment.

### ORAL EXERCISE

*Find the total of each group of numbers, then find the difference between the two totals:*

**1.** (24 26 32 30 35 25) – (18 13 19 12 20 30).
**2.** (13 27 45 25 21 19) – (15 14 21 32 18 22).
**3.** (11 29 35 15 24 16) – (27 13 18 22 25 20).
**4.** (17 14 29 32 22 26) – (23 13 16 24 20 16).
**5.** (45 25 30 40 32 18) – (25 35 33 17 20 30).
**6.** (34 24 35 30 15 32) – (21 39 14 15 11 30).
**7.** (15 25 33 27 14 36) – (13 30 16 14 20 16).
**8.** (14 16 30 10 40 50) – (11 19 18 12 20 30).
**9.** (19 10 11 20 30 32) – (15 11 14 30 32 18).
**10.** (33 17 22 11 17 50) – (21 19 31 12 17 40).
**11.** (25 30 15 40 15 20) – (15 16 19 21 29 30).
**12.** (17 23 25 26 15 44) – (24 20 26 27 13 20).
**13.** (11 39 52 18 10 20) – (12 18 40 22 28 12).
**14.** (35 15 27 23 34 16) – (21 17 12 42 13 15).
**15.** (22 18 34 26 60 10) – (35 15 20 11 19 31).
**16.** (33 17 22 18 40 60) – (14 26 23 17 40 12).

### ORAL EXERCISE

*State the amount of change in each of the following problems:*

| | Cost of Items Purchased | Amount Paid | | Cost of Items Purchased | Amount Paid |
|---|---|---|---|---|---|
| **1.** | 17¢, 13¢, 42¢ | $2 | **14.** | $1.25, $0.75, $2.18 | $20 |
| **2.** | 27¢, 23¢, 14¢ | $2 | **15.** | $1.50, $2.70, $1.18 | $20 |
| **3.** | 45¢, 55¢, 13¢ | $5 | **16.** | $4.60, $1.40, $2.13 | $20 |
| **4.** | 64¢, 16¢, 87¢ | $5 | **17.** | $1.50, $1.20, $2.30 | $10 |
| **5.** | 23¢, 14¢, 27¢ | $2 | **18.** | $3.17, $4.11, $4.98 | $50 |
| **6.** | 63¢, 17¢, 59¢ | $5 | **19.** | $4.25, $0.75, $3.18 | $20 |
| **7.** | 49¢, 84¢, 37¢ | $5 | **20.** | $1.29, $2.17, $1.50 | $20 |
| **8.** | 78¢, 42¢, 67¢ | $5 | **21.** | $1.64, $1.66, $2.50 | $20 |
| **9.** | 52¢, 69¢, 88¢ | $5 | **22.** | $1.59, $23.41, $118 | $200 |
| **10.** | 75¢, 86¢, 54¢ | $5 | **23.** | $24.17, $20.83, $15 | $100 |
| **11.** | 89¢, 46¢, 72¢ | $5 | **24.** | $11.48, $10.52, $50 | $100 |
| **12.** | 76¢, 54¢, 29¢ | $5 | **25.** | $18.91, $12.09, $45 | $100 |
| **13.** | 75¢, 25¢, 89¢ | $10 | **26.** | $21.27, $2.73, $50.50 | $100 |

**39.** $19 - 7 = 9$ (the minuend minus 10) $+ 3$ (the complement of the subtrahend); $191 - 17 = 91$ (the minuend minus 100) $+ 83$ (the complement of the subtrahend); $1912 - 178 = 912$ (the minuend minus 1000) $+ 822$ (the complement of the subtrahend), and so on.

**40.** This principle makes it a simple matter to find the difference between a subtrahend and several minuends.

**41. Examples.** The following examples illustrate the application of the principle:

| **1.** | **2.** | **3.** |
|---|---|---|
| 316 | 299 | 311 |
| +478 | +488 | +111 |
| −118 | −111 | −219 |
| =676 | =676 | =203 |

Solutions. **1.** 2 (the complement of 8), 10, 16; 16 − 10 = **6.** 9 (the complement of 1), 16, 17; 17 − 10 = **7.** 9, 13, 16; 16 − 10 = **6.**

**2.** 9, 17, 26; 26 − 10 = 16; that is, **6** and 1 to add to the minuends. 9, 18 (9 + 8 + 1), 27; 27 − 10 = 17; that is, **7** and 1 to add to the minuends. 9, 14, 16; 16 − 10 = **6.**

**3.** 1, 2, **3.** 3 − 10 is impossible, so subtract 1 ten from the minuend (or add 1 ten to the subtrahend). 9, 10. 10 − 10 = **0.** 8, 9, 12. 12 − 10 = **2.**

**42. Example.** The following problem shows a concrete application of the foregoing principle:

DEPOSITORS' LEDGER

| DEPOSITOR | BALANCE | CHECKS | DEPOSITS | BALANCE |
|---|---|---|---|---|
| A | $ 74 | $ 25 | $ 86 | $ 135 |
| B | $ 86 | $ 11 | $ 99 | $ 174 |
| C | $ 92 | $ 79 | $ 81 | $ 94 |

SOLUTION. Here is a depositors' ledger. The data in the first three columns being given, it is required to find the new balance.

The process is as follows: A. 6, 11, 15, **5**; 8, 16, 23, **13**; balance, $135. B. 9, 18, 24, **4** and 1 to add to the minuend. 10, 19, 27, **17**; balance, $174. C. 1, 2, **4** and 1 to take away from the minuend. 7, 10, 19, **9**; balance, $94.

### WRITTEN EXERCISE

*Find the new balances, the total old balance, the total checks, the total deposits, the total new balances, and check the work:*

**1.**

| DEPOSITOR | BAL. | CHECKS | DEPOSITS | BAL. |
|---|---|---|---|---|
| A | $ 758 | $ 128 | $ 421 | *a* |
| B | 921 | 154 | 175 | *b* |
| C | 934 | 214 | 122 | *c* |
| D | 862 | 162 | 218 | *d* |
| E | 478 | 187 | 126 | *e* |
| F | 921 | 215 | 124 | *f* |
| G | 756 | 157 | 137 | *g* |
| H | 864 | 128 | 142 | *h* |
| I | 926 | 214 | 121 | *i* |
| J | 752 | 221 | 124 | *j* |
| K | 878 | 162 | 218 | *k* |
|  | *l* | *m* | *n* | *o* |

**2.**

| DEPOSITOR | BAL. | CHECKS | DEPOSITS | BAL. |
|---|---|---|---|---|
| A | $ 428 | $ 125 | $ 718 | *a* |
| B | 726 | 128 | 296 | *b* |
| C | 832 | 279 | 318 | *c* |
| D | 456 | 154 | 421 | *d* |
| E | 298 | 275 | 568 | *e* |
| F | 728 | 178 | 188 | *f* |
| G | 762 | 218 | 215 | *g* |
| H | 837 | 316 | 176 | *h* |
| I | 493 | 121 | 219 | *i* |
| J | 862 | 128 | 188 | *j* |
| K | 925 | 125 | 211 | *k* |
|  | *l* | *m* | *n* | *o* |

**43.** $48 - 29 = \overline{48 + 1}$ (30, the next higher order of units than 29, $-$ 29) $-$ 30, or 19; $128 - 59 = \overline{128 + 1} - 60$, or 69.

**44.** This principle may be applied to advantage in billing items in which the gross weights and the tares are recorded.

The **gross weight** is the weight of merchandise, together with bag, cask, or other covering; the **tare** is the weight of the bag, cask, or other covering of merchandise; the **net weight** is the difference between the gross weight and the tare.

**45. Example.** The gross weights and tares, in pounds, of 3 bbl. of sugar are: 332 – 19, 337 – 18, 335 – 18. Find the total net weight.

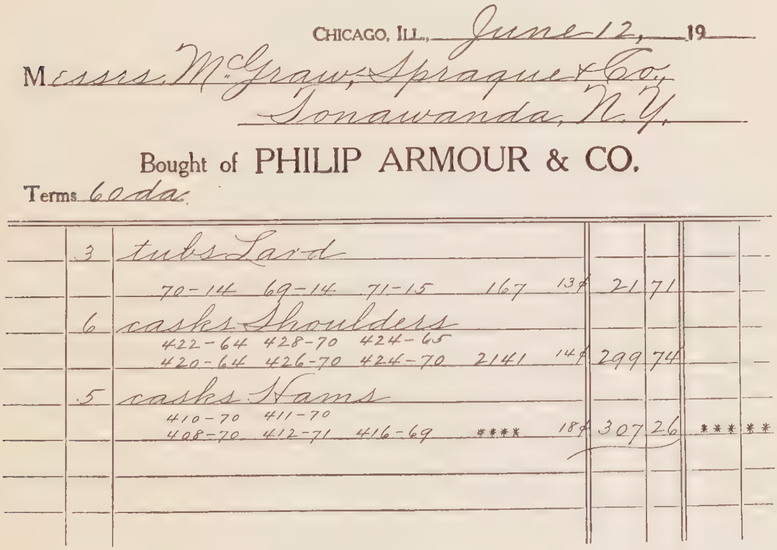
Solution. The numbers would be written on the bill horizontally, as shown in the margin. Adding the units of the tare, the result is 25; 30 (the next higher order of units than 25) minus 25 equals 5; 5 added to the units of the gross weight equals 19; 19 – 30 is impossible, so write **9** and subtract 2 tens (the difference between the tens in 30 and 19) from the gross weight or add 2 tens to the tens of the tare. Adding **2** tens to the tens of the tare, the result is 5; 10 – 5 = 5; 5 added to the tens of the gross weight equals 14; 14 – 10 = **4**. Adding the hundreds in the gross weight, the result is **9**. Net weight is 949 lb.

332 – 19 337 – 18 335 – 18 949 #

### WRITTEN EXERCISE

*Copy the following bills. Verify the net weights given and supply all missing terms.*

**1.**



CHICAGO, ILL., June 12, 19

Messrs. McGraw, Sprague & Co.,
Tonawanda, N. Y.

Bought of PHILIP ARMOUR & CO.

Terms 60 da.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 3 | tubs Lard | | | | |
| | | 70-14 69-14 71-15 167 13¢ | 21 | 71 | | |
| | 6 | casks Shoulders | | | | |
| | | 422-64 428-70 424-65<br>420-64 426-70 424-70 2141 14¢ | 299 | 74 | | |
| | 5 | casks Hams | | | | |
| | | 410-70 411-70<br>408-70 412-71 416-69 **** 18¢ | 307 | 26 | *** | ** |
| | | | | | | |
| | | | | | | |

2.

Chicago, Ill., July 20, 19

Messrs. A. M. THOMPSON & CO.
Rochester, N.Y.

Bought of **Nelson, Morris & Co.**

Terms 30 da.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | tubs Lard | | | | | | |
| | 72-17 70-14 | | | | | | |
| | 69-14 71-14 | | | | | | |
| | 71-15 70-16 | *** | $0.13 | 43 | 29 | | |
| 6 | casks Shoulders | | | | | | |
| | 421-65 426-70 | | | | | | |
| | 424-72 422-64 | | | | | | |
| | 427-72 421-60 | **** | .14 | 299 | 32 | | |
| 6 | casks Hams | | | | | | |
| | 409-72 412-70 | | | | | | |
| | 414-71 410-73 | | | | | | |
| | 412-70 416-71 | **** | .18 | 368 | 28 | *** | ** |

3. The gross weights and tares of 6 casks of shoulders are as follows: 428 – 68, 419 – 70, 423 – 65, 432 – 72, 436 – 69, 434 – 65 lb. Find the total net weight.

4. The gross weight and tares of 12 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 71 – 15, 72 – 17, 73 – 17, 69 – 15, 71 – 16, 72 – 15, 73 – 16, 74 – 17, 75 – 17 lb. Find the total net weight.

5. The gross weights and tares of 10 bbl. of sugar are as follows: 319 – 18, 331 – 19, 329 – 17, 334 – 20, 338 – 21, 325 – 18, 326 – 16, 325 – 19, 327 – 19, 321 – 17 lb. Find the total net weight.

### WRITTEN EXERCISE

*Copy each of the following examples; complete the work, and check the result. Time, approximately, thirty minutes. Face of paper, minus discount and collection and exchange, equals proceeds.*

| Face of Paper | Discount | Coll. and Exch. | Proceeds |
|---|---|---|---|
| 1. $376.25 | $3.76 | $0.38 | ? |
| 255.78 | 1.28 | 0.26 | ? |
| 176.44 | 1.76 | —— | ? |
| 259.86 | 2.60 | 0.26 | ? |
| 492.71 | 2.46 | 0.49 | ? |
| 149.52 | 0.75 | 0.15 | ? |
| 643.15 | 6.43 | 0.64 | ? |
| 319.21 | 0.80 | —— | ? |
| ? | ? | ? | ? |

| Face of Paper | Discount | Coll. and Exch. | Proceeds |
|---|---|---|---|
| 2. $186.73 | $1.87 | $0.19 | ? |
| 237.50 | 2.38 | 0.24 | ? |
| 412.88 | 4.13 | —— | ? |
| 337.95 | 3.38 | 0.34 | ? |
| 360.90 | 0.90 | —— | ? |
| 245.91 | 1.23 | 0.25 | ? |
| 307.55 | 3.08 | 0.31 | ? |
| 250.40 | 3.76 | 0.25 | ? |
| ? | ? | ? | ? |

| Face of Paper | Discount | Coll. and Exch. | Proceeds |
|---|---|---|---|
| 3. $421.95 | $2.11 | $0.42 | ? |
| 112.70 | 1.13 | 0.11 | ? |
| 324.50 | 1.62 | —— | ? |
| 245.60 | 2.46 | 0.25 | ? |
| 194.75 | 0.92 | 0.19 | ? |
| 217.50 | 1.09 | 0.22 | ? |
| 311.59 | 3.12 | 0.31 | ? |
| 289.72 | 1.45 | 0.29 | ? |
| ? | ? | ? | ? |

### WRITTEN EXERCISE

*Copy each of the following examples; find the new balance, the total old balance, the total checks, the total deposits, and check the work:*

**1.**

| Name | Bal. | Checks | Deposits | Bal. |
|---|---|---|---|---|
| A | $516 | $423 | $313 | ? |
| B | 203 | 507 | 398 | ? |
| C | 195 | 461 | 412 | ? |
| D | 204 | 165 | — | ? |
| E | 335 | 515 | 296 | ? |
| F | 411 | 309 | 156 | ? |
| G | 135 | 257 | 145 | ? |
| H | — | 295 | 512 | ? |
| I | 500 | 316 | 104 | ? |
| J | 450 | 225 | 117 | ? |
| K | 650 | 751 | 211 | ? |
| L | 512 | 337 | 206 | ? |
| M | 242 | 176 | 105 | ? |
|  | ? | ? | ? | ? |

**2.**

| Name | Bal. | Checks | Deposits | Bal. |
|---|---|---|---|---|
| A | $309 | $423 | $267 | ? |
| B | 476 | 379 | 112 | ? |
| C | 277 | 407 | 321 | ? |
| D | 255 | — | 126 | ? |
| E | 167 | 217 | 178 | ? |
| F | 213 | 453 | 329 | ? |
| G | 208 | 196 | 114 | ? |
| H | 126 | 240 | 114 | ? |
| I | 255 | 153 | — | ? |
| J | — | 212 | 317 | ? |
| K | 455 | 235 | 103 | ? |
| L | 275 | 230 | 115 | ? |
| M | 315 | 275 | 144 | ? |
|  | ? | ? | ? | ? |

**3.**

| Name | Bal. | Checks | Deposits | Bal. |
|---|---|---|---|---|
| A | $311 | $242 | $301 | ? |
| B | 235 | 115 | 118 | ? |
| C | 178 | — | 212 | ? |
| D | 142 | 188 | 206 | ? |
| E | — | 268 | 315 | ? |
| F | 447 | 397 | 109 | ? |
| G | 214 | 375 | 226 | ? |
| H | 154 | 106 | — | ? |
| I | 256 | 400 | 144 | ? |
| J | 512 | 613 | 199 | ? |
| K | 245 | 237 | 155 | ? |
| L | 146 | — | 114 | ? |
| M | 565 | 743 | 248 | ? |
|  | ? | ? | ? | ? |

**4.**

| Name | Bal. | Checks | Deposits | Bal. |
|---|---|---|---|---|
| A | $267 | $133 | $145 | ? |
| B | — | 247 | 315 | ? |
| C | 256 | 119 | 228 | ? |
| D | 116 | — | 177 | ? |
| E | 215 | 411 | 196 | ? |
| F | 423 | 375 | 116 | ? |
| G | 198 | 213 | 132 | ? |
| H | 245 | 157 | 109 | ? |
| I | 233 | 334 | 168 | ? |
| J | 443 | 337 | 155 | ? |
| K | 704 | 856 | 266 | ? |
| L | 231 | 250 | 119 | ? |
| M | 178 | 252 | 107 | ? |
|  | ? | ? | ? | ? |

### A WRITTEN REVIEW TEST

*Write problems 1 and 2 from dictation, and then complete the work. Time, approximately, forty minutes, including the dictation.*

**1.** Add the following and check the results:

| | | | |
|---|---|---|---|
| 3412 | 4571 | 3582 | ? |
| 6243 | 8132 | 2738 | ? |
| 1729 | 2642 | 9810 | ? |
| 4572 | 7235 | 1345 | ? |
| 2643 | 6254 | 4165 | ? |
| 3414 | 8149 | 3542 | ? |
| 7145 | 6127 | 4713 | ? |
| 3328 | 2170 | 2312 | ? |
| 5331 | 4262 | 1745 | ? |
| 3473 | 1208 | 8052 | ? |
| ? | ? | ? | ? |

**2.** Find the new balance, the total old balance, the total checks, the total deposits, and check the work:

| NAME | BALANCE | CHECKS | DEPOSITS | BALANCE |
|---|---|---|---|---|
| A | $235 | $142 | $217 | ? |
| B | 312 | 254 | 122 | ? |
| C | 288 | 300 | 212 | ? |
| D | 542 | 250 | 106 | ? |
| E | — | 350 | 500 | ? |
| F | 600 | 325 | — | ? |
| G | 750 | — | 252 | ? |
| H | 219 | 200 | 151 | ? |
| I | 224 | 100 | 216 | ? |
| J | 116 | 330 | 214 | ? |
| K | 255 | 225 | 110 | ? |
| ? | ? | ? | ? | ? |

**3.** The text, page 14, problem 4.

**4.** The text, page 14, problem 5.

**5.** The text, page 14, problem 6.

# CHAPTER IV

## MULTIPLICATION

### ORAL EXERCISE

**1.** Which of the following numbers are **concrete**; that is, refer to some particular kind of object or measure? 12; $5\frac{1}{2}$; 12 ft.; 2.5 da.; 15 yd.; 18 men; 200; $12; $172\frac{4}{5}$.

**2.** Which of the above numbers are **abstract**; that is, do not refer to any particular kind of object or measure?

**3.** $5+4+2+8+9=?$

**4.** $9+9+9+9+9=?$ 5 times $9=?$

**5.** Could the sum of the numbers in problem 3 be found by any shorter process?

**6.** What is the first process in problem 4 called? the second?

**7.** 9 times $27=?$ 9 times 29 bu. $=?$

**8.** If 1 bu. of rye weighs 56 lb., what will 12 bu. weigh?

**46.** In problems 7 and 8 it is seen that *the multiplier is always an abstract number;* and *the multiplicand and product are like numbers.*

**47.** Three 5's are equal to five 3's; $3 multiplied by 5 is equal to $5 multiplied by 3; 4 trees multiplied by 125 is equal to 125 trees multiplied by 4.

**48.** It is therefore seen that *the product is not affected by changing the order of the factors regarded as abstract numbers.*

**49.** The multiplicand and multiplier together are called **factors** (makers) of the product; the product of two abstract integers is sometimes called a **multiple** of either of the factors.

**50.** Sometimes a number is used several times as a factor. Numbers so used are indicated by a small figure, called an **exponent**, written above and at the right of the factor.

Thus, 4 used *twice* as a factor is written $4^2$, 5 used *four times* as a factor is written $5^4$, and 6 used *five times* as a factor is written $6^5$.

**51.** The product arising from using a number two or more times as a factor is called a **power** of that number.

Thus, 4 is the second power of 2; 64 is the third power of 4 and the sixth power of 2.

Too much attention should not be given to the definitions like the above. They are valuable only as they help to make clear the matter in the exercises. They are rarely heard in business and therefore should not be memorized.

### ORAL EXERCISE

**1.** Multiply at sight each number below by 2; by 3; by 4; by 5; by 6; by 7; by 8; by 9.

Name the products by lines from left to right and from right to left; also by columns from left to right and from right to left. Name results only. Thus, to multiply lines by 4 say 20, 36, 8, 24, 40, 12, 28, 44, 16, 48, 32, 52, 68, 84, and so on up to 100; and backwards, 100, 80, 96, 64, and so on back to 20. To multiply columns by 4 say 20, 68, 36, 84, and so on to 52, 100; and backwards 100, 52, 80, 32, and so on to 68, 20. Continue the work until results can be named at the rate of 120 or more per minute.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 9 | 2 | 6 | 10 | 3 | 7 | 11 | 4 | 12 | 8 | 13 |
| 17 | 21 | 14 | 18 | 22 | 15 | 19 | 23 | 16 | 24 | 20 | 25 |

**2.** Multiply as instructed in problem 1 and add 8 (carried) to each product. Also multiply as instructed and add 6, 4, 7, 2, 5, 3, and 9 to each product.

Name results only. Thus, to multiply by lines say 20, 28; 36, 44; 8, 16; and so on.

**3.** Multiply by 2: 27, 35, 81, 36, 28, 32, 47, 93, 56, 39, 54, 45, 52, 86, 75, 67, 59. Also by 4, 3, 5, 8, 6, 7, 9.

**4.** Find the cost of each of the following: 20 lb. crackers at 8¢; 9 lb. coffee at 34¢; 7 lb. tea at 57¢; 11 lb. beef at 17¢; 120 lb. sugar at 4¢; 134 lb. sugar at 5¢.

**5.** Find the cost of each of the following: 44 yd. at 9¢; 37 yd. at 8¢; 123 yd. at 6¢; 214 yd. at 4¢; 52 yd. at 12¢; 29 yd. at 8¢; 8 yd. at $1.03; 7 yd. at $1.01; 5 yd. at $1.35.

**6.** Beginning at 0 count by 9's to 81; by 10's to 150; by 11's to 154; by 12's to 108; by 13's to 117; by 14's to 126; by 15's to 135; by 16's to 144; by 17's to 153; by 18's to 162; by 19's to 171; by 20's to 180.

**52. Examples.** 1. Find the cost of 2150 lb. at 5¢.

Solution. Since 1 lb. costs 5¢, 2150 lb. will cost 2150 times 5¢; but 2150 times 5¢ is equal to 5 times 2150¢. 5 times $21.50 (2150¢) equals $107.50, the required result.

$$\begin{array}{r} \$21.50 \\ 5 \\ \hline \$107.50 \end{array}$$

2. Multiply 224 by 46.

Solution. In multiplying one number by another, there is no practical advantage in beginning with the lowest order of units of the multiplier; in fact, in some multiplications there is a decided advantage in beginning with the highest order. The arrangement of work for both methods is shown in the margin.

$$\begin{array}{r} 224 \\ 46 \\ \hline 1344 \\ 896\phantom{0} \\ \hline 10304 \end{array} \qquad \begin{array}{r} 224 \\ 46 \\ \hline 896\phantom{0} \\ 1344 \\ \hline 10304 \end{array}$$

Check. The work may be checked by multiplying first by one method and then by the other, or by interchanging the multiplier and multiplicand and remultiplying. (See also pages 73 and 74.)

3. Multiply 2004 by 1275.

Solution. When one of two numbers to be multiplied contains a number of zeros or ones, it is always easier to take that number as the multiplier. Since the product of any number multiplied by 0 is 0, the product of 1275 multiplied by the tens and hundreds of the multiplier need not be written.

$$\begin{array}{r} 1275 \\ 2004 \\ \hline 5100 \\ 2550\phantom{000} \\ \hline 2555100 \end{array} \qquad \begin{array}{r} 1275 \\ 2004 \\ \hline 2550\phantom{000} \\ 5100 \\ \hline 2555100 \end{array}$$

Check. The problem may be checked the same as problem 2.

When two numbers are to be multiplied, it is generally easier to take as the multiplier the number having the least number of places. Thus, to find the cost of 1647 A. of land at $27 per acre, take 27 as the multiplier.

If one of the two numbers to be multiplied has two or more digits alike, it is easier to take that number as the multiplier. Thus, to multiply together 6729 and 7777, it is easier to take 7777 as the multiplier.

### ORAL EXERCISE

1. Find the value of 51 T. of hay at $17 per ton.
2. Find the cost of 175 lb. of sugar at 5¢ per pound.
3. How much will a boy earn in 87 hr. at 9¢ an hour?
4. What is the cost of a flock of 52 sheep at $7 per head?
5. At the rate of 47 mi. an hour, how far will a person travel in 12 hr.?
6. What is the cost of 12 pr. of shoes at $4.50 per pair, and 8 pr. of boots at $3.50 per pair?

**7.** What must be paid for handling 12 loads of freight at \$2.25 per load?

**8.** In an orchard there are 13 rows of trees, each containing 21 trees. How many trees in the orchard?

**9.** If you buy 5 pencils at 9¢ each and 9 penholders at 5¢ each, and some stationery costing 25¢, how much change should you receive from a two-dollar bill? from a ten-dollar bill?

**10.** I bought 6 cd. of wood at \$5.75 per cord. If a fifty-dollar bill is offered in payment, how much change should be received?

**11.** I bought 12 bu. of wheat at \$1.05. If I gave in payment two ten-dollar bills, what change should I receive?

**12.** My average marketing expenses per day are \$2.10. If I offer a twenty-dollar bill in payment for 7 days' expenses, what change should I receive?

**13.** I sold 16 chairs at \$7 each, and 5 tables at \$9 each. If two one-hundred-dollar bills are offered in payment, how much change should I return? If a one-hundred-dollar bill, a fifty-dollar bill, and a twenty-dollar bill are offered in payment, how much change should I return?

### WRITTEN EXERCISE

*In the following problems find the missing numbers by multiplying across and adding down. Check the results by comparing the sum of the line products with the sum of the multiplicands multiplied by one of the multipliers.*

| 1. | 2. | 3. |
|---|---|---|
| 15 × 211 = ? | 9 × 1475 = ? | 12 × \$16.50 = ? |
| 15 × 346 = ? | 9 × 2618 = ? | 12 × \$27.75 = ? |
| 15 × 318 = ? | 9 × 1575 = ? | 12 × \$14.95 = ? |
| 15 × 721 = ? | 9 × 1792 = ? | 12 × \$29.86 = ? |
| 15 × 936 = ? | 9 × 4936 = ? | 12 × \$49.88 = ? |
| 15 × 849 = ? | 9 × 7289 = ? | 12 × \$39.62 = ? |
| 15 × 218 = ? | 9 × 8728 = ? | 12 × \$86.99 = ? |
| 15 × ? = ? | 9 × ? = ? | 12 × ? = ? |

| **4.** | **5.** | **6.** |
|---|---|---|
| 12 × 192 = ? | 98 × 2178 = ? | 16 × $18.10 = ? |
| 12 × 721 = ? | 98 × 1692 = ? | 16 × 17.20 = ? |
| 12 × 836 = ? | 98 × 2536 = ? | 16 × 21.40 = ? |
| 12 × 456 = ? | 98 × 2892 = ? | 16 × 25.85 = ? |
| 12 × ? = ? | 98 × ? = ? | 16 × ? = ? |

Problems such as the above are very helpful. They afford a variety of work and suggest a simple method by which the student may test the correctness of his results. The instructor should add as many more problems as circumstances require.

**7.** A produce dealer bought 2145 bu. of potatoes at 83¢ a bushel, and sold them at $1.05 a bushel. What did he gain?

**8.** A drover bought 125 head of cattle at $15.75 per head. He sold 65 head at $23.40, 15 head at $13.75, and 45 head at $17.75. Did he gain or lose, and how much?

**9.** A grocer bought 14 bu. of apples at $1.35 per bushel and 12 bu. of potatoes at 84¢ per bushel. He sold the apples at 40¢ a peck and the potatoes at 25¢ a peck. What did he gain?

**10.** A bought 1247 bbl. of apples at $3.10 per barrel. After holding them for three months he sold them at $4.80 per barrel. If he paid $74.82 for storage, and his loss by decay was 37 bbl. of apples, what was his gain?

**11.** The gross weight in pounds, and tare in pounds, of 25 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 72 – 16, 71 – 14, 72 – 15, 70 – 15, 69 – 14, 71 – 15, 70 – 15, 69 – 14, 71 – 16, 71 – 15, 71 – 14, 70 – 15, 68 – 14, 73 – 16, 73 – 15, 70 – 14, 70 – 14, 71 – 15, 73 – 16, 74 – 18, 71 – 13, 73 – 16. Find the cost at 13¢ per pound.

**12.** The gross weight in pounds, and the tare in pounds, of 25 casks of hams are as follows: 400 – 78, 420 – 68, 420 – 71, 403 – 71, 409 – 71, 418 – 68, 412 – 72, 407 – 67, 423 – 69, 419 – 67, 426 – 68, 403 – 70, 399 – 69, 400 – 69, 425 – 71, 413 – 72, 399 – 67, 412 – 72, 418 – 68, 409 – 71, 408 – 70, 412 – 68, 402 – 71, 421 – 71, 403 – 71. Find the cost at 18¢ per pound.

## SHORT METHODS IN MULTIPLICATION

**53.** There are many short methods in multiplication, but of these only a few are practical, either because they apply generally to problems that in themselves are not practical or because they have been supplanted by the elaborate use of tables and mechanical devices. A great many practical and helpful tables are in use for figuring pay rolls, interest, discount, and the like. These tables are great time savers.

**54.** The machines that are used for adding, subtracting, multiplying, dividing, and for setting forth results in interest and discount are now in such common use that a chapter is devoted to their consideration in Appendix A at the close of this volume. These machines are found in business offices, especially where extensive operations are to be performed. Both in accuracy and in the saving of time they are most valuable.

**55.** The short methods given herewith have a wide application. They are not dependent upon formal rules, and are suggestive of other ways in which the student may exercise his own ingenuity to shorten his work in multiplication.

### Multiplication by Powers and Multiples of Ten

#### ORAL EXERCISE

**1.** 40 is how many times 4? 60 is how many times 6? 100 is how many times 10? 150 is how many times 15?

**2.** Give a short method for multiplying an integer by 10.

**3.** 400 is how many times 4? 600 is how many times 6? 1000 is how many times 10? 1500 is how many times 15?

**4.** Give a short method for multiplying an integer by 100; by 1000; by 10000.

**5.** How does the product of $40 \times 66$ compare with the product of $4 \times 66 \times 10$? the product of $400 \times 59$ with the product of $4 \times 59 \times 100$?

**6.** Give a short method for multiplying an integer by any number of 10's, 100's, or 1000's.

**7.** Multiply 270 by 300.

Solution. In the accompanying illustration it will be seen that $270 \times 300 = 27 \times 3 \times 1000$ or 81,000.

$$\begin{array}{rcl} 270 & = & 27 \times 10 \\ \underline{300} & = & \underline{3} \times \underline{100} \\ 81000 & = & 81 \times 1000 \end{array}$$

**8.** Formulate a rule for finding the product when there are zeros on the right of both factors.

**9.** $7 is how many times $0.70? $90 is how many times $0.90? $500 is how many times $0.50?

**10.** State a short method for multiplying United States money by 10; by 100; by 1000.

**11.** Read aloud the following, supplying the missing words:

(*a*) Annexing a cipher to an integer multiplies the integer by ——; annexing two ciphers to an integer —— the integer by ——.

(*b*) Removing the decimal point in United States money one place to the right —— the number by 10; removing the decimal point two places to the right —— the number by ——.

**12.** Multiply $14.70 by 10; by 100; by 1000.

**56.** In the above exercise it is clear that

*Annexing a cipher to an integer multiplies the integer by 10;* and

*Removing the decimal point one place to the right multiplies the number by 10.*

### ORAL EXERCISE

**1.** Read aloud the following numbers multiplied by 10; by 100; by 1000: 17; 285; 3712; $413.45; $1926.75; 4165.95.

**2.** Read each of the following numbers multiplied by 20; by 400; by 600; by 5000: 16; 19; 37; 49¢; 64¢; $122; $2.60.

**3.** By inspection find the cost of:

*a.* 750 lb. coffee at 30¢.
*b.* 500 lb. cocoa at 40¢.
*c.* 650 lb. chocolate at 50¢.
*d.* 300 bbl. lump salt at $3.
*e.* 200 bbl. oatmeal at $4.50.
*f.* 170 bx. wool soap at $3.
*g.* 650 yd. silk at $1.20.
*h.* 140 bu. beans at $3.50.
*i.* 500 bu. beans at $2.50.
*j.* 240 gro. jet buttons at $3.
*k.* 500 doz. half hose at $5.50.
*l.* 800 yd. taffeta silk at $1.20.

**57.** When the multiplier is a number a little less than 10, 100, or 1000, the multiplication may be shortened as shown in the following examples.

**58. Examples.** **1.** Multiply 123 by 99.

Solution. *99 is 100 diminished by 1; hence, multiply 123 by 100 and then by 1 and subtract the results.* The product is 12,177. Check by retracing the steps in the process.

$$\begin{array}{r} 12300 \\ \underline{123} \\ 12177 \end{array}$$

**2.** Multiply 145 by 96.

Solution. *96 is 100 diminished by 4; hence, multiply 145 by 100 and then by 4 and subtract the results.* The product is 13,920. Check by retracing the steps in the process.

$$\begin{array}{r} 14500 \\ \underline{580} \\ 13920 \end{array}$$

### WRITTEN EXERCISE

**1.** Find the total cost of :

5260 bu. rye at 99¢.
1521 bu. rye at 92¢.
1640 bu. wheat at 98¢.
2994 bu. millet at 97¢.
1112 bu. wheat at 97¢.
2160 bu. millet at 96¢.
834 bu. millet at 95¢.
246 bu. wheat at 92¢.
998 bu. millet at $1.04.
998 bbl. apples at $1.05.
893 bkt. peaches at $1.05.
993 bu. clover seed at $3.35.

## MULTIPLICATION BY 11 AND MULTIPLES OF 11

**59. Example.** Multiply 237 by 11.

Solution. To multiply by 11 is to multiply by 10 + 1. Hence, annex a cipher to 237 and add 237 ; or, better still, add the digits as follows : **7** ; 3 + 7 = **10** ; 3 + 2 + 1 (carried) = **6** ; bring down **2** ; therefore, the result is 2607.

### ORAL EXERCISE

**1.** Multiply each of the following by 11:

14 ; 26 ; 45 ; 19 ; 16 ; 34 ; 36 ; 49 ; 64 ; 125 ; 112 ; 115 ; 128 ; 192 ; 175 ; 116 ; 142 ; $4.95 ; $9.62 ; $4.41 ; $6.82 ; $5.21 ; $3.65 ; $4.31 ; $21.12 ; $14.21 ; $18.32 ; $3.26.

**2.** Find the cost of 11 yd. at 27¢ ; at 91¢ ; at 86¢ ; at 95¢ ; at $1.49 ; at $1.23 ; at $2.17 ; at $2.31 ; at $2.40 ; at $2.50 ; at $2.75 ; at $4.35 ; at $3.15 ; at $3.10 ; at $8.13.

**60. Examples.** 1. Multiply 46 by 22.

Solution. 22 is 11 times 2. Multiply 46 by 11 and by 2, as follows: 2 × 6 = 12; write 2 and carry 1. 4 + 6 = 10; 2 × 10 + 1 (carried) = 21; write 1 and carry 2. 2 × 4 + 2 (carried) = 10; write 10. The result is 1012.

$$\begin{array}{r} 46 \\ \underline{22} \\ 1012 \end{array}$$

2. Find the cost of 122 bu. of potatoes at 66¢ per bu.

Solution. 6 × 2 = 12; write 2 and carry 1. 2 + 2 = 4; 6 × 4 + 1 (carried) = 25; write 5 and carry 2. 1 + 2 = 3; 6 × 3 + 2 (carried) = 20; write 0 and carry 2. 6 × 1 + 2 (carried) = 8. Write 8. The result is $80.52.

$$\begin{array}{r} 122 \\ \underline{.66} \\ 80.52 \end{array}$$

WRITTEN EXERCISE

*In the following problems make all the extensions mentally.*

1. Find the total cost of:

11 lb. coffee at 42¢.
14 doz. eggs at 21¢.
64 lb. cheese at 22¢.
33 bu. carrots at 56¢.
11 bu. potatoes at 85¢.
88 bu. wheat at 88¢.
115 bu. rye at 99¢.
215 bu. peas at 77¢.
344 bu. oats at 44¢.
300 bu. grain at 85¢.
115 bu. barley at 88¢.
400 bbl. apples at $3.25.

2. Find the total cost of:

77 bu. peaches at $1.85.
151 bu. corn at 66¢.
265 bu. onions at 80¢.
135 bu. apples at 82¢.
241 bu. turnips at 44¢.
112 bu. tomatoes at 55¢.
820 bu. rye at 88¢.
327 bu. oats at 33¢.
314 bu. peas at 66¢.
110 bu. pears at $1.66.
880 bu. barley at $1.17.
100 bu. quinces at $1.60.

A careful computer checks his work *at every step.* The student who forms the habit of doing this in all his computations will soon find himself in no need of printed answers to problems involving only numerical calculation.

Checks for multiplication have already been mentioned. To guard against large errors, it is also important to form a rough estimate of an answer before beginning the solution. Thus, in finding the cost of 211 yd. of lining at 32¢, at once see that the result will be a little more than $63.00 (210 times 30¢); this will do away with such absurd results as $6752, $675.20, or $6.75.

**3.** Copy and find the amount of the following bill:

Boston, Mass., July 21, 19

Mrs. GEORGE W. MUNSON
168 Huntington Ave., City

Bought of S. S. PIERCE COMPANY

Terms Cash

| | | | |
|---|---|---|---|
| 15 | cs. Horse-radish | $0.66 | |
| 25 | lb. Huyler's Cocoa | .44 | |
| 31 | gal. N. O. Molasses | .63 | |
| 55 | lb. Japan Tea | .48 | |
| 212 | " Raisins | .11 | |

## MULTIPLICATION BY 25, 50, AND 75

**61.** Annexing two ciphers to an integer multiplies it by 100. Removing the decimal point two places to the right multiplies the decimal by 100.

**62. Example.** Multiply 76 by 100.

Solution. $76 \times 100 = 7600$. (Annexing the two ciphers gives the required result without the necessity for a written solution.)

**63. Example.** Multiply 148 by 25.

Solution. $148 \times 100 = 14{,}800$. $14{,}800 \div 4 = 3700$.
Hence, to multiply an integer by 25, annex two ciphers to the multiplicand and then divide by 4.

**64. Example.** Multiply 278 by 50.

Solution. $278 \times 100 = 27{,}800$. $27{,}800 \div 2 = 13{,}900$.
Hence, to multiply an integer by 50, annex two ciphers to the multiplicand and then divide by 2.

**65. Example.** Multiply 48 by 75.

Solution. $48 \times 100 = 4800$. $4800 \div 4 = 1200$; $3 \times 1200 = 3600$.
Hence, to multiply an integer by 75, annex two ciphers to the multiplicand, divide that product by 4, and then multiply by 3.

### ORAL EXERCISE

*State the product of:*

**1.** 36 × 25.
**2.** 27 × 50.
**3.** 28 × 75.
**4.** 97 × 25.
**5.** 248 × 25.
**6.** 126 × 50.
**7.** 164 × 25.
**8.** 48 × 50.
**9.** 52 × 75.
**10.** 67 × 50.
**11.** 89 × 50.
**12.** 186 × 50.
**13.** 146 × 25.
**14.** 204 × 50.
**15.** 64 × 75.
**16.** 63 × 25.
**17.** 69 × 25.
**18.** 56 × 75.
**19.** 240 × 75.
**20.** 184 × 75.
**21.** 144 × 75.

### WRITTEN EXERCISE

*In the following problems make all the extensions mentally.*

**1.** Find the total cost of:

42 lb. cocoa at 40¢.
45 lb. cocoa at 50¢.
50 lb. coffee at 28¢.
25 lb. raisins at 15¢.
23 lb. tea at 40¢.
27 bx. salt at 50¢.
23 lb. coffee at 25¢.
21 lb. candy at 75¢.
33 lb. chocolate at 50¢.
85 lb. Oolong tea at 45¢.

**2.** Find the total cost of:

36 yd. wash silk at 25¢.
25 doz. whalebones at 92¢.
97 yd. cloth at 75¢.
25 gro. buttons at 35¢.
29 yd. gunner's duck at 19¢.
87 yd. flannel at 50¢.
21 yd. cottonade at 18¢.
25 yd. denim at 19¢.
17 yd. dress goods at 50¢.
23 yd. cheviot at 21¢.

### Multiplication by an Even Number of Hundreds

**66. Example.** Multiply 468 by 300.

Solution. 468 × 100 = 46,800; 46,800 × 3 = 140,400.

Hence, to multiply an integer by an even number of hundreds, annex two ciphers to the multiplicand and then multiply by the significant figure in the multiplier.

**67.** The value of many short methods is that they enable one to write results quickly without performing the mechanical operations.

**68.** Many short methods in multiplication are not practical because they require one to remember so many things, or they apply to so few numbers that it is impossible for an ordinary person to remember them. The short methods given in this text are practical.

### ORAL EXERCISE

*Find the product of:*

1. 234 × 200.
2. 175 × 600.
3. 335 × 800.
4. 216 × 900.
5. 648 × 100.
6. 452 × 500.
7. 753 × 300.
8. 845 × 400.
9. 453 × 200.
10. 256 × 400.
11. 145 × 800.
12. 333 × 700.
13. 964 × 200.
14. 554 × 300.
15. 181 × 700.
16. 312 × 800.
17. 237 × 600.
18. 122 × 900.

## MULTIPLICATION BY NUMBERS FROM 101 TO 109 INCLUSIVE

**69. Examples.** 1. Find the cost of 64 bu. of wheat at $1.02.

$$\begin{array}{r} 64 \\ 1.02 \\ \hline 65.28 \end{array}$$

SOLUTION. 2 × 64 = 128; write 28 and carry 1. 1 × 64 + 1 = 65; write 65. The result is $65.28.

Some persons may prefer to work this problem as follows: 64 bu. at $1 = $64; 64 bu. at 2 ¢ = $1.28; $64 + $1.28 = $65.28.

2. Find the cost of 251 bu. of barley at $1.04.

$$\begin{array}{r} 251 \\ 1.04 \\ \hline 261.04 \end{array}$$

SOLUTION. 4 × 51 = 204; write 04 in the product and carry 2. 4 × 2 + 2 (carried) + 1 (the right-hand figure of the multiplicand) = 11; write 1 and carry 1. 1 × 25 + 1 (carried) = 26; write 26. The result is $261.04.

**70.** Similarly multiply by such numbers as 201, 302, and 405.

**71. Example.** Find the cost of 124 bu. of beans at $2.05.

$$\begin{array}{r} 124 \\ 2.05 \\ \hline 254.20 \end{array}$$

SOLUTION. 5 × 24 = 120. Write 20 and carry 1. 5 × 1 + 1 (carried) + 2 × 4 (the right-hand figure of the multiplicand) = 14; write 4 and carry 1. 2 × 12 + 1 (carried) = 25; write 25. The result is $254.20.

Some persons may prefer the following solution: 124 bu. at $2 = $248; 124 bu. at 5 ¢ = $6.20; $248 + $6.20 = $254.20. The student should try to exercise his own ingenuity in all this work.

### WRITTEN EXERCISE

*Find the value of:*

1. 215 T. coal at $6.05.
2. 224 bu. rye at $1.02.
3. 215 bu. wheat at $1.02.
4. 318 bu. barley at $1.05.
5. 124 bbl. apples at $2.05.
6. 116 bbl. onions at $1.08.
7. 232 bbl. potatoes at $2.05.
8. 302 bu. peas at 74¢.
9. 104 bu. corn at 89¢.
10. 103 bu. beets at 85¢.
11. 205 bu. turnips at 54¢.
12. 215 bu. pears at $1.05.
13. 411 bu. plums at $1.08.
14. 206 bu. parsnips at 93¢.

## MISCELLANEOUS SHORT METHODS

**72.** When one part of the multiplier is contained in another part a whole number of times, the multiplication may be shortened as shown in the following examples.

**73. Example.** Multiply 412 by 357.

SOLUTION. 35 is 5 times 7. 7 × 412 = 2884, which write as the first partial product. 5 × 2884 = 14,420, which write as the second partial product.

CHECK. Interchange the multiplier and multiplicand and remultiply. 4 × 357 = 1428; 3 × 1428 = 4284. Add. Since the results by both multiplications agree, the work is probably correct.

$$\begin{array}{r} 412 \\ \underline{357} \\ 2884 \\ \underline{14420\phantom{0}} \\ 1470.84 \end{array} \qquad \begin{array}{r} 357 \\ \underline{412} \\ 1428\phantom{00} \\ \underline{4284} \\ 147084 \end{array}$$

**74. Example.** Multiply 214 by 756.

SOLUTION. 56 is 8 times 7. 7 × 214 = 1498, which we write as the first partial product. 8 × 1498 = 11,984, which we write as the second partial product. The sum (161,784) of these partial products is the entire product.

Check as in problem 1. (See also pages 73 and 74.)

$$\begin{array}{r} 214 \\ \underline{756} \\ 1498\phantom{00} \\ \underline{11984} \\ 161784 \end{array}$$

### WRITTEN EXERCISE

*Find the product of:*

1. 319 × 248.
2. 927 × 279.
3. 728 × 287.
4. 848 × 369.

The above short methods are practical in a limited number of problems.

### WRITTEN REVIEW EXERCISE

**1.** Use 6 as a multiplier for each column. Check. (See page 39.)

| *a.* | *b.* | *c.* | *d.* | *e.* |
|---|---|---|---|---|
| 56 | 74 | 39 | 126 | 215 |
| 48 | 63 | 58 | 232 | 175 |
| 73 | 52 | 82 | 311 | 243 |
| 49 | 65 | 72 | 135 | 223 |
| 45 | 55 | 85 | 144 | 183 |
| 66 | 47 | 19 | 225 | 253 |
| 77 | 88 | 92 | 245 | 127 |

**2.** Use 8 as a multiplier for each column. Check.

**3.** I bought 15 A. of land at $275 per acre and laid it out in 100 city lots. After expending $6750 for grading and taxes, $257 for ornamental trees, and $250 for advertising, I sold 15 lots at $625 each, 35 lots at $415 each, and exchanged the remainder for a farm of 120 A., which I immediately sold at $195 per acre. Did I gain or lose, and how much?

**4.** Copy and find the amount of the following bill:

Rochester, N.Y., July 26, 19

Mr. F. C. GORHAM
120 Spring Street, City

Bought of C. E. Ferguson & Son

Terms 30 da.

| | | | | | |
|---|---|---|---|---|---|
| | 37 bu. Oats | $0.40 | | | |
| | 50 " Corn | .67 | | | |
| | 76 " Wheat | 1.02 | | | |
| | 75 " Rye | 1.04 | | | |
| | 95 " Beans | 4.00 | | | |
| | 16 " Clover Seed | 3.50 | | | |
| | 26 " Millet | .99 | | | |

## WRITTEN REVIEW

Copy these examples; add the checks in the *Checks in Detail* column and enter the totals in the *Total Checks* column; find the new balance, the total old balance, the total checks, the total deposits, and check the work.

1.

| Name | Balance | Checks in Detail | Total Checks | Deposits | Balance |
|---|---|---|---|---|---|
| | | $180.55 | | | |
| A | $313.25 | 211.15 | ? | $278.40 | ? |
| | | 165.43 | | | |
| | | 208.19 | | | |
| B | 285.67 | 100.55 | ? | 327.44 | ? |
| | | 145.97 | | | |
| C | 186.53 | 200.12 | ? | 198.45 | ? |
| | | 45.67 | | | |
| | | 118.95 | | | |
| D | 276.65 | 205.18 | ? | 210.50 | ? |
| E | —— | 612.40 | ? | 918.75 | ? |
| | | 64.25 | | | |
| F | 347.85 | 103.86 | ? | —— | ? |
| | | 6.84 | | | |
| | ? | ? | ? | ? | ? |

2.

| Name | Balance | Checks in Detail | Total Checks | Deposits | Balance |
|---|---|---|---|---|---|
| A | $195.63 | $214.70 | ? | $174.25 | ? |
| | | 71.20 | | | |
| | | 8.50 | | | |
| B | 98.40 | 102.45 | ? | 115.68 | ? |
| | | 74.65 | | | |
| | | 123.52 | | | |
| C | 153.30 | 10.55 | ? | 89.48 | ? |
| | | 75.20 | | | |
| | | 55.34 | | | |
| D | 386.54 | 7.35 | ? | 275.40 | ? |
| | | 172.38 | | | |
| | ? | ? | ? | ? | ? |

### A WRITTEN REVIEW TEST

*Write the following problems from dictation, and complete the work. Time, approximately, forty minutes, including the dictation. Mental extensions only.*

**1.** Write in one column, and find the total value:

78 yd. at 11¢
69 yd. at 25¢
60 yd. at 85¢
45 yd. at 98¢
37 yd. at 97¢
112 yd. at 99¢
55 yd. at 55¢
91 yd. at 50¢
89 yd. at 99¢
75 yd. at 90¢
76 yd. at 70¢
125 yd. at 98¢

**2.** Write in one column, and find the total value:

76 yd. at $1.10
55 yd. at $1.06
108 yd. at $1.11
56 yd. at $1.25
88 yd. at $1.04
67 yd. at $1.02
82 yd. at $1.05
65 yd. at $1.20
130 yd. at $1.09
83 yd. at $1.50
97 yd. at $1.03
137 yd. at $1.01

**3.** Write in one column; use 11 as the multiplier, and check the results:

49, 16, 34, 78, 57, 73, 85, 94, 59, 64, 56, 81.

**4.** Write in one column; use 6 as the multiplier, and check the results:

125, 212, 350, 175, 162, 224, 319, 452, 133, 145, 121, 142.

**5.** Write in one column; use 8 as the multiplier, and check the results:

45, 75, 62, 29, 76, 61, 19, 34, 85, 92, 27, 77.

**6.** Write in one column; square each number, and total the products:

25, 55, 15, 45, 75, 35, 85, 65, 95.

**7.** The text, page 49, problem 3, in the Written Review Exercise.

# CHAPTER V

## DIVISION

### ORAL EXERCISE

**1.** What is the product of 12 times 15? How many times is 15 contained in 180? What is $\frac{1}{12}$ of 180?

**2.** How much is 11 times \$17? How many times is \$17 contained in \$187? What is $\frac{1}{11}$ of \$187?

**3.** What is the product of 9 times 12 ft.? How many times is 12 ft. contained in 216 ft.? What is $\frac{1}{15}$ of 225 ft.?

**4.** When one factor and the product are given, how is the other factor found?

**75.** The process of finding either factor when the product and the other factor are given is called **division.**

**76.** The known product is called the **dividend**; the known factor, the **divisor**; the unknown factor, when found, the **quotient.**

**77.** The part of the dividend remaining when the division is not exact is called the **remainder.**

While definitions such as the above should not be memorized, the ideas which they express should be thoroughly understood.

**78.** Since 6 times 7 ft. = 42 ft., 42 ft. ÷ 7 ft. = 6, and 42 ft. ÷ 6 = 7 ft. It is therefore clear that

1. *If the dividend and divisor are concrete numbers, the quotient is an abstract number;* and

2. *If the dividend is concrete and the divisor abstract, the quotient is a concrete number like the dividend.*

In §106 it will be seen that there are **two kinds of division**: 42 ft. ÷ 7 ft. = 6 is sometimes called *measuring*, because 42 ft. is measured by 7 ft.; 42 ft. ÷ 6 = 7 ft. is sometimes called *partition*, because 42 ft. is divided into 6 equal parts.

### ORAL EXERCISE

**1.** Divide by 2: 18, 32, 78, 450, 642, 964, 893.
**2.** Divide by 3: 27, 57, 72, 423, 642, 963, 845.
**3.** Divide by 4: 64, 88, 92, 488, 192, 396, 728.
**4.** Divide by 5: 65, 85, 95, 135, 275, 495, 725.
**5.** Divide by 6: 84, 96, 54, 246, 546, 672, 846, 636.
**6.** Divide by 7: 63, 84, 91, 217, 497, 714, 791, 921.
**7.** Divide by 8: 72, 56, 88, 248, 640, 128, 144, 152.
**8.** Divide by 4: 56, 96, 77, 241, 168, 128, 920, 848.
**9.** Divide by 6: 78, 96, 56, 272, 848, 190, 725, 966.
**10.** Divide by 9: 98, 72, 49, 279, 819, 720, 189, 918.

### ORAL EXERCISE

**1.** 16 ft. ÷ 2 = ? 24 ft. ÷ 8 ft. = ?

**2.** $25 ÷ 5 = ? $29.75 ÷ 5 = ? $129.78 ÷ 9 = ? 13.40 ÷ 4 = ?

**3.** 126 yd. ÷ 3 yd. = ? $125 ÷ 25 = ? $6.25 ÷ $1.25 = ?

**4.** If 9 T. of coal cost $49.50, what is the cost per ton?

Solution. $49.50 ÷ 9 = $5; subtracting 9 times $5, the result is $4.50 undivided; $4.50 ÷ 9 = $0.50. Therefore the quotient is $5.50.

$$\begin{array}{r} \$5.50 \\ 9\overline{)\$49.50} \end{array}$$

**5.** At $1.75 a yard, how many yards can be bought for $35?

Solution. The divisor contains cents and it is therefore better to first change both dividend and divisor to cents. It is found that $35 would buy 20 times as many yards as $1.75, or 20 yd.

$$\begin{array}{r} 20 \\ 175\overline{)3500} \end{array}$$

**6.** If 5 T. of coal cost $31.25, what is the cost per ton?

**7.** At $2.50 per yard how many yards can be bought for $550?

### ORAL EXERCISE

**1.** How many weeks in 98 da.?

**2.** What is $\frac{1}{25}$ of 2250 bbl. of apples? $\frac{1}{15}$? $\frac{1}{5}$? $\frac{1}{45}$?

**3.** The quotient is 8 and the dividend 128. What is the divisor?

**4.** How many times can 18 be subtracted from 75, and what will remain?

**5.** At 15¢ per pound, how many pounds of beef can be bought for $6.30?

**6.** The quotient is 5, the divisor 23, and the remainder 2. What is the dividend?

**7.** If 5 men earn $17.50 a day, how much can 8 men earn in 2 da. at the same rate?

**8.** What is the nearest number to 150 that can be divided by 9 without a remainder?

**9.** If there are 960 sheets in 40 qr. of paper, how many sheets in 5 qr.? in 11 qr.?

**10.** If 6 bbl. of apples are worth $21, what are 24 bbl. worth at the same rate? 36 bbl.?

**11.** If 17 bbl. of flour cost $85, what will 25 bbl. cost at the same rate? 32 bbl.? 48 bbl.? 34 bbl.?

**12.** If 8 be added to a certain number, 7 can be subtracted from that number 7 times. What is the number?

**13.** If 20 yd. of cloth cost $60, for how much per yard must it be sold to gain $25? to gain $15?

**14.** A grocer sold 250 oranges at 5¢ each and gained $5. How much did he pay a dozen for the oranges?

**15.** A grocer pays $3 for 20 doz. of eggs. At what price per dozen must he sell them in order to gain $1.50?

**16.** At $2.50 per yard, how many yards of cloth can be bought for $75? for $150? for $2500? for $750?

**17.** How many days' labor at $3.50 per day will pay for 2 T. of coal at $7 a ton and 5 lb. of tea at 70¢ per pound?

**18.** A clothier pays $96 for a dozen overcoats. At how much apiece must he retail them to gain $48 on the lot?

**19.** A man exchanged 1140 bu. of wheat at $1 per bushel for flour at $6 per barrel. How many barrels did he receive?

**20.** It was found that after 15 had been subtracted 5 times from a certain number the remainder was 4. What was the number?

**21.** A man contracts a debt of $175 which he promises to pay in weekly installments of $3.50 each. After paying $35, how many more payments has he to make?

**79. Examples.** **1.** Divide 4285 by 126.

COMPLETE OPERATION

$$
\begin{array}{rl}
 & 34\tfrac{1}{126} \\
126\overline{)4285} & \\
378 & =3 \text{ times } 126 \\
\hline
505 & \text{undivided} \\
504 & =4 \text{ times } 126 \\
\hline
1 & \text{undivided}
\end{array}
$$

REQUIRED WORK

$$
\begin{array}{r}
34\tfrac{1}{126} \\
126\overline{)4285} \\
378 \\
\hline
505 \\
504 \\
\hline
1
\end{array}
$$

CHECK. $\overline{34 \times 126} + 1 = 4285$

The remainder cannot always be written as a part of the quotient. Thus in the problem, "At $7 per head how many sheep can be bought for $37," we cannot say, "5$\frac{2}{7}$ sheep," but "5 sheep and $2 remaining."

**2.** A farmer received $283.25 in payment for 275 bu. of wheat. How much was received per bushel for the wheat?

SOLUTION. $283.75 ÷ 275 = $1 and $8.25 undivided. $8.25 ÷ 275 = $0.03. $1.03 per bushel was therefore received for the wheat.

CHECK. 275 times $1.03 = $283.25.

$$
\begin{array}{r}
\$1.03 \\
275\overline{)\$283.25} \\
275\phantom{.25} \\
\hline
8\ 25 \\
8\ 25 \\
\hline
\end{array}
$$

**80.** Work in division may be abridged by omitting the partial products and writing only the partial dividends.

**81. Example.** Divide $614.80 by 232.

SOLUTION. Omit writing the products; subtract mentally and write the remainder only: 2 × 232 = 464; 464 subtracted from 614 equals 150; omit the writing of the 464. Proceed as follows: 2 times 2 plus **0** = 4; $\overline{\text{2 times 3}}$ plus **5** = 11. 2 times 2 + 1 = 5, and 5 plus **1** = 6. Bring down 8. $\overline{\text{6 times 2}}$ plus **6** = 18; 6 times 3 plus 1 = 19, and 19 + **1** = 20; $\overline{\text{6 times 2}}$ plus 2 = 14, and 14 plus **1** = 15. Bring down 0 and proceed as before.

$$
\begin{array}{r}
\$2.65 \\
232\overline{)\$614.80} \\
150\ 8\phantom{0} \\
11\ 60 \\
0\ 00
\end{array}
$$

### WRITTEN EXERCISE

**1.** Find the cost of 8800 lb. of oats at 45¢ per bushel of 32 lb.

**2.** How many automobiles, at $650 each, can be purchased for $4,225,000?

**3.** By what number must 8656 be multiplied to make the product 8,223,200?

**4.** If 120 bbl. of flour cost $660, what will 829 bbl. cost at the same rate?

**5.** The product of two numbers is 1,928,205. If one of them is 621, what is the other?

**6.** If 380 T. of coal can be bought for $3040, how many tons can be bought for $3600?

**7.** How many cords of 128 cu. ft. in a pile of wood containing 235,820 cu. ft.? What is it worth at $4.50 per cord?

**8.** A speculator sold a quantity of apples that cost $2500 for $4750. If his gain per barrel was $1.12½, how many barrels did he buy?

**9.** A man received a legacy of $11,375 which he invested in railroad stock. He paid a broker $125 to buy stock at $112.50 per share. How many shares were bought?

**10.** A dealer bought 250 T. of coal by the long ton of 2240 lb. at $6.50 per ton. He retailed the same at $8.25 per short ton of 2000 lb. What was the total gain?

**11.** In a recent year there were produced in the United States 730,627,000 bu. of wheat on 45,814,000 A. What was the yield per A.? What was the total yield worth at 90¢ per bu.?

**12.** Copy and complete the following table of corn statistics. Check the work. (The total yield multiplied by the price per bushel should equal the total valuation.)

PRINCIPAL CORN-GROWING STATES IN A RECENT YEAR

| STATE | YIELD IN BUSHELS | FARM PRICE PER BUSHEL | FARM VALUATION |
|---|---|---|---|
| Illinois | 426 320 000 | 62¢ | 264 318 400 |
| Iowa | ? | 62¢ | 267 853 020 |
| Nebraska | ? | 62¢ | 113 221 920 |
| Missouri | ? | 62¢ | 151 220 480 |
| Indiana | 199 364 000 | 62¢ | ? |
| Kansas | 174 225 000 | 62¢ | ? |
| Total | ? | 62¢ | ? |

**13–15.** Make and solve three self-checking problems in division.

## SHORT METHODS IN DIVISION

### POWERS AND MULTIPLES OF 10

#### ORAL EXERCISE

**1.** How many times is 10 contained in 50? 100 in 800? 1000 in 9000?

**2.** Cutting off a cipher in 30 divides it by what number?

**3.** Cutting off two ciphers in 800 divides it by what number?

**4.** Cutting off three ciphers in 11,000 divides it by what number?

**5.** Read aloud, supplying the missing words:

*a.* The number of 10's in any number may be found by cutting off the units' figure; the number of 100's by cutting off the —— and —— figures; the number of —— by cutting off the hundreds' and tens' and units' figures.

*b.* In 4561 there are 456 tens and 1 unit, or $456\frac{1}{10}$ tens; 45 —— and 61 units, or $45\frac{61}{100}$ hundreds; and —— thousands and 561 units, or $4\frac{561}{1000}$ thousands.

**6.** How many times is $0.10 contained in $1? $0.01 in $1? $0.001 in $1?

**7.** What is $\frac{1}{10}$ of $1? $\frac{1}{100}$ of $1? $\frac{1}{1000}$ of $1?

**8.** Read aloud, supplying the missing words: $0.60 is —— of $6; $0.06 is —— of $6; $0.006 is —— of $6.

**9.** Formulate a short method for dividing United States money by 10; by 100; by 1000.

**10.** By inspection find the quotient of:

*a.* 736 ÷ 10.
*b.* 1531 ÷ 100.
*c.* 16351 ÷ 1000.
*d.* 311219 ÷ 10000.
*e.* $271 ÷ 100.
*f.* $519.50 ÷ 10.
*g.* $84.50 ÷ 100.
*h.* $2150 ÷ 1000.
*i.* 2140 lb. ÷ 100.
*j.* 3145 lb. ÷ 100.
*k.* 3416 ft. ÷ 1000.
*l.* 1279 posts ÷ 100.

**11.** Read aloud, supplying the missing amounts:

*a.* 6400 ÷ 1600 = ——; 640 ÷ 10 = ——.

*b.* 27000 ÷ 9000 = ——; 2700 ÷ 900 = ——; 270 ÷ 90 = ——; 27 ÷ 9 = ——.

*c.* 18801 ÷ 90 = —— ÷ 9; 214200 ÷ 700 = 2142 ÷ ——.

**12.** How is the quotient affected by like changes in both the dividend and divisor?

**13.** Divide 1323 by 400.

Solution. Cut off the two ciphers in the divisor and two digits in the right of the dividend, thus dividing both dividend and divisor by 100. 4 is contained in 13 three times with a remainder 1 *hundred.* Adding to this remainder the 23 units remaining in the dividend after dividing by 100, the true remainder is 123, which write in fractional form.

$$\begin{array}{r} 3\frac{123}{400} \\ 4|00\overline{)13|23} \\ \underline{12\phantom{|23}} \\ 1\,23 \end{array}$$

**14.** Read aloud, supplying the missing amounts: 1611 ÷ 400 = ——; 2847 ÷ 700 = ——; 1531 ÷ 300 = ——; 16139 ÷ 4000 = ——.

**15.** Formulate a rule for dividing a number by any multiple of ten.

**16.** State the quotient of:

*a.* 1231 ÷ 30.
*b.* 9647 ÷ 40.
*c.* 6551 ÷ 50.
*d.* 4273 ÷ 70.
*e.* 8197 ÷ 90.
*f.* 96131 ÷ 400.
*g.* 84199 ÷ 700.
*h.* 64137 ÷ 800.
*i.* 45117 ÷ 900.
*j.* 25121 ÷ 500.
*k.* 63571 ÷ 3000.
*l.* 16657 ÷ 4000.
*m.* 36119 ÷ 6000.
*n.* 18177 ÷ 9000.
*o.* 42113 ÷ 7000.

## ORAL REVIEW EXERCISE

The diagram on the opposite page is a portion of the New York Central time-table giving the distances between many of the stations from New York City to Suspension Bridge, and the time taken by two different trains to travel this route.

**1.** How many miles between New York City and Poughkeepsie? between Poughkeepsie and Utica? between Utica and Syracuse? between Syracuse and Rochester? between Rochester and Buffalo? between Buffalo and Niagara Falls?

**2.** What is the distance between New York City and Syracuse? between Poughkeepsie and Niagara Falls? between Rochester and Suspension Bridge?

**3.** How many miles between Ludlow and each station below it? between Poughkeepsie and each station below it? between Tarrytown and each station below it?

**4.** How many miles between Montrose and each station below it? between Oscawana and each station below it?

**5.** At 2¢ per mile, what is the fare from New York to Niagara Falls? from Poughkeepsie to Syracuse? from Buffalo to Utica? from Troy to Yonkers?

**6.** At 2¢ per mile, what is the fare from Rochester to Syracuse? from Rensselaer to Suspension Bridge? from Albany to Niagara Falls? from Syracuse to Buffalo? to Albany?

**7.** How long does it take train No. 93 to travel the first 30 mi. toward Poughkeepsie? the first 74 mi. toward Albany?

**8.** How long is train No. 93 in making the run from Fishkill Landing to Camelot? This is approximately how many miles an hour?

**9.** How long does it take train No. 73 to make the run from Utica to Syracuse? How long does it take train No. 73 to make the run from Fishkill Landing to Chelsea? This is approximately how many miles an hour?

**10.** Add each number in the column marked "Miles" to the one immediately below it.

Thus, 9, 12, 16, 24, 34, 45, 58, etc. In adding 89 and 95 think of 179 and 5, or 184; in adding 143 and 149 think first of 243 and 49 and then of 283 and 9, or 292.

| Miles | NORTH AND WEST BOUND | | Midnight Express | Poughkeepsie Local |
|---|---|---|---|---|
| | | | 73 | 93 |
| 0 | **New York**<br>**Grand Cent. Sta.** | Lv. | 12¶10 | 6†01 |
| 4 | **125th St. Sta.** | " | 12$^{\text{A}}_{\text{M}}$23 | 6$^{\text{A}}_{\text{M}}$13 |
| 5 | **138th St. Sta.** | " | See Note A | 6.15 |
| 7 | High Bridge | " | | 6.21 |
| 8 | Morris Heights | " | | 6.25 |
| 10 | Kings Bridge | " | | 6.29 |
| 11 | Spuyten Duyvil | " | | 6.33 |
| 13 | Riverdale | " | | ...... |
| 14 | Mt. St. Vincent | " | | ...... |
| 15 | Ludlow | " | | ...... |
| 16 | Yonkers | " | 12.46 | 6.43 |
| 18 | Glenwood | " | ...... | 6.46 |
| 20 | Hastings-on-Hudson | " | ...... | 6.52 |
| 21 | Dobbs' Ferry | " | ...... | 6.59 |
| 22 | Ardsley-on-Hudson | " | ...... | 7.01 |
| 23 | Irvington | " | ...... | 7.05 |
| 26 | Tarrytown | " | 1.09 | 7.12 |
| 30 | Scarborough | " | ...... | 7.19 |
| 31 | Ossining | " | 1.25 | 7.25 |
| 35 | Croton-on-Hudson | " | ...... | 7.31 |
| 37 | Oscawana | " | ...... | 7.34 |
| 38 | Crugers | " | ...... | 7.37 |
| 39 | Montrose | " | ...... | 7.41 |
| 42 | Peekskill | " | 1.47 | 7.49 |
| 47 | Highlands | " | ...... | 7.59 |
| 50 | Garrison | " | x | 8.06 |
| 53 | Cold Spring | " | x | 8.12 |
| 56 | Storm King | " | ...... | 8.16 |
| 58 | Dutchess Junc. | " | ...... | 8.21 |
| 59 | Fishkill Landing | " | 2.24 | 8.27 |
| 63 | Chelsea | " | 2.31 | 8.34 |
| 65 | New Hamburg | " | ...... | 8.40 |
| 69 | Camelot | " | ...... | 8.46 |
| 74 | **Poughkeepsie** | Ar | 2.53 | 8$^{\text{A}}_{\text{M}}$55 |
| 74 | **Poughkeepsie** | Lv. | 3.05 | ...... |
| 80 | Hyde Park | " | ...... | ...... |
| 84 | Staatsburgh | " | ...... | ...... |
| 89 | Rhinecliff (Rh'b'k) | " | ...... | ...... |
| 95 | Barrytown | " | ...... | ...... |
| 99 | Tivoli | " | ...... | ...... |
| 105 | Germantown | " | ...... | ...... |
| 109 | Linlithgo | " | ...... | ...... |
| 111 | Greendale | " | ...... | ...... |
| 115 | Hudson | " | 4.47 | ...... |
| 119 | Stockport | " | ...... | ...... |
| 122 | Newton Hook | " | ...... | ...... |
| 125 | Stuyvesant | " | ...... | ...... |
| 131 | Schodack Landing | " | ...... | ...... |
| 135 | Castleton | " | ...... | ...... |
| 142 | Rensselaer | " | ...... | ...... |
| 143 | **Albany** | Ar. | 5.50 | ...... |
| 149 | **Troy** | " | 6$^{\text{A}}_{\text{M}}$50 | |
| 238 | **Utica** | Ar. | 8$^{\text{A}}_{\text{M}}$40 | ...... |
| 291 | **Syracuse** | " | 9.55 | ...... |
| 371 | **Rochester** | " | 11.38 | ...... |
| 440 | **Buffalo** | " | 1$^{\text{P}}_{\text{M}}$15 | ...... |
| 463 | **Niagara Falls** | Ar. | 2$^{\text{P}}_{\text{M}}$13 | ...... |
| 464 | **Suspension Bridge** | " | 2$^{\text{P}}_{\text{M}}$20 | ...... |

**11.** Multiply each number in the column marked "Miles" by 5; by 8; by 3; by 7; by 6; by 4; by 9.

The numbers in the portion of the time-table illustrated may be used for such other exercises as may seem necessary at this point. Students should be impressed with the importance of being able to add, subtract, multiply, and divide numbers in any relative position.

**12.** Five parts of 120 are 15, 18, 32, 10, and 20. Find the sixth part, and multiply it by 15.

**13.** From a flock of 170 sheep I sold at different times 12, 18, 32, and 9. How many sheep remained?

**14.** Multiply by 11 each of the following numbers: 21, 32, 43, 54, 65, 76, 87, 98, 61, 28, 37, 14, 21, 62.

**15.** At 22¢ per yard, what will 18 yd. cost? 21 yd.? 36 yd.? 56 yd.? 29 yd.? 73 yd.? 94 yd.? 72 yd.?

**16.** Multiply each number in problem 15 by 33; by 44.

### WRITTEN REVIEW EXERCISE

**1.** Find the total cost of the articles in problem 3 of the oral exercise, page 42. Find the total of the products in the oral exercise, page 46.

**2.** A mechanic earns $125 per month and his monthly expenses average $72. If he saves the remainder, how long will it take him to save $4352?

**3.** I spent $24,800 for apples at $2.50 per barrel. The loss from decay was equal to 74 bbl. What was my gain, if the remainder of the apples sold for $3.75 per barrel, and my expenses for storage were $675.80?

**4.** During a certain week a contractor employed help as follows: 34 hands, 8 hr. per day, for 5 da., at 15¢ per hour; 16 hands, 9 hr. per day, for 6 da., at 25¢ per hour; 29 hands, 10 hr. per day, for 6 da., at 18¢ per hour. Find the amount due.

**5.** In a recent year there were produced on 37,917,000 A. in the United States 1,418,337,000 bu. oats, valued on the farm at 31.3¢ per bushel. What was the average yield per acre? What was the value of the year's crop?

**6.** Without copying find (*a*) the total number of railway employees in the United States in 1910 and (*b*) the total number per one hundred miles of line in the same year.

RAILWAY EMPLOYEES IN THE UNITED STATES

| Class | 1910 | | | 1911 | | |
|---|---|---|---|---|---|---|
| | Total Number | Number per 100 Mi. | Average Daily Wages | Total Number | Number per 100 Mi. | Average Daily Wages |
| General officers | 5,476 | 2 | $13.27 | 5,628 | 2 | $12.99 |
| Other officers | 9,392 | 4 | 6.22 | 10,196 | 4 | 6.27 |
| General office clerks | 76,329 | 32 | 2.40 | 76,513 | 31 | 2.49 |
| Station agents | 37,379 | 16 | 2.12 | 38,277 | 16 | 2.17 |
| Other station men | 153,104 | 64 | 1.84 | 153,117 | 62 | 1.89 |
| Engineers | 64,691 | 27 | 4.55 | 63,390 | 26 | 4.79 |
| Firemen | 68,321 | 28 | 2.74 | 66,376 | 27 | 2.94 |
| Conductors | 48,682 | 20 | 3.91 | 48,200 | 20 | 4.16 |
| Other trainmen | 136,938 | 57 | 2.69 | 133,221 | 54 | 2.88 |
| Machinists | 55,193 | 23 | 3.08 | 55,207 | 22 | 3.14 |
| Carpenters | 68,085 | 28 | 2.51 | 65,989 | 27 | 2.54 |
| Other shopmen | 225,196 | 94 | 2.18 | 226,785 | 92 | 2.24 |
| Section foremen | 44,207 | 18 | 1.99 | 44,466 | 18 | 2.07 |
| Other trackmen | 378,955 | 157 | 1.47 | 363,028 | 147 | 1.50 |
| All other employees | 229,806 | 95 | 2.01 | 227,779 | 93 | 2.08 |

**7.** Without copying find (*a*) the total number of railway employees in the United States in 1911 and (*b*) the total number per one hundred miles of line in the same year.

**8.** Find the total salaries paid to railway employees in 1910; in 1911.

**9.** Find the average daily wages paid to railway employees in 1910; in 1911.

**10.** In a recent year four leading railway systems had outstanding bonds as follows:

*a.* $761,963,000.
*c.* $1,096,773,410.
*b.* $576,300,000.
*d.* $428,649,000.

Find the average amount of the bonds outstanding.

### WRITTEN REVIEW

*In these problems divide across, and then add the dividend column and the quotient column. Check: divide the total of the dividend column by the divisor, and this quotient should equal the sum of the individual quotients. Time, approximately, fifteen minutes.*

**1.**

$36 \div 4 = ?$
$48 \div 4 = ?$
$56 \div 4 = ?$
$24 \div 4 = ?$
$84 \div 4 = ?$
$44 \div 4 = ?$
$64 \div 4 = ?$
$76 \div 4 = ?$
$\overline{?} \div \overline{4} = \overline{?}$

**2.**

$45 \div 5 = ?$
$75 \div 5 = ?$
$95 \div 5 = ?$
$65 \div 5 = ?$
$35 \div 5 = ?$
$55 \div 5 = ?$
$15 \div 5 = ?$
$40 \div 5 = ?$
$\overline{?} \div \overline{5} = \overline{?}$

**3.**

$66 \div 6 = ?$
$72 \div 6 = ?$
$84 \div 6 = ?$
$78 \div 6 = ?$
$48 \div 6 = ?$
$36 \div 6 = ?$
$54 \div 6 = ?$
$42 \div 6 = ?$
$\overline{?} \div \overline{6} = \overline{?}$

**4.**

$98 \div 7 = ?$
$84 \div 7 = ?$
$63 \div 7 = ?$
$49 \div 7 = ?$
$91 \div 7 = ?$
$56 \div 7 = ?$
$77 \div 7 = ?$
$42 \div 7 = ?$
$\overline{?} \div \overline{7} = \overline{?}$

**5.**

$88 \div 2 = ?$
$76 \div 2 = ?$
$58 \div 2 = ?$
$64 \div 2 = ?$
$82 \div 2 = ?$
$94 \div 2 = ?$
$52 \div 2 = ?$
$74 \div 2 = ?$
$\overline{?} \div \overline{2} = \overline{?}$

**6.**

$48 \div 3 = ?$
$54 \div 3 = ?$
$69 \div 3 = ?$
$72 \div 3 = ?$
$84 \div 3 = ?$
$93 \div 3 = ?$
$87 \div 3 = ?$
$54 \div 3 = ?$
$\overline{?} \div \overline{3} = \overline{?}$

**7.**

$126 \div 2 = ?$
$152 \div 2 = ?$
$134 \div 2 = ?$
$168 \div 2 = ?$
$184 \div 2 = ?$
$156 \div 2 = ?$
$172 \div 2 = ?$
$138 \div 2 = ?$
$\overline{?} \div \overline{2} = \overline{?}$

**8.**

$144 \div 4 = ?$
$124 \div 4 = ?$
$152 \div 4 = ?$
$148 \div 4 = ?$
$176 \div 4 = ?$
$184 \div 4 = ?$
$136 \div 4 = ?$
$180 \div 4 = ?$
$\overline{?} \div \overline{4} = \overline{?}$

**9.**

$129 \div 3 = ?$
$114 \div 3 = ?$
$108 \div 3 = ?$
$147 \div 3 = ?$
$189 \div 3 = ?$
$165 \div 3 = ?$
$195 \div 3 = ?$
$138 \div 3 = ?$
$\overline{?} \div \overline{3} = \overline{?}$

**82.** All mailable matter for transmission by the United States mails within the United States or to Cuba, Mexico, Hawaii, Porto Rico, Canada, and the Philippine Islands is divided into four classes: first-, second-, third-, and fourth-class matter.

*First-class matter* includes letters, postal cards, and anything sealed or otherwise closed against inspection. The rate for first-class matter is 2 ¢ per ounce or fraction thereof; for a postal card, 1 ¢; for a reply postal card, 2 ¢. Written or typewritten matter is of the first class, whether sealed or unsealed.

*Second-class matter* includes newspapers and periodicals entirely in print. When sent by publishers or news agents, the rate is 1 ¢ per pound or fraction thereof; when sent by others, 1 ¢ for each 4 oz. or fraction thereof.

*Third-class matter* includes books and catalogues (weighing 8 oz. or less), circulars, pamphlets, proof sheets and manuscript copy accompanying the same, and engravings. The rate is 1 ¢ for each 2 oz. or fraction thereof. The limit of weight is 4 lb.

All postal matter of the first, second, or third class may be *registered* at the rate of 10 ¢ for each package in addition to the regular rates of postage.

The rates on *special delivery letters* are 10 ¢ per letter in addition to the regular postage. Any matter on which a special delivery stamp is affixed is entitled to special delivery within certain limits.

*Foreign rates of postage* are as follows: letters, 5 ¢ per ounce for the first ounce, and 3 ¢ for each additional ounce. (Double rate is charged at delivery office for any deficiency in prepayment.) Postal cards, 2 ¢ each; newspapers and other printed matter, 1 ¢ for 2 oz.

Some foreign countries, as Germany and Great Britain, come under the letter rate of 2 ¢ per ounce.

All *fourth-class matter* is now included in the domestic parcel post, by a law which became effective January 1, 1913. The following are some of the principal features of this law:

The country is divided into zones, the rate of postage being dependent on the zone where the parcel is to be delivered. The zone center is the point of mailing. Regular postage stamps are used on parcel-post packages.

Parcels weighing 4 oz. or less are mailable at the rate of 1¢ for each ounce or fraction of an ounce, regardless of distance. Parcels weighing more than 4 oz. are mailable at the pound rates shown in the table, a fraction of a pound being considered as a full pound.

Books and catalogues weighing in excess of 8 oz. may be sent by parcel post.

The weight limit for zones 1 and 2 is 50 lb.; for *all* the other zones, 20 lb.

The local rate applies to parcels to be delivered at the office of mailing, or on a rural route starting from that office.

A parcel may be insured against loss to the amount of its actual value not exceeding $50.

A special delivery of a parcel will be made on the payment of an additional 10¢, at the mailing office.

In the table, the rates are complete up to 11 lb. From this point on only illustrative weights and rates are given; omissions are indicated by stars.

The local post office can furnish a parcel-post map of the United States showing the regions included in the different zones.

| | | 50 mi. | 50–150 mi. | 150–300 mi. | 300–600 mi. | 600–1000 mi. | 1000–1400 mi. | 1400–1800 mi. | All over 1800 mi. |
|---|---|---|---|---|---|---|---|---|---|
| Weight | First Zone | | Second Zone Rate | Third Zone Rate | Fourth Zone Rate | Fifth Zone Rate | Sixth Zone Rate | Seventh Zone Rate | Eighth Zone Rate |
| | Local Rate | Zone Rate | | | | | | | |
| 1 lb. | $0.05 | $0.05 | $0.05 | $0.06 | $0.07 | $0.08 | $0.09 | $0.11 | $0.12 |
| 2 lb. | .06 | .06 | .06 | .08 | .11 | .14 | .17 | .21 | .24 |
| 3 lb. | .06 | .07 | .07 | .10 | .15 | .20 | .25 | .31 | .36 |
| 4 lb. | .07 | .08 | .08 | .12 | .19 | .26 | .33 | .41 | .48 |
| 5 lb. | .07 | .09 | .09 | .14 | .23 | .32 | .41 | .51 | .60 |
| 6 lb. | .08 | .10 | .10 | .16 | .27 | .38 | .49 | .61 | .72 |
| 7 lb. | .08 | .11 | .11 | .18 | .31 | .44 | .57 | .71 | .84 |
| 8 lb. | .09 | .12 | .12 | .20 | .35 | .50 | .65 | .81 | .96 |
| 9 lb. | .09 | .13 | .13 | .22 | .39 | .56 | .73 | .91 | 1.08 |
| 10 lb. | .10 | .14 | .14 | .24 | .43 | .62 | .81 | 1.01 | 1.20 |
| 11 lb. | .10 | .15 | .15 | .26 | .47 | .68 | .89 | 1.11 | 1.32 |
| * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * |
| 15 lb. | .12 | .19 | .19 | .34 | .63 | .92 | 1.21 | 1.51 | 1.80 |
| * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * | * * * |
| 20 lb. | .15 | .24 | .24 | .44 | .83 | 1.22 | 1.61 | 2.01 | 2.40 |
| * * * | * * * | * * * | * * * | | | | | | |
| 30 lb. | .20 | .34 | .34 | | | | | | |
| * * * | * * * | * * * | * * * | | | | | | |
| 40 lb. | .25 | .44 | .44 | | | | | | |
| * * * | * * * | * * * | * * * | | | | | | |
| 50 lb. | .30 | .54 | .54 | | | | | | |

### ORAL EXERCISE

**1.** What is the postage on a letter weighing $\frac{1}{2}$ oz.? $4\frac{1}{2}$ oz.? $1\frac{1}{2}$ oz.? $3\frac{1}{4}$ oz.? $2\frac{1}{4}$ oz.? $4\frac{1}{4}$ oz.?

**2.** What will be the cost of postage on the following articles at your post office to points within the United States: an ordinary letter weighing $2\frac{3}{4}$ oz.; a registered letter weighing $1\frac{1}{2}$ oz.; a bundle of papers weighing 10 oz.?

**3.** Find the total cost of postage on the following to points within the United States: a special delivery letter weighing $1\frac{1}{4}$ oz.; some printers' proofs weighing 18 oz.; some separate matter for the printer weighing 12 oz.; a pamphlet weighing 6 oz.

**4.** Use a zone map and find the cost of mailing each of the following articles:

| Article | Weight | Destination |
|---|---|---|
| A pair of opera glasses | 2 lb. 8 oz. | Kansas City, Mo. |
| A pair of ladies' gloves | 6 oz. | Indianapolis, Ind. |
| A copy of Star-Land | 1 lb. 8 oz. | Macon, Ga. |
| A copy of Whittier's Poems | 1 lb. 12 oz. | Pittsburgh, Pa. |
| A copy of Lowell's Poems | 1 lb. 10 oz. | Denver, Colo. |
| A box of merchandise | 3 lb. 8 oz. | Chicago, Ill. |
| A box containing a pair of shoes | 3 lb. 6 oz. | Austin, Tex. |
| A piece of hardware | 6 lb. 9 oz. | Detroit, Mich. |

**5.** A publisher sends 20,000 copies of his magazine by mail. If each magazine and wrapper weighs $14\frac{1}{2}$ oz. and the total number is weighed at the post office in bulk, what will the publisher have to pay for postage?

**6.** A subscriber mailed two copies of the above magazine to a friend. What was the cost for postage?

**7.** 25,000 copies of a monthly magazine weighing $14\frac{1}{4}$ oz. were sent by mail. What was the cost to the publisher for postage?

**8.** Find the total cost for mailing the following: printers' proof weighing $18\frac{1}{4}$ oz.; manuscript and printers' proof in one package, weighing $28\frac{1}{2}$ oz.; a special delivery letter, weighing $\frac{3}{4}$ oz.

## PRICE LISTS AND INVENTORIES

### PRICE LISTS

*These price lists are to be used in making out the inventories which are found on the four pages following:*

| ARTICLE | 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|---|
| Bedsteads | $6.25 | $8.50 | $11.75 | $5.25 | $14.50 |
| Bookcases | 42.00 | 38.00 | 25.00 | 35.00 | 32.00 |
| Bureaus | 18.50 | 27.25 | 22.50 | 16.50 | 34.20 |
| Cabinets: | | | | | |
| China | 22.00 | 20.00 | 27.50 | 32.50 | 37.50 |
| Medicine | 2.25 | 1.75 | 2.50 | 3.25 | 2.75 |
| Music | 10.00 | 11.00 | 9.00 | 8.50 | 10.50 |
| Parlor | 25.50 | 21.50 | 25.00 | 36.50 | 38.00 |
| Chairs: | | | | | |
| Easy | 12.50 | 10.50 | 15.00 | 22.25 | 18.50 |
| Morris | 10.50 | 12.25 | 12.00 | 9.75 | 8.25 |
| Piano | 5.00 | 6.50 | 7.25 | 9.00 | 12.00 |
| Typewriter | 3.50 | 4.50 | 4.00 | 5.50 | 5.00 |
| Cheval Mirrors | 20.00 | 17.25 | 18.50 | 16.50 | 21.25 |
| Chiffoniers | 24.00 | 18.20 | 31.75 | 27.50 | 40.00 |
| Davenports | 62.50 | 50.00 | 60.00 | 45.50 | 37.50 |
| Desks: | | | | | |
| Flat-top | 21.20 | 17.60 | 16.80 | 22.30 | 18.60 |
| Roll-top | 23.50 | 21.25 | 25.50 | 27.25 | 30.50 |
| Typewriter | 11.25 | 12.25 | 14.25 | 10.25 | 13.25 |
| Dinner Trays | 5.50 | 6.25 | 7.25 | 5.25 | 8.25 |
| Footrests | 1.75 | 1.50 | 1.60 | 2.00 | 2.25 |
| Hall Racks | 14.25 | 13.50 | 17.25 | 18.50 | 12.25 |
| Lounges | 25.00 | 17.50 | 21.50 | 32.00 | 23.50 |
| Mattresses | 11.40 | 12.50 | 13.25 | 17.50 | 21.00 |
| Ottomans | 5.25 | 4.50 | 4.25 | 5.50 | 4.75 |
| Parlor Suites | 63.00 | 52.50 | 75.00 | 67.50 | 62.50 |
| Pillows | 2.50 | 3.00 | 3.50 | 2.00 | 3.25 |
| Sideboards | 60.00 | 50.00 | 45.00 | 37.50 | 72.50 |
| Tables: | | | | | |
| Dining | 21.50 | 17.50 | 22.50 | 24.50 | 25.00 |
| Dressing | 37.50 | 32.25 | 21.50 | 35.25 | 24.75 |
| Serving | 13.50 | 11.00 | 14.50 | 10.50 | 12.25 |
| Work | 10.00 | 9.25 | 9.00 | 10.25 | 11.00 |
| Wardrobes | 20.50 | 25.25 | 15.50 | 21.75 | 25.00 |
| Washstands | 6.00 | 7.50 | 5.50 | 8.00 | 10.50 |

Each inventory may be worked out *five* times, using the above price lists. This work may be done by copying the data and making the extensions, or by making the extensions only and then totaling each inventory.

## INVENTORIES

**1.**

7 Bureaus
19 Bedsteads
12 Chiffoniers
5 Dressing Tables
21 Washstands
8 Mattresses
23 Pillows
6 Bookcases
3 Davenports
13 Lounges
17 Easy Chairs
7 Morris Chairs
5 Parlor Suites
8 Music Cabinets
12 Piano Chairs
15 Parlor Cabinets
5 Sideboards
13 Dining Tables
8 China Cabinets
4 Serving Tables
9 Work Tables
12 Dinner Trays
4 Medicine Cabinets
8 Wardrobes
11 Cheval Mirrors
15 Ottomans
12 Footrests
5 Hall Racks
6 Roll-top Desks
3 Flat-top Desks
9 Typewriter Desks
8 Typewriter Chairs

**2.**

8 Bureaus
17 Bedsteads
15 Chiffoniers
3 Dressing Tables
18 Washstands
11 Mattresses
21 Pillows
5 Bookcases
4 Davenports
12 Lounges
20 Easy Chairs
3 Morris Chairs
6 Parlor Suites
10 Music Cabinets
11 Piano Chairs
21 Parlor Cabinets
2 Sideboards
16 Dining Tables
5 China Cabinets
6 Serving Tables
7 Work Tables
15 Dinner Trays
8 Medicine Cabinets
5 Wardrobes
14 Cheval Mirrors
12 Ottomans
17 Footrests
7 Hall Racks
4 Roll-top Desks
2 Flat-top Desks
10 Typewriter Desks
6 Typewriter Chairs

**3.**

8 Bureaus
15 Bedsteads
14 Chiffoniers
4 Dressing Tables
19 Washstands
12 Mattresses
18 Pillows
8 Bookcases
5 Davenports
16 Lounges
22 Easy Chairs
6 Morris Chairs
7 Parlor Suites
11 Music Cabinets
14 Piano Chairs
12 Parlor Cabinets
6 Sideboards
10 Dining Tables
7 China Cabinets
6 Serving Tables
8 Work Tables
11 Dinner Trays
7 Medicine Cabinets
9 Wardrobes
13 Cheval Mirrors
19 Ottomans
12 Footrests
6 Hall Racks
8 Roll-top Desks
4 Flat-top Desks
14 Typewriter Desks
10 Typewriter Chairs

**4.**

9 Bureaus
13 Bedsteads
11 Chiffoniers
7 Dressing Tables
17 Washstands
9 Mattresses
23 Pillows
6 Bookcases
3 Davenports
21 Lounges
18 Easy Chairs
9 Morris Chairs
4 Parlor Suites
9 Music Cabinets
13 Piano Chairs
18 Parlor Cabinets
4 Sideboards
12 Dining Tables
9 China Cabinets
5 Serving Tables
11 Work Tables
14 Dinner Trays
5 Medicine Cabinets
7 Wardrobes
17 Cheval Mirrors
15 Ottomans
18 Footrests
3 Hall Racks
5 Roll-top Desks
6 Flat-top Desks
11 Typewriter Desks
7 Typewriter Chairs

**5.**

12 Bureaus
23 Bedsteads
13 Chiffoniers
8 Dressing Tables
16 Washstands
14 Mattresses
22 Pillows
4 Bookcases
4 Davenports
17 Lounges
14 Easy Chairs
12 Morris Chairs
10 Parlor Suites
14 Music Cabinets
17 Piano Chairs
22 Parlor Cabinets
7 Sideboards
14 Dining Tables
4 China Cabinets
11 Serving Tables
5 Work Tables
19 Dinner Trays
6 Medicine Cabinets
12 Wardrobes
20 Cheval Mirrors
11 Ottomans
21 Footrests
4 Hall Racks
12 Roll-top Desks
5 Flat-top Desks
13 Typewriter Desks
12 Typewriter Chairs

**6.**

14 Bureaus
21 Bedsteads
16 Chiffoniers
11 Dressing Tables
14 Washstands
15 Mattresses
17 Pillows
7 Bookcases
7 Davenports
19 Lounges
25 Easy Chairs
11 Morris Chairs
9 Parlor Suites
12 Music Cabinets
21 Piano Chairs
16 Parlor Cabinets
9 Sideboards
18 Dining Tables
6 China Cabinets
9 Serving Tables
10 Work Tables
13 Dinner Trays
10 Medicine Cabinets
15 Wardrobes
16 Cheval Mirrors
19 Ottomans
14 Footrests
8 Hall Racks
9 Roll-top Desks
8 Flat-top Desks
7 Typewriter Desks
15 Typewriter Chairs

**7.**

19 Bureaus
21 Bedsteads
11 Chiffoniers
9 Dressing Tables
15 Washstands
11 Mattresses
24 Pillows
9 Bookcases
6 Davenports
22 Lounges
12 Easy Chairs
8 Morris Chairs
9 Parlor Suites
14 Music Cabinets
9 Piano Chairs
17 Parlor Cabinets
3 Sideboards
13 Dining Tables
4 China Cabinets
11 Serving Tables
13 Work Tables
24 Dinner Trays
6 Medicine Cabinets
13 Wardrobes
14 Cheval Mirrors
6 Ottomans
8 Footrests
11 Hall Racks
3 Roll-top Desks
7 Flat-top Desks
16 Typewriter Desks
9 Typewriter Chairs

**8.**

17 Bureaus
20 Bedsteads
16 Chiffoniers
6 Dressing Tables
21 Washstands
15 Mattresses
25 Pillows
10 Bookcases
2 Davenports
16 Lounges
8 Easy Chairs
13 Morris Chairs
3 Parlor Suites
10 Music Cabinets
7 Piano Chairs
22 Parlor Cabinets
5 Sideboards
9 Dining Tables
10 China Cabinets
7 Serving Tables
17 Work Tables
21 Dinner Trays
9 Medicine Cabinets
11 Wardrobes
18 Cheval Mirrors
9 Ottomans
10 Footrests
12 Hall Racks
2 Roll-top Desks
9 Flat-top Desks
10 Typewriter Desks
11 Typewriter Chairs

# CHAPTER VI

## CHECKING RESULTS

**83.** It has been seen in the preceding exercises on statistics, time sheets, etc., that various ruled forms provide for practical and convenient methods of checking results. While it is possible to give a great variety of these problems it is also necessary to give a great many problems that do not furnish such a check.

**84.** It is very important that all results be checked. The most common methods of checking addition, subtraction, and division have already been mentioned. Multiplication may be proved by dividing the product by either factor, or as explained on page 38.

The properties of 9 and 11 may also be applied to advantage in checking results, especially results in multiplication and division.

## PROPERTIES OF 9 AND 11

### Properties of 9

**85.** Any number of 10's is equal to the same number of 9's plus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 999's plus the same number of units; and so on.

Thus, 10 = one 9 + 1; 40 = four 9's + 4; 100 = one 99 + 1; 300 = three 99's + 3; 500 = five 99's + 5.

**86.** Any number may be resolved into one less than as many multiples of 10 as it contains digits.

Thus, 946 = 900 + 40 + 6; 42175 = 40000 + 2000 + 100 + 70 + 5.

**87.** The excess of 9's in any power of 10 or in any multiple of a power of 10 is the same as the significant figure (unless that figure is 9, then there is no excess) in that number. Hence,

*The excess of 9's in any number is equal to the excess of 9's in the sum of its digits.*

Thus, the excess of 9's in $241 = 2 + 4 + 1$, or 7. The excess of 9's in $946 = 9 + 4 + 6$, or 19; but 19 contains 9, and the excess of 9's in $19 = 1 + 9$, or 10; but 10 contains 9, and the excess of 9's in $10 = 1 + 0$, or 1; the excess of 9's in 946 is therefore shown to be 1.

**88.** In finding the excess of 9's in any number, omit all 9's and all combinations of two or three digits which it is seen at a glance will make 9 or some multiple of 9.

Thus, in finding the excess of 9's in 9458, begin at the left, reject the first digit 9, the sum of the next two digits, 9, and the single 8 will be the excess of 9's in the entire number.

## PROPERTIES OF 11

**89.** Any number of 10's is equal to the same number of 11's minus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 1001's minus the same number of units; and so on.

Thus, 40 = four 11's − 4; 500 = five 99's + 5; 7000 = seven 1001's − 7.

**90.** It is therefore clear that even powers of 10 are multiples of 11 plus 1 and odd powers of 10 are multiples of 11 minus 1.

Thus, $10^2$ or 100 = nine 11's + 1; $10^3$ or 1000 = ninety-one 11's − 1; $10^4$ or 10,000 = nine hundred nine 11's + 1.

**91.** From the foregoing it is evident that:

*The excess of 11's in any number is equal to the sum of the digits in the odd places (increased by 11 or a multiple of 11 if necessary) minus the sum of the digits in the even places.*

Thus, the excess of 11's in 45 is 1 $(5 - 4)$; the excess of 11's in 125 is 4 $(\overline{5 - 2} + \overline{1 - 0})$; the excess of 11's in 2473 is 9 $(\overline{3} + \overline{4 + 11} - \overline{7 + 2} = 9)$; the excess of 11's in 14,206 is 5.

## CHECKING ADDITION AND SUBTRACTION

**92. Examples.** 1. By casting out the 9's, show that the sum of 935, 651, 782, and 465 is 2833.

SOLUTION. The sum of the digits in 935 is 17; but since 17 contains 9, find the sum of the digits in 17 and the result, 8, is the excess of 9's in the entire number. In like manner find the excess of 9's in 651, 782, and 465. Since 935 is a multiple of 9 + 8, 651 a multiple of 9 + 3, 782 a multiple of 9 + 8, 465 a multiple of 9 + 6, the sum of these numbers, 2833, should equal a multiple of 9 + (8 + 3 + 8 + 6), or 9 + 25. 25 is a multiple of 9 + 7, and 2833 is a multiple of 9 + 7; hence, the addition is probably correct.

$$\begin{array}{r} 935 = 8 \\ 651 = 3 \\ 782 = 8 \\ 465 = 6 \\ \hline 2833 = 7 \end{array}$$

2. By casting out the 11's, show that the sum of 648, 217, 451, and 688 is 2004.

SOLUTION. $\overline{8-4}+\overline{6-0}=10$, the excess of 11's in 648. $\overline{7-1}+\overline{2-0}=8$, the excess of 11's in 217. 12 (11 + 1) − 5 + $\overline{4-0}=11$; but 11 contains 11, hence, the excess of 11's in 451 is 0. $\overline{8-8}+\overline{6-0}=6$, the excess of 11's in 688. Since 648 is a multiple of 11 + 10, 217 a multiple of 11 + 8, 451 a multiple of 11, and 688 a multiple of 11 + 6, the sum of these numbers, 2004, should be a multiple of 11 + (10 + 8 + 6), or 11 + 24. 24 is a multiple of 11 + 2 and 2004 is a multiple of 11 + 2; hence, the addition is probably correct.

$$\begin{array}{r} 648 = 10 \\ 217 = 8 \\ 451 = 0 \\ 688 = 6 \\ \hline 2004 = 2 \end{array}$$

**93.** Subtraction may be proved either by casting out the 9's or 11's in practically the same manner as addition.

The difference between the excess of 9's or 11's in the minuend and subtrahend should equal the excess of 9's or 11's in the remainder; or the sum of the excess of 9's or 11's in the subtrahend and remainder should equal the excess of 9's or 11's in the minuend.

These methods are but little used for checking addition and subtraction. Addition is generally checked as explained on page 12, and subtraction as explained on page 24. On the other hand, long multiplications and divisions are almost always checked by applying the properties of 9 or 11.

## CHECKING MULTIPLICATION AND DIVISION

**94. Examples.** 1. By casting out the 9's show that the product of 64 × 95 is 6080.

SOLUTION. The excess of 9's in 95 is 5, and in 64, 1. Since 95 is a multiple of 9 + 5 and 64 a multiple of 9 + 1, the product of 64 × 95 should be a multiple of 9 plus (1 × 5). 1 × 5 or 5 equals the excess of 9's in 6080; hence, the work is probably correct.

$$\begin{array}{r} 95 = 5 \\ 64 = 1 \\ \hline 6080 = 5 \end{array}$$

**2.** By casting out the 11's show that the product of 46 × 95 is 4370.

Solution. The excess of 11's in 95 is 7, and in 46, 2. Since 95 is a multiple of 11 + 7 and 46 a multiple of 11 + 2, the product of 46 × 95 should be a multiple of 11 plus (2 × 7) or 14; but 14 is a multiple of 11 + 3. Since the product 4370 is a multiple of 11 + 3, the work is probably correct.

$$\begin{array}{r} 95 = 7 \\ \underline{46} = \underline{2} \\ 4370 = 3 \end{array}$$

**95.** Division may be proved either by casting out the 9's or 11's in practically the same manner as multiplication. The excess of 9's or 11's in the quotient multiplied by the excess of 9's or 11's in the divisor should equal the excess of 9's or 11's in the dividend, minus the excess of 9's or 11's in the remainder, if any.

Casting out the 9's will not show an error caused by a transposition of figures; but casting out the 11's will show such an error. The method of casting out the 11's is therefore considered the better proof.

### WRITTEN EXERCISE

**1.** Determine without dividing whether $2.64 is the quotient of $1375.44 ÷ 521.

**2.** Determine without multiplying whether $1807.50 is the product of 482 times $3.75.

**3.** Determine without adding whether 4231 is the sum of 296, 348, 924, 862, 956, and 845.

**4.** Multiply 34,125 by 729 in two lines of partial products and verify the work by casting out the 9's.

**5.** Find the cost of 173,000 shingles at $4.27 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

**6.** Find the cost of 126,000 ft. of clear pine at $24.60 per thousand, in two lines of partial products, and verify the work by casting out the 9's.

**7.** Find the cost of 2,191,000 ft. of flooring at $32.08 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

# FRACTIONS

## CHAPTER VII

### DECIMAL FRACTIONS

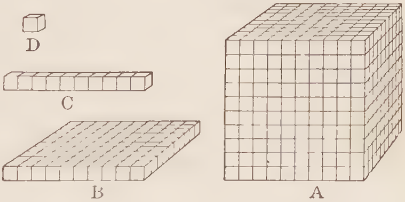
#### ORAL EXERCISE

**1.** In the number $7.62 what figure stands for the dollars? the tenths of a dollar? the hundredths of a dollar?

**2.** What name is given to the point which separates the whole number of dollars from the part of a dollar?

**3.** Read: 3.5 dollars; 3.5 ft.; 27.5 lb.; .7 of a dollar; .5 of a ton; 16.6; .9; 9.25 dollars; 7.25 ft.; 8.75 rd.; .95 of a dollar; .85 of a pound sterling; .57.

**4.** What is the first place at the right of the decimal point called? the second place?



**5.** In the accompanying diagram what part of *A* is *B*? What part of *B* is *C*? What part of *C* is *D*?

**6.** What part of *A* is *C*? What part of *A* is *D*?

**7.** If *A* is a cubic inch, what is *B*? *C*? *D*?

**8.** In a pile of 10,000 bricks one brick is what part of the whole pile? 10 bricks is what part of the whole pile? 100 bricks is what part of the whole pile? 1000 bricks is what part of the whole pile?

**9.** How may one tenth be written besides $\frac{1}{10}$? one hundredth besides $\frac{1}{100}$? one thousandth besides $\frac{1}{1000}$?

**96.** Units expressed by figures at the right of the decimal point are called **decimal units.**

**97.** A number containing one or more decimal units is called a **decimal fraction** or a **decimal.**

## NOTATION AND NUMERATION

### ORAL EXERCISE

**1.** Read: 0.7; 0.03; 0.25. How many places must be used to express completely any number of hundredths?

**2.** Read: 0.004; 0.025; 0.725. How many places must be used to express completely any number of thousandths?

**3.** Read: .0005; .00007; .000009; .0037; .00045; .000051; .0121; .00376; .000218; .1127; .01525; .004531; .16067.

**4.** How many places must be used to express completely any number of ten-thousandths? any number of hundred-thousandths? any number of millionths?

**98.** In **reading decimals** pronounce the word *and* at the decimal point and omit it in all other places.

Thus, in reading 0.605 or .605 say *six hundred five thousandths;* in reading 600.005 say *six hundred and five thousandths.*

**99.** The relation of integers and decimals with their increasing and decreasing orders to the left and to the right of the decimal point is shown in the following

### NUMERATION TABLE

| **PERIODS:** | Millions | | | Thousands | | | Units | | | | Thousandths | | | Millionths | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **ORDERS:** | Hundred-millions | Ten-millions | Millions | Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Units | Decimal Point | Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths | Millionths |
| | **9** | **8** | **7,** | **6** | **5** | **4,** | **3** | **2** | **1** | **.** | **2** | **3** | **4** | **5** | **6** | **7** |

**100.** Hundredths are frequently referred to as **per cent**, a phrase originally meaning *by the hundred.*

**101.** The symbol % stands for hundredths and is read *per cent.*

Thus 45% = .45; 48% of a number = .48 of it.

### ORAL EXERCISE

*Read:*

**1.** 0.073.
**2.** 0.00073.
**3.** 3004.025.
**4.** 300.4025.
**5.** 532.002.
**6.** 60.0625.
**7.** 63.3125.
**8.** 126.8125.
**9.** 31.08%.
**10.** 126.75%.
**11.** 2150.1875.
**12.** 3165.00625.
**13.** 131.3125 T.
**14.** 240.0125 A.
**15.** A tax of 1.0625 mills.
**16.** A tax of 9.1875 mills.

**17.** Read the number in the foregoing numeration table.

**18.** Read the following, using the words "per cent": .17; 28; .85; .67; .425; .37½.

**19.** Read the following as decimals, not using the words "per cent": 25%; 75%; 87%; 62½%; 27.15%.

**20.** Read aloud the following:

*a.* The value of a pound sterling in United States money is $4.8665.

*b.* A meter (metric system of measures) is equal to 39.37079 in.; a kilometer, to 0.62137 mi.

*c.* 1 metric ton is equal to 1.1023 ordinary tons; 1.5 metric tons are equal to 1.65345 ordinary tons.

*d.* A flat steel bar 3 in. wide and 0.5 in. thick weighs 5.118 lb.

*e.* The circumference of a circle is 3.14159 times the length of its diameter.

### WRITTEN EXERCISE

*Write decimally:*

**1.** Five tenths; fifty hundredths; five hundred thousandths.

**2.** Nine hundred and eleven ten-thousandths; nine hundred eleven ten-thousandths; five hundred and two thousandths.

**3.** One hundred seventy-four millionths; one hundred seventy-four million and seven millionths; seven million and one hundred seventy-four millionths.

**4.** Seven thousand and seventy-five ten-thousandths; two hundred fifty-seven ten-millionths; two hundred and forty-six millionths; two hundred forty-six millionths.

**5.** Four million ten thousand ninety-seven ten-millionths; four million ten thousand and ninety-seven ten-millionths; five hundred millionths; five hundred-millionths.

**6.** Six hundred six and five thousand one hundred-thousandths; six hundred six and fifty-one hundred-thousandths; fifty-six and one hundred twenty-eight ten-billionths.

**7.** Seventeen thousand and eighteen hundred seventy-six millionths; seventeen thousand and eighteen hundred seventy-six ten-thousandths; twenty-one hundred sixteen hundredths.

**102.** In the number 2.57 there are 2 integral units, 5 tenths of a unit, and 7 hundredths of a unit. In the number 2.5700 there are 2 integral units, 5 tenths of a unit, 7 hundredths of a unit, 0 thousandths of a unit, and 0 ten-thousandths of a unit. 2.5700 is therefore equal to 2.57. That is,

*Decimal ciphers may be annexed to or omitted from the right of any number without changing its value.*

### ORAL EXERCISE

*Read the following (a) as printed and (b) in their simplest decimal form:*

**1.** 0.700.
**2.** 5.2450.
**3.** 16.010.
**4.** 18.210.
**5.** 0.50.
**6.** 0.00950.
**7.** 0.7000.
**8.** 12.9010.

## ADDITION

### ORAL EXERCISE

**1.** What is the sum of 0.4, 0.05, 0.0065?

**2.** What is the sum of 0.3, 0.021, 0.008?

**3.** Find the sum of seven tenths, forty-four hundredths, and two; of four tenths, twenty-one hundredths, and six thousandths.

**103. Example.** Find the sum of 12.021, 256.12, and 27.5.

Solution. Write the numbers so that their decimal points stand in the same vertical column. Units then come under units, tenths under tenths, and so on. Add as in integral numbers and *place the decimal point in the sum directly under the decimal points in the several numbers added.*

$$\begin{array}{r} 12.021 \\ 256.12\phantom{0} \\ 27.5\phantom{00} \\ \hline 295.641 \end{array}$$

### WRITTEN EXERCISE

*Find the sum of:*

**1.** 7.5, 165.83, 5.127, 6.0015, and 71.215.

**2.** 257.15, 27.132, 5163, 8.000125, and 4100.002.

**3.** 0.175, 5.0031, .00127, 70.2116001, and 21.00725.

**4.** 51.6275, 19.071, 0.000075, 21.00167, and 40,000.01.

**5.** 2.02157, 2.1785, 2500.00025, 157.2165, and 7.0021728.

**6.** Copy, find the totals as indicated, and check:

| | | | | |
|---:|---:|---:|---:|---:|
| $1241.50 | $9215.45 | $1421.12 | $1421.32 | ? |
| 1.52 | 1275.92 | 1.46 | 1618.40 | ? |
| 349.21 | 3725.41 | 2.18 | 1920.41 | ? |
| 2975.47 | 7286.95 | 7.96 | 10.20 | ? |
| 27.14 | 8276.92 | 14.21 | 41.64 | ? |
| 9218.49 | 7271.44 | 1240.80 | 126.18 | ? |
| 5.17 | 8926.95 | 7216.80 | 24.17 | ? |
| 12627.85 | 8972.76 | 4.75 | 240.20 | ? |
| 721.92 | 7214.25 | 8.16 | 960.80 | ? |
| 11.41 | 8142.76 | .47 | 1860.45 | ? |
| 1.21 | 8436.14 | .92 | 9270.54 | ? |
| .72 | 8435.96 | 9.26 | 75.86 | ? |
| 14178.21 | 7926.14 | 1490.75 | 45.95 | ? |
| 2172.14 | 9214.72 | 1860.54 | 75.86 | ? |
| 726.95 | 1241.16 | 9265.80 | 72.18 | ? |
| 85.21 | 4214.71 | 625.50 | 9260.14 | ? |
| 75.92 | 8726.19 | 240.75 | 1.20 | ? |
| 72604.25 | 2140.12 | 60.50 | 7.40 | ? |
| 124.61 | 7146.14 | 120.41 | 8.32 | ? |
| 2114.62 | 7214.86 | 4101.08 | 2860.14 | ? |
| ? | ? | ? | ? | ? |

**7.** Find the sum of twenty-one hundred sixty-five and one hundred sixty-five ten-thousandths, thirty-nine and twelve hundred sixty-five millionths, twenty-seven hundred thirty-six and one millionth, four and six tenths, six hundred and six thousandths, and six hundred sixty-five thousandths.

## SUBTRACTION

### ORAL EXERCISE

**1.** From the sum of 0.7 and 0.4 take 0.5.

**2.** From the sum of 0.07 and 0.21 take 0.006.

**3.** From seventy-four hundredths take six thousandths.

**4.** To the difference between .43 and .03 add the sum of .45 and .007.

**5.** Goods on hand at the beginning of a week, $24.50; goods purchased during the week, $35.50; goods sold during the week, $36; goods on hand at the close of the week, $36.50. What was the gain or loss for the week?

**104. Example.** From 14.27 take 5.123.

$$\begin{array}{r} 14.27 \\ \underline{5.123} \\ 9.147 \end{array}$$

Solution. Write the numbers so that the decimal points stand in the same vertical column. The minuend has not as many places as the subtrahend; hence suppose decimal orders to be annexed until the right-hand figure is of the same order, then subtract as in integers and *place the decimal point in the remainder directly under the decimal points in the numbers subtracted.*

### WRITTEN EXERCISE

*Find the difference between:*

**1.** 7.2154 and 2.8576.

**2.** 17.2157 and 1.0002.

**3.** 1.0005 and .889755.

**4.** $1265.45 and $87.99.

**5.** 9 and 5.2675.

**6.** 16 and 5.0000271.

**7.** .0002 and .000004.

**8.** 24.503 and 17.00021.

**9.** The sum of two numbers is 166.214. If one of the numbers is 40.21, what is the difference between the numbers?

**10.** The minuend is 127.006 and the remainder 15.494. What is the sum of the minuend, subtrahend, and remainder?

**11.** From the sum of ninety-nine ten-thousandths, one hundred fifty-one and five thousandths, two hundred fifty-two and twenty-five millionths, six tenths, and eighteen and one hundred seventy-five thousandths take the sum of twelve hundred fifteen millionths, and one hundred eighty-eight thousandths.

**12.** From the sum of two hundred fifty-seven thousandths and eight and one hundred twenty-six millionths take the sum of five hundred ten thousandths and two and one hundred twenty-four ten-thousandths.

**13.** A merchant had, at the beginning of a year, goods amounting to $8165.95. During the year his purchases amounted to $5265.90 and his sales to $9157.65. At the close of the year he took an account of stock and found that the goods on hand were worth $7216.56. What was his gain or loss for the year?

**14.** A provision dealer had on hand Jan. 1, goods worth $4127.60. His purchases for the year amounted to $4165.95 and his sales to $6256.48. Dec. 31 of the same year his inventory showed that the goods on hand were worth $3972.50. If the amount paid for freight on the goods bought amounted to $237.50, what was his gain or loss on provisions?

**15.** I had on hand Jan. 1, lumber amounting to $4210.60. During the year my purchases amounted to $3126.50, and my sales to $4165.85. I lost by fire lumber valued at $506.75, for which I received from an insurance company $500. Dec. 31, my inventory showed the lumber to be worth $5209.08. How much did I gain or lose on lumber during the year?

**16.** At the beginning of a year my resources were as follows: cash on hand, $1262.50; goods in stock, $1742.85; account against A. M. Eaton, $146.50. At the same time my liabilities were as follows: note outstanding, $156.85; account in favor of Robert Wilson, $521.22. During the year I made an additional investment of $1250.65, and withdrew for private use $275. I sold for cash during the year goods amounting to $1250.75, and bought for cash goods amounting to $530.90; I also paid Robert Wilson $320 to apply on account. At the close of the year my inventory showed goods in stock valued at $750.48. What was my gain or loss for the year and my present worth at the close of the year?

Do not fail to check all problems. No phase of arithmetic is more important.

## MULTIPLICATION

### ORAL EXERCISE

**1.** How many times .4 is 4? .77 is 7.7? .999 is 9.99?

**2.** 44 is how many times .44? 22 is how many times .022? 1 is how many times .001? .01 is how many times .0001?

**3.** Read aloud the following, supplying the missing terms: Removing the decimal point one place to the right multiplies the value of the decimal by —— ; two places, —— the value by —— ; three places, —— the value by ——.

**4.** Multiply 12.1252 by 1000; by 100; by 100,000.

**5.** Multiply $9.375 by 100; by 10,000; by 100,000.

**6.** Multiply 5.15 by 10; by 100; by 1000; by 10,000.

**7.** Multiply .000016 by 1000; by 100,000; by 1,000,000.

**8.** Multiply $67.50 by 10; by 100; by 1000; by 10,000.

**9.** Multiply .0037 by 10; by 100; by 1000; by 10,000,000.

**10.** What part of 1 is .1? of 7 is .7? of 29 is 2.9?

**11.** What part of 84 is .84? of 129 is 1.29? of 1275 is 12.75?

**12.** What part of .6 is .006? of .64 is .0064?

Read aloud the following, supplying the missing terms:

*a.* Each removal of the decimal point one place to the left —— the value of the decimal by 10.

*b.* To divide a decimal by —— is to find one tenth (.1) of it, or to —— it by .1.

**13.** Give a short method for multiplying a number by .1; by .01; by .001; by .0001.

**14.** Multiply .009 by .1; by .01; by .001.

**15.** Multiply 217.59 by .1; by .01; by .001.

**16.** Multiply 54.65 by .01; by .00001; by .000001.

**17.** Multiply 2.375 by .1; by .01; by .001; by .0001.

**18.** Multiply 25.215 by .1; by .01; by .001; by .0001.

**19.** Multiply 2111 by .01; by .001; by .0001; by .00001.

**20.** Compare 2400 × $0.06 with 100 × 24 × $0.06 or with 24 × $6.

**21.** Compare 3000 × 612.251 with 1000 × 3 × 612.251, or with 3 × 612251.

**22.** Multiply 21.25 by 2400.

SOLUTION. 2400 is 24 times 100. Multiply by 100 by removing the decimal point two places to the right. The result is 2125. 24 times 2125 equals 51,000, the required product.

In multiplying begin with either the lowest or the highest digit in the multiplier as shown in the margin.

$$\begin{array}{r} 2125 \\ 24 \\ \hline 8500 \\ 4250\phantom{0} \\ \hline 51000 \end{array} \qquad \begin{array}{r} 2125 \\ 24 \\ \hline 4250\phantom{0} \\ 8500 \\ \hline 51000 \end{array}$$

**23.** Formulate a brief rule for multiplying a decimal by any number of 10's, 100's, 1000's, etc.

**24.** Find the cost of:

*a.* 500 lb. at 18¢.
*b.* 150 lb. at 14¢.
*c.* 200 lb. at 26¢.
*d.* 600 lb. at 29¢.
*e.* 300 lb. at 41¢.
*f.* 400 lb. at $12\frac{1}{2}$¢.
*g.* 900 lb. at 34¢.
*h.* 700 lb. at 51¢.
*i.* 1400 lb. at 5¢.

**105. Examples.** **1.** Multiply 41.127 by 4.

SOLUTION. 41.127 is equal to 41,127 thousandths. 41,127 thousandths multiplied by 4 equals 164,508 thousandths, or 164.508. That is, thousandths multiplied by a whole number must equal thousandths.

$$\begin{array}{r} 41.127 \\ 4 \\ \hline 164.508 \end{array}$$

**2.** Multiply 41.127 by .04.

SOLUTION. The multiplier, .04, is equal to 4 times .01; therefore, multiply by 4 and by .01. Multiplying by 4, as in problem 1, the result is 164.508. Multiplying by .01, by simply moving the decimal point in the product two places to the left, the result is 1.64508.

$$\begin{array}{r} 41.127 \\ .04 \\ \hline 1.64508 \end{array}$$

It will be seen that *the number of decimal places in the product is equal to the decimal places in the multiplicand and multiplier.*

It should not be necessary to memorize the above rule. The student should know at a glance that the product of tenths and tenths is hundredths, of tenths and hundredths is thousandths, and so on.

### ORAL EXERCISE

**1.** In multiplying 24.05 by 3.14 can you tell before multiplying how many integral places there will be in the product? how many decimal places? Explain.

**2.** How many integral places will there be in each of the following products: 2.5 × 4.015? 27.51 × 3.1416? 321.1 × 201.51? 1.421 × 42.267? 126.5 × .01? 1020 × 5.01? .105 × 6? 2.41 × 10.05? How many decimal places will there be in each of the above products?

**3.** What are 400 bbl. of apples worth at $2.12 per barrel? at $1.27½ per barrel?

**4.** I bought 60 lb. of sugar at $0.04½ and gave in payment a five-dollar bill. How much change should I receive?

**5.** A and B are partners in a manufacturing business, A receiving 52 % and B 48 % of the yearly profits. The profits for a certain year are $5000. Of this sum how much should A and B, respectively, receive?

### WRITTEN EXERCISES

*Find the product of:*

**1.** 3.121 × 152.
**2.** 3121 × .152.
**3.** 31.21 × 15.2.
**4.** 12.14 × 265.
**5.** 9.004 × .021.
**6.** .3121 × .0152.
**7.** 2.531 × 31000.
**8.** .1724 × 18000.
**9.** .15539 × 2002.

**10.** A man owned 75% of a gold mine and sold 50% of his share. What is the remainder worth if the value of the whole mine is $425,000?

**11.** A man bought a farm of 240 A. at $137.50 per acre. He sold 75 % of it at $150 per acre, and the remainder at $175 per acre. What was his gain?

**12.** Copy and complete the following table of statistics. Check the results. (The total yield multiplied by the price per bushel should equal the total valuation.)

LARGEST WHEAT-GROWING STATES IN A RECENT YEAR

| STATE | YIELD IN BUSHELS | FARM PRICE PER BUSHEL | FARM VALUATION |
|---|---|---|---|
| North Dakota | 143,820,000 | 92.4 ¢ | ? |
| Kansas | 92,290,000 | 92.4 ¢ | ? |
| Minnesota | 67,038,000 | 92.4 ¢ | ? |
| South Dakota | 52,185,000 | 92.4 ¢ | ? |
| Total | ? | ? | ? |

**13–15.** Make and solve three self-checking problems in multiplication of decimals.

## DIVISION

### ORAL EXERCISE

**1.** Divide by 8: 64 ft., .64, .064, 6.4.
**2.** Divide by 9: 63 in., .63, .063, 6.3.
**3.** Divide by 16: $640, $6.40, 6.4, .64, .064.
**4.** Divide by 15: $15.75, $7.50, $0.75, 30.45, 3.045, .3045.
**5.** Divide 337.5 by 45.

$$
\begin{array}{r l}
7.5 & \\
45\overline{)337.5} & \\
315\phantom{.5} & = 45 \text{ times } 7 \\
\overline{22.5} & \text{undivided} \\
22.5 & = 45 \text{ times } .5
\end{array}
$$

CHECK. 45 times 7.5 = 337.5; hence, the work is correct.

**106.** In the above exercise it is clear that *when the divisor is an integer. each quotient figure is of the same order of units as the right-hand figure of the partial dividend used to obtain it.*

### ORAL EXERCISE

**1.** 500 is how many times 50? $75 is how many times $7.50?

**2.** Divide 50 by 5; 500 by 50. How do the quotients compare?

**3.** Divide 7.50 by 15; $75 by 150. How do the quotients compare?

**4.** 720 is how many times 72? 9 is how many times .9?

**5.** Divide 720 by 9; 72 by .9; 7.2 by .09; .72 by .009.

**107.** It has been seen that multiplying both dividend and divisor by the same number does not change the quotient.

**108.** Therefore, to divide decimals when the divisor is not an integer:

*Multiply both dividend and divisor by the power of 10 that will make the divisor an integer, and divide as in United States money.*

**109.** Divide 0.3375 by 0.45.

.3375 ÷ .45 = 33.75 ÷ 45. 33.75 ÷ 45 = .7, with a remainder of 2.25. 2.25 ÷ 45 = .05. The quotient is therefore .75.

```
      .75
45)33.75
   31 5
    2 25
    2 25
```

Observe that *the divisor may always be made an integer if the decimal point in the dividend is carried to the right as many places as there are decimal places in the divisor.*

Should there be a remainder after using all the decimal places in the dividend, annex decimal ciphers and continue the division as far as is desired.

### ORAL EXERCISE

*Divide :*

**1.** 1 by 1.
**2.** 1 by .1.
**3.** 1 by 10.
**4.** .1 by .1.
**5.** 1 by .01.
**6.** 1 by 100.
**7.** 1 by .001.
**8.** .10 by .10.
**9.** .01 by .01.
**10.** 1 by 1000.
**11.** 1 by .0001.
**12.** 1 by 10,000.
**13.** 1 by .00001.
**14.** .001 by .001.
**15.** 1 by 100,000.
**16.** 1 by .000001.
**17.** .0001 by .0001.
**18.** .00001 by .00001.
**19.** 33 by .11.
**20.** 33 by 110.
**21.** .33 by .11.
**22.** 3.3 by 1.1.
**23.** .0001 by 1.
**24.** 33 by .011.
**25.** 33 by 1100.
**26.** .0001 by .1.
**27.** 3300 by .11.
**28.** 330 by .011.
**29.** 33 by .0011.
**30.** 33 by 11000.
**31.** .0001 by .01.
**32.** .033 by .011.
**33.** .0001 by .001.
**34.** .0033 by .0011.
**35.** .0001 by .0001.
**36.** .0001 by .00001.

### WRITTEN EXERCISE

*Divide :*

**1.** 5842 by .046.
**2.** 2.592 by .108.
**3.** 1.750 by 8750.
**4.** .00338 by .013.
**5.** 1.728 by .0024.
**6.** 2200 by .44.
**7.** 231.6 by 579.
**8.** 950 by 19,000.
**9.** 81.972 by .00009.
**10.** 115.814 by .00079.
**11.** 16 by .0064.
**12.** 1.86 by 31,000.
**13.** 1600 by 64,000.
**14.** .0004 by 20,000.
**15.** 100 by .000001.

*Find the sum of the quotients:*

**16.**
8.1 ÷ .9.
81 ÷ .09.
8.1 ÷ .09.
.81 ÷ 900.
.0081 ÷ 9.
8.1 ÷ 900.
810 ÷ .009.
.0081 ÷ 9000.
81000 ÷ .009.
81 ÷ .000009.
8100 ÷ 90000.
.00081 ÷ 90000.

**17.**
72 ÷ 8.
72 ÷ .8.
7.2 ÷ .8.
72 ÷ .08.
.72 ÷ .08.
72 ÷ .008.
72 ÷ 8000.
72 ÷ .0008.
.072 ÷ .008.
72 ÷ .00008.
.0072 ÷ .0008.
.00072 ÷ .00008.

**18.**
125 ÷ 250.
12.5 ÷ 2.5.
1.25 ÷ 2.5.
12.5 ÷ 250.
125 ÷ 2500.
.125 ÷ .025.
12500 ÷ .25.
125 ÷ 25000.
12500 ÷ .025.
125 ÷ 250000.
.125 ÷ .00025.
12500 ÷ .0025.

**19.**
8.8 ÷ 2.2.
.88 ÷ .22.
88 ÷ .0022.
8.8 ÷ 2200.
880 ÷ 2200.
8.8 ÷ 2.200.
880 ÷ .2200.
8800 ÷ 2200.
880 ÷ 22000.
880 ÷ .00022.
88000 ÷ .0022.
88000 ÷ .00022.

**20.**
17 ÷ 68.
1.7 ÷ 6.8.
.17 ÷ .68.
1.7 ÷ 680.
170 ÷ 680.
.017 ÷ .068.
1.7 ÷ 68000.
1700 ÷ 6800.
1700 ÷ 68000.
.0017 ÷ .0068.
.00017 ÷ .00068.
.000017 ÷ .000068.

**21.**
36 ÷ .072.
3.6 ÷ .072.
.36 ÷ .072.
360 ÷ .072.
.036 ÷ .072.
3.6 ÷ 72000.
36 ÷ 720000.
360 ÷ .00072.
3600 ÷ .0072.
.0036 ÷ .0072.
3.6 ÷ .000072.
.00036 ÷ .00072.

**22.** The product of two numbers is 0.00025. If one of the numbers is 0.0025, what is the other?

**23.** A retailer bought 450 yd. of cloth for $1237.50 and sold it at $3.25 per yard. How much did he gain per yard?

**24.** A drover bought a flock of sheep at the rate of $3.30 per head. He sold them at a profit of $0.20 per head and received $700. How many sheep were there in the flock and what was his gain?

**25.** Copy and complete the following table. Check the results.

LARGEST OAT-GROWING STATES IN A RECENT YEAR

| STATE | YIELD IN BUSHELS | FARM PRICE PER BUSHEL | FARM VALUATION |
|---|---|---|---|
| Illinois | ? | 31 ¢ | 54,818,000 |
| Iowa | ? | 31 ¢ | 58,811,000 |
| Wisconsin | ? | 31 ¢ | 27,119,000 |
| Minnesota | ? | 31 ¢ | 31,926,000 |
| Total | ? | 31 ¢ | ? |

**26–28.** Make and solve three self-checking problems in division of decimals.

## DIVIDING BY POWERS AND MULTIPLES OF TEN

### ORAL EXERCISE

**1.** 6.4 is what part of 64? \$0.17 is what part of \$1.70?

**2.** Compare (as in problem 1) \$240.60 with \$24,060; 17.75 ft. with 1775 ft.

**3.** Compare (as in problem 1) .1 with 1; .01 with 1; .001 with 1; .0001 with 1.

**4.** Read aloud the following, supplying the missing terms: Removing the decimal —— place to the —— divides the value of the decimal by 10; two places, —— the value of the decimal by ——; three places, —— the value of the decimal by ——.

**5.** Compare the quotient of $28 \div .7$ with the quotient of $\overline{28 \times 10} \div \overline{.7 \times 10}$; the quotient of $28 \div .7$ with the quotient of $280 \div 7$.

**6.** Compare the quotient of $16.4 \div 40$ with the quotient of $\overline{16.4 \div 10} \div \overline{40 \div 10}$; the quotient of $16.4 \div 40$ with the quotient of $1.64 \div 4$. What is the quotient of 56.77 divided by 7000?

SOLUTION. Removing the decimal point three places to the left and dropping the ciphers of the divisor is equivalent to dividing both dividend and divisor by 1000 and does not change the value of the quotient.

$$\begin{array}{r} .00811 \\ 7\overline{).05677} \end{array}$$

## Buying and Selling by the Hundred

### ORAL EXERCISE

**1.** Compare $460 \div 100 \times \$2$ with $4.60 \times \$2$.

**2.** Find the cost of 450 lb. of guano at $4 per cwt.

**3.** Find the cost of 600 lb. of wire nails at 34¢ per cwt.

**4.** Find the cost of 4950 paving stones at $8 per C.

Solution. C stands for 100. 4950 paving stones are 49.5 times 100 paving stones. Since 1 hundred paving stones cost $8, 49.5 hundred paving stones will cost 49.5 times $8, or $396.

$$\begin{array}{r} 49.5 \\ 8 \\ \hline 396.0 \end{array}$$

### WRITTEN EXERCISE

*Find the cost:*

| | Quantity | Price per Hundredweight | | Quantity | Price per Hundredweight |
|---|---|---|---|---|---|
| **1.** | 450 lb. | 55¢ | **5.** | 1600 lb. | 71¢ |
| **2.** | 510 lb. | 77¢ | **6.** | 2600 lb. | 15¢ |
| **3.** | 640 lb. | 60¢ | **7.** | 4900 lb. | 70¢ |
| **4.** | 330 lb. | 56¢ | **8.** | 3100 lb. | 88¢ |

## Buying and Selling by the Thousand

### ORAL EXERCISE

**1.** Compare $\overline{3500 \div 1000} \times \$9$ with $3.500 \times \$9$.

**2.** Compare $\overline{12200 \div 1000} \times \$5$ with $12.2 \times \$5$.

**3.** Find the cost of 7150 feet of lumber at $11 per M.

Solution. M stands for thousand. 7150 feet are 7.15 times 1000 feet. Since 1 thousand feet of lumber cost $11, 7.15 thousand feet will cost 7.15 times $11, or $78.65.

$$\begin{array}{r} 7.15 \\ 11 \\ \hline 78.65 \end{array}$$

*Find the cost of:*

**4.** 8500 tiles at $8 per M; at $9 per M.

**5.** 4500 bricks at $6 per M; at $7 per M.

**6.** 7500 shingles at $12 per M; at $14 per M.

**7.** 3200 ft. lumber at $14 per M; at $12 per M.

**8.** 15,000 ft. lumber at $11 per M; at $12 per M.

**9.** 12,000 ft. lumber at $16 per M; at $15 per M.

## Buying and Selling by the Ton of 2000 Pounds

### ORAL EXERCISE

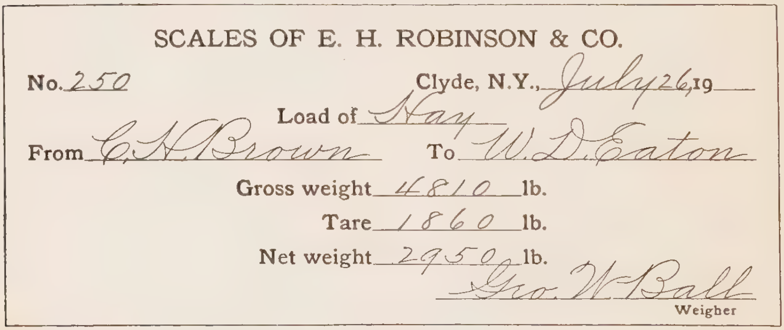
1. Compare $\overline{8000 \div 2000} \times 8$ with $\overline{8000 \div 1000} \times 4$.
2. Compare $\overline{7000 \div 2000} \times 18$ with $7 \times 9$.
3. Find the cost of 4250 lb. coal at $8 per ton.

Solution. 4250 lb. is 4.25 times 1000 lb. If the cost of 2 thousand pounds is $8, the cost of 1 thousand pounds is $4. Since 1 thousand pounds of coal cost $4, 4.25 thousand pounds will cost 4.25 times $4, or $17.

$$\begin{array}{r} 4.25 \\ 4 \\ \hline 17.00 \end{array}$$

### WRITTEN EXERCISE

1. At $9 per ton, find the cost of the hay in the following weigh ticket. Also find the cost at $8.75 per ton.



SCALES OF E. H. ROBINSON & CO.

No. 250 Clyde, N.Y., July 26, 19

Load of Hay

From C. H. Brown To W. D. Eaton

Gross weight 4810 lb.

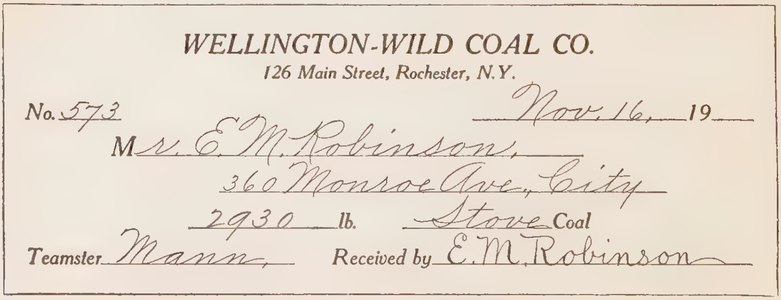
Tare 1860 lb.

Net weight 2950 lb.

Geo. W. Ball

Weigher

2. At $7.50 per ton find the cost of the coal in the following weigh ticket. Also find the cost at $6.95 per ton.



*WELLINGTON-WILD COAL CO.*

*126 Main Street, Rochester, N.Y.*

*No.* 573 Nov. 16, *19*

*M*r. E. M. Robinson,

360 Monroe Ave., City

2930 *lb.* Stove *Coal*

*Teamster* Mann, *Received by* E. M. Robinson

**3.** What will 8650 lb. of hay cost at $12 per ton?

**4.** Find the cost of 2150 lb. of coal at $6 per ton.

**5.** At $32 per ton, what is the cost of 26,480 lb. of phosphate?

**6.** Find the cost of 54,260 pounds of coal at $5.80 per ton.

**7.** Find the cost of 12 loads of coal weighing 4100, 3900, 4306, 4100, 4060, 4300, 3286, 3980, 3850, 4130, 3700, 3950 lb. net, at $5.20 per ton.

**8.** Find the total cost of: 5265 lb. hard coal at $8.40 per ton; 12,200 lb. soft coal at $3 per ton; 8275 lb. cannel coal at $11.75 per ton; 34,160 lb. egg coal at $6.20 per ton; 12,275 lb. nut coal at $5.75 per ton; 8753 lb. grate coal at $5.80 per ton; 24,160 lb. stove coal at $6.50 per ton.

**9.** During the month of January, in a recent year, there were consumed in a manufacturing plant 72 loads of coal weighing as follows: 6100, 6500, 6700, 6840, 7210, 6680, 7250, 8400, 6100, 6100, 6250, 6380, 6480, 6300, 6500, 6160, 6410, 6370, 6410, 6570, 6480, 6240, 6370, 6430, 6480, 6300, 7400, 7580, 7620, 7240, 7110, 7220, 7420, 7480, 6390, 6100, 6250, 6250, 6900, 6270, 6280, 6290, 6270, 6390, 6420, 6120, 6120, 6200, 6300, 6120, 6430, 6430, 8100, 6100, 6200, 6310, 6204, 6160, 6170, 6240, 6390, 6140, 6240, 7190, 7240, 7140, 7200, 6340, 8420, 6310, 7420, 6120 lb. net. Find the cost at $5.87½ per ton.

### WRITTEN REVIEW EXERCISE

**1.** Of what number is 25.56 both the divisor and quotient?

**2.** The sum of the divisor and quotient is 414.06. If the divisor is .6, what is the dividend?

**3.** In what time will 3 boys at $.75 per day earn as much as 2 men earn in 75 da. at $2.25 per day?

**4.** A merchant sold a quantity of flour for $370 and realized a gain of $34. If the selling price was $7.40 per barrel, what was the cost per barrel?

**5.** What number is that which is 165 times as great as 82.5?

**6.** If 450 bbl. of beef sold for $5872.50, what was the selling price per hundred barrels?

**7.** What will be the cost, at 15¢ per yard, of a paper border for a room 8 yd. wide and 12 yd. long?

**8.** An article was sold for $22.50; if $\frac{1}{6}$ of the cost was lost, what was the loss?

**9.** Wood costing $3.50 per cord is sold for $4.10 per cord. How many cords must be handled to gain $240?

**10.** Find the cost of 8 bbl. of pork weighing 280, 281, 286, 290, 285, 277, 285, and 290 lb. net, at $8.50 per hundred pounds.

**11.** A flock of 200 sheep was bought for $700. Ten of the sheep died, and the remainder of the flock was sold at $3.95 per head. What was the gain or loss?

**12.** If the actual cost of the necessities of life for one person, in a given year, amounted to $81.45, and 10 yr. later, owing to the advance in prices, the same necessities cost $108.60, what was the fractional increase in the cost of living?

**13.** A, B, and C bought a stock of goods for $7500, A contributing $2500, B $3000, and C the remainder. They sold the goods for $8400 and divided the profits equally. How much of the $8400 should A, B, and C, respectively, receive?

**14.** The following were the transactions of a merchant for 1 mo.: merchandise on hand July 1, $3378.50, sold for cash, $2374.20; bought for cash, $1945.35; sold on account, $2276.30; bought on account, $876.40; on hand July 31, $2056.35. Expenses for the month, $284.25. What was the net gain for the month?

**15.** The following were the transactions of a merchant for 1 mo.: merchandise on hand January 1, $8120.90; bought for cash, $3265.90; bought on account, $2845.10; sold for cash, $5157.65; sold on account, $4218.25; on hand January 31, $7253.25. The sale to Jas. S. Greet, on account, cannot be collected, $51.20. What was the net gain for the month?

**16.** What is the total freight on 12,250 lb. of hardware at \$.65 per hundredweight and 15,670 lb. of hardware at \$.60 per hundredweight?

**17.** A merchant bought 250 yd. of cloth at \$3.50 per yard, and 150 yd. at \$4.25. At what average price per yard should the whole be sold to realize an average profit of \$1 per yard?

**18.** What is the cost of 25 bbl. of sugar containing 312, 304, 309, 317, 330, 325, 315, 318, 317, 305, 319, 320, 325, 330, 335, 330, 325, 315, 315, 320, 320, 330, 330, 315, 315 lb. net, at $5\frac{3}{8}$¢ per pound?

**19.** A received \$1088 from the sale of his barley crop. If he received \$0.85 per bushel for the barley and his farm produced an average of 32 bu. to the acre, how many acres did it take to produce the barley?

**20.** A manufacturing pay roll shows that 40 hands are employed at \$1.45 per day, 50 hands at \$1.40 per day, 10 hands at \$3 per day, 40 hands at \$2.50 per day, and 5 hands at \$8 per day. Find the average daily wages.

**21.** A hardware merchant found that his stock of goods, Jan. 1, amounted to \$34,350.65. During the year he bought goods amounting to \$211,165.45, and sold goods amounting to \$220,540.45. Dec. 31, he took an account of stock and found that the goods on hand at cost prices were worth \$81,275.64. What was his gain or loss for the year?

**22.** Without copying the following figures, find (*a*) the sum of each line, and (*b*) the sum of each column. Prove the work by adding the line totals and comparing the sum with the sum of the column totals.

| | | | | |
|---:|---:|---:|---:|---:|
| 17.035 | 18.0135 | 186.02 | 126.42 | 6.009 |
| 8.005 | 5.07 | 142.004 | .0634 | 3.14 |
| 32.972 | 18.0981 | 165.42 | 1.7538 | 9.314 |
| 126.83 | 4.931 | .628 | 6.75 | .048 |
| 95.16 | 6.815 | .8467 | 8.41 | .062 |
| 101.215 | 21.214 | 21.221 | 2.61 | 18.641 |

### A REVIEW TEST

*Without copying, find the quotients and the sum of the quotients, in each problem. Time, approximately, forty minutes.*

In these problems the quotient in each division is apparent at a glance, hence the attention is fixed on placing the decimal point.

**1.**

12.5 ÷ 5
1.25 ÷ .5
.125 ÷ .5
.125 ÷ .05
.0125 ÷ 5
125 ÷ .5
12.5 ÷ .05
.0125 ÷ .005
12.5 ÷ .005

**2.**

96 ÷ .4
.96 ÷ .004
9.6 ÷ 4
.96 ÷ .04
9.6 ÷ .04
960 ÷ .4
.096 ÷ .04
960 ÷ .004
9.6 ÷ .04

**3.**

3.9 ÷ .3
39 ÷ .03
3.9 ÷ .003
.0039 ÷ .003
.039 ÷ .0003
390 ÷ .3
3.9 ÷ 3
390 ÷ .03
39 ÷ .003

**4.**

.0065 ÷ 1.3
6.5 ÷ 1.3
.65 ÷ .13
65 ÷ .013
6.5 ÷ 130
65 ÷ 130
6.5 ÷ 1300
.65 ÷ 13
6500 ÷ 130

**5.**

.69 ÷ 23
690 ÷ 2.3
6.9 ÷ 23
6.9 ÷ 2.3
.69 ÷ 230
.0069 ÷ .23
6.9 ÷ 230
.069 ÷ .0023
690 ÷ 2300

**6.**

.085 ÷ .17
.85 ÷ .017
8.5 ÷ .017
.85 ÷ .0017
85 ÷ .17
.085 ÷ .0017
8.5 ÷ 1.7
85 ÷ 1700
8.5 ÷ 170

**7.**

5.7 ÷ .19
57 ÷ .019
570 ÷ 1.9
57 ÷ 190
.0057 ÷ 1.9
5.7 ÷ 190
57 ÷ .19
.57 ÷ 190

**8.**

75 ÷ .25
75 ÷ 2.5
.75 ÷ 2.5
7.5 ÷ 250
.75 ÷ .25
75 ÷ .025
.75 ÷ 250
7.5 ÷ .025

**9.**

55 ÷ 1.1
550 ÷ 1.1
.055 ÷ 110
5.5 ÷ 110
55 ÷ 110
.055 ÷ .011
550 ÷ .11
550 ÷ 1100

# CHAPTER VIII

## FACTORS, DIVISORS, AND MULTIPLES

### FACTORS

#### ORAL EXERCISE

**1.** Name two factors of 63; of 88; of 144; of 128.

**2.** What are the factors of 49? of 77? of 35? of 21?

**3.** Name three factors of 45; of 66; of 24; of 60; of 80.

**4.** Name a factor that is common to 35 and 77; 36, 63, and 81.

**5.** Name three factors that are common to 30, 60, and 210.

**6.** Which of the following numbers have no factors except itself and one? 11, 27, 15, 37, 49, 62, 73, 81, 23.

**110.** An **even number** is an integer of which two is a factor. An **odd number** is an integer of which two is not a factor. A **prime number** is a number that has no integral factor except itself and one. A **composite number** is a number that has one or more integral factors besides itself and one.

Numbers are **mutually prime** when they have no common factor greater than one.

#### WRITTEN EXERCISE

**1.** Make a list of all the odd numbers from 1 to 100 inclusive; of all the prime numbers; of all the even numbers; of all the composite numbers.

#### ORAL EXERCISE

**1.** Is 2 a factor of 28? of 125? of 42? of 49? By what means do you readily determine this?

**2.** Is 5 a factor of 125? of 170? of 224? of 1255? of 1056? By what means do you readily determine this?

**3.** When is a number divisible by 10? by 3? by 9?

## Tests of Divisibility of Numbers

**111.** A number is divisible by:

1. *Two*, when it is even, that is, when it ends with 0, 2, 4, 6, or 8.
2. *Three*, when the sum of its digits is divisible by 3.
3. *Four*, when the number expressed by its two right-hand figures is divisible by 4.
4. *Five*, when it ends with 0 or 5.
5. *Six*, when it is even and the sum of its digits is divisible by 3.
6. *Eight*, when the number expressed by the last three right-hand figures is divisible by 8.
7. *Nine*, when the sum of its digits is divisible by 9.
8. *Ten*, when its right-hand figure is a cipher.

### ORAL EXERCISE

*Name one or more factors of each of the following numbers:*

**1.** 184.
**2.** 2781.
**3.** 1449.
**4.** 638172.
**5.** 6984.
**6.** 2750.
**7.** 8975.
**8.** 71168.
**9.** 51625.
**10.** 83870.
**11.** 13599.
**12.** 123125.
**13.** 14128.
**14.** 66438.
**15.** 31284.
**16.** 17375.

## Factoring

**112. Factoring** is the process of separating a number into its factors.

**113. Example.** Find the prime factors of 780.

$$\begin{array}{r|r} 5 & 780 \\ \hline 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline & 13 \end{array}$$

Solution. Since the number ends in a cipher, divide it by the prime factor 5; since the resulting quotient is an even number, divide it by 2. Since 78 is an even number, divide it by 2; since the sum of the digits in the resulting quotient is divisible by 3, divide by 3. The prime factors are then found to be 5, 2, 2, 3, and 13.

### WRITTEN EXERCISE

*Find the prime factors of:*

**1.** 112.
**2.** 126.
**3.** 288.
**4.** 786.
**5.** 392.
**6.** 315.
**7.** 968.
**8.** 689.
**9.** 1098.
**10.** 408.
**11.** 650.
**12.** 762.
**13.** 2718.
**14.** 3240.
**15.** 3205.
**16.** 6900.
**17.** 2064.
**18.** 7400.

## CANCELLATION

### ORAL EXERCISE

**1.** $(4 \times 15) \div (4 \times 3) = 15 \div 3$. Why?

**2.** Divide $\overline{2 \times 5 \times 7}$ by $\overline{5 \times 2}$; $\overline{8 \times 7 \times 5}$ by $\overline{5 \times 2 \times 7}$.

**3.** $\frac{3 \times 7 \times 8}{7 \times 3} = ?$ $\quad \frac{5 \times 2 \times 8 \times 3}{2 \times 8 \times 3} = ?$ $\quad \frac{2 \times 9 \times 7 \times 5}{5 \times 7 \times 2 \times 3} = ?$

**4.** What effect on the quotient has rejecting equal factors in both dividend and divisor?

**114. Cancellation** is the process of shortening computations by rejecting or canceling equal factors from both dividend and divisor.

**115. Example.** Divide the product of 6, 8, 12, 32, and 84 by the product of 3, 4, 6, and 24.

$$\frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}} \times \overset{2}{\cancel{12}} \times \overset{4}{\cancel{32}} \times \overset{28}{\cancel{84}}}{\cancel{3} \times \cancel{4} \times \cancel{6} \times \underset{\cancel{3}}{\cancel{24}}} = 2 \times 2 \times 2 \times 4 \times 28 = 896.$$

SOLUTION. Do not form the products, but indicate the multiplication by the proper signs and write the divisor below the dividend as shown above. 3, 4, and 6 in the divisor are factors of 6, 8, and 12, respectively, in the dividend; hence, reject 3, 4, and 6 in the divisor and write 2, 2, and 2, respectively, in the dividend; then cancel the common factor 8 from 24 in the divisor and 32 in the dividend, retaining the factors 3 and 4, respectively; next cancel the common factor 3 in the divisor from 84 in the dividend and there remains the uncanceled factors 2, 2, 2, 4, and 28 in the dividend. Hence, the quotient is $2 \times 2 \times 2 \times 4 \times 28$, or 896.

### WRITTEN EXERCISE

**1.** $\overline{14 \times 21 \times 48} \div \overline{7 \times 21 \times 6} = ?$

**2.** $\overline{128 \times 48 \times 88} \div \overline{64 \times 24 \times 4} = ?$

**3.** Divide $128 \times 18 \times 36$ by $64 \times 18 \times 12$.

**4.** $\frac{12 \times 16 \times 24 \times 8 \times 92 \times 28}{6 \times 8 \times 23 \times 7} = ?$

**5.** If 18 T. of hay cost $270, what will 25 T. cost at the same rate?

**6.** How many days' work at $2.75 will pay for 2 A. of land at $110 per acre?

**7.** If 75 bbl. of flour may be made from 375 bu. of wheat, how many bushels will be required to make 120 bbl. of flour?

**8.** If 45 men can complete a certain piece of work in 120 da., how many men can complete the same piece of work in 30 da.?

**9.** The freight on 350 lb. of evaporated apricots is $1.47. At that rate how much freight should be paid on 7350 lb. of evaporated apricots?

**10.** If 15 rm. of paper are required to print 400 copies of a book of 300 pp., how many reams will be required to print 32,000 copies of a book of 300 pp.?

## DIVISORS AND MULTIPLES

### Common Divisors

#### ORAL EXERCISE

**1.** Name a factor that is common to 35 and 49.

**2.** Name two factors that are common to both 48 and 64.

**3.** Name the greatest factor that is common to 75 and 100.

**116.** A **common divisor** is a factor that is common to two or more given numbers. The **greatest common divisor** (g. c. d.) is the greatest factor that is common to two or more given numbers.

**117. Example.** Find the g. c. d. of 24, 84, and 252.

Solutions. (*a*) Separate each of the numbers into its prime factors. The factor 2 occurs twice in all the numbers and the factor 3 once in all the numbers. None of the other factors occur in all the numbers; hence, 2 × 2 × 3, or 12, is the greatest common divisor of 24, 84, and 252.

(*a*)

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ 84 &= 2 \times 2 \times 3 \times 7 \\ 252 &= 2 \times 2 \times 3 \times 3 \times 7 \end{aligned}$$

(*b*) The common prime factors of two or more given numbers may be found by dividing the numbers by their prime factors successively until the quotients contain no common factor, as shown in the margin.

(*b*)

$$\begin{array}{r|l} 2 & 24 - 84 - 252 \\ \hline 2 & 12 - 42 - 126 \\ \hline 3 & 6 - 21 - \phantom{0}63 \\ \hline & 2 - \phantom{0}7 - \phantom{0}21 \end{array}$$

Ever since decimal fractions came into quite general use the subject of greatest common divisor has been stripped of most of its practical value. When fractions like $\frac{1261}{1649}$ were quite generally used, it was necessary to reduce them to their lowest terms before they could be conveniently handled in an operation. For this purpose, the greatest common divisor (here 97) was found and canceled from each term, thus greatly simplifying the fraction (here $\frac{13}{17}$). Now, however, the greatest common divisor of the terms of the fractions used in business is easily found by inspection, and the need for finding the greatest common divisor is slight.

### ORAL EXERCISE

**1.** What is the greatest common divisor of 65 and 75? of 12 and 32? of 75 and 125?

**2.** What is the greatest common divisor of 12, 30, and 96? of 8, 24, and 42? of 36, 90, and 96?

**3.** What divisor should be used in reducing $\frac{75}{125}$ to its lowest terms? $\frac{128}{640}$? $\frac{64}{320}$? $\frac{16}{160}$? $\frac{48}{240}$? $\frac{72}{240}$?

### WRITTEN EXERCISE

*Find the greatest common divisor of:*

**1.** 48, 240. **2.** 42, 28, 144. **3.** 88, 144, 220.

**4.** A real estate dealer has four plots of land which he wishes to divide into the largest number of building lots of the same size. If the plots contain 168, 280, 182, and 252 square rods, respectively, how many square rods will there be in each building lot?

## COMMON MULTIPLES

### ORAL EXERCISE

**1.** Name a multiple of 7; of 9; of 16; of 64.

**2.** Name two other multiples of each of the above numbers.

**3.** Name two multiples that are common to 3 and 4; to 5 and 9; to 8 and 12. Which of the multiples just named is the least common multiple?

**118.** A **common multiple** is any integral number of times two or more given numbers. The **least common multiple** (l. c. m.) of two or more numbers is the least number which is an integral number of times each of the given numbers.

**119. Example.** Find the l. c. m. of 28, 42, and 84.

SOLUTIONS. (*a*) Resolve each of the numbers into its prime factors. The factor 2 occurs twice in 28 and in 84, the factor 3 occurs once in 42 and 84, the factor 7 occurs once in each of the numbers. Therefore, the least common multiple is 2 × 2 × 3 × 7, or 84 ; or

(*a*)

$$\begin{aligned} 28 &= 2 \times 2 \times 7 \\ 42 &= 2 \times 3 \times 7 \\ 84 &= 2 \times 2 \times 3 \times 7 \end{aligned}$$

(*b*) Arrange the numbers in a horizontal line and divide by any prime factor that will exactly divide any two of them. Divide the numbers in the resulting quotient by any prime factor that will divide any two of them, and so continue the operation until quotients are found that are prime to each other. Find the product of the several divisors and the last quotients and the result is the l.c.m. $2 \times 2 \times 3 \times 7 = 84$, the l. c. m.

(*b*)

$$\begin{array}{r|rrr} 2) & 28 & 42 & 84 \\ \hline 2) & 14 & 21 & 42 \\ \hline 3) & 7 & 21 & 21 \\ \hline 7) & 7 & 7 & 7 \\ \hline & 1 & 1 & 1 \end{array}$$

All numbers that are factors of other given numbers may be disregarded in finding the l. c. m. Thus the common multiples of 4, 8, 16, 32, 64, and 80 are the same as the multiples of 64 and 80.

### ORAL EXERCISE

*State the least common multiple of:*

1. 6, 5, and 3.
2. 6, 8, 12, and 24.
3. 4, 5, 15, and 30.
4. 2, 4, 7, 8, 48, 24.
5. 6, 42, 84, 168, 336.
6. 5, 15, 75, 150, 300.

### WRITTEN EXERCISE

*Find the least common multiple of:*

1. 6, 7, 8, and 5.
2. 6, 18, 24, and 84.
3. 12, 24, 36, and 96.
4. 32, 46, 92, and 128.
5. 4, 20, 12, and 48.
6. 62, 78, 30, and 142.
7. 35, 105, 125, and 225.
8. 114, 240, 72, and 320.
9. What number is that of which 2, 3, 5, and 11 are the only prime factors?

# CHAPTER IX

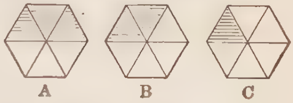
## COMMON FRACTIONS

### ORAL EXERCISE

**1.** When a quantity is divided into 3 equal parts, what is each part called? into 8 equal parts? into 12 equal parts?

**2.** The shaded part of A is what part of the whole hexagon? the shaded part of B? the shaded part of C?

**3.** In the shaded part of A how many sixths? in the shaded part of B?



**4.** One half of the hexagon is how many sixths of it? How many sixths in the whole hexagon?

**5.** In the unshaded part of B how many thirds? Two thirds are how many sixths?

**6.** In the unshaded part of C how many sixths?

**7.** Read the following fractions in the order of their size, the largest first: $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$.

**8.** Complete the following statement: Such parts of a unit as .5, .25, $\frac{1}{3}$, $\frac{5}{6}$, etc., are called ——.

**120. Common fractions** are expressed by two numbers, one written above and one below a short horizontal line.

**121.** The number written above the line is called the **numerator** of the fraction, and the number written below, the **denominator** of the fraction.

**122.** The *numerator* tells the *number of parts* expressed by the fraction; the *denominator names the parts* expressed by the fraction.

Thus, in the fraction $\frac{3}{4}$, 4 tells that a number has been divided into four equal parts and 3 shows that three of these parts have been taken.

**123.** It is clear that the *greater the number* of equal parts into which a unit is divided, the *smaller* is each part; and the *fewer* equal parts into which a unit is divided, the *greater* the size of each part. Hence,

*Of two fractions having the same denominator, the one having the greater numerator expresses the greater value;* and

*Of two fractions having the same numerator, the one having the smaller denominator expresses the greater value.*

**124.** The **terms of a fraction** are the numerator and denominator.

**125.** A **unit fraction** is a fraction whose numerator is one.

Thus $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ are unit fractions. $\frac{1}{3}$ in. is read *one third of an inch.*

**126.** An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator.

Thus, $\frac{6}{6}$, $\frac{5}{3}$, and $\frac{25}{5}$ are improper fractions. The value of an improper fraction is always equal to or greater than one.

**127.** A **mixed number** is the sum of a whole number and a fraction.

Thus, $2\frac{1}{7}$ and $4\frac{2}{5}$, read *two and one seventh* and *four and two fifths*, are mixed numbers.

### ORAL EXERCISE

**1.** What takes the place of the denominator in .5? in .25?

**2.** Read aloud the following fractions in the order of their size, the largest first: $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{100}$.

**3.** Read aloud the following fractions in the order of their size, the smallest first: $\frac{2}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{7}$, $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{10}$, $\frac{1}{6}$, $\frac{4}{7}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{7}$.

**4.** Read aloud the following: $\frac{1}{2}$ mi.; $\frac{3}{4}$ T.; $27\frac{1}{2}$ yd.; $\frac{1}{1728}$ cu. ft.; $275\frac{3}{4}$ A.; $250\frac{5}{16}$ lb.; £ $18\frac{9}{20}$; £ $271\frac{1}{4}$; $\frac{1}{144}$ sq. ft.

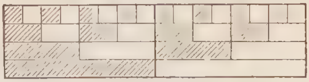
**5.** Of all the cotton produced in the United States in a recent year the principal cotton-growing states contributed approximately as follows: North Carolina, $\frac{1}{14}$; South Carolina, $\frac{1}{8}$; Georgia, $\frac{1}{6}$; Oklahoma and Indian Territory, $\frac{1}{15}$; Alabama, $\frac{1}{9}$; Mississippi, $\frac{1}{13}$; Louisiana, $\frac{1}{40}$; Texas, $\frac{1}{4}$; Arkansas, $\frac{1}{17}$; Tennessee, $\frac{1}{36}$. Name the principal cotton-growing states, in the order of production, for that year.

## REDUCTION

### To Higher Terms

#### ORAL EXERCISE

**1.** How many halves in 1? how many fourths? how many eighths? how many sixteenths?



**2.** How many fourths in $\frac{1}{2}$? how many eighths? how many sixteenths?

**3.** How many eighths in $\frac{1}{4}$? how many sixteenths?

**4.** How many fourths in $\frac{12}{16}$? how many eighths in $\frac{14}{16}$? how many halves in $\frac{8}{16}$?

**5.** What effect is produced upon the value of a fraction by multiplying or dividing both terms of a fraction by the same number?

**6.** Change 14 gal. to quarts. Compare the size of the units in 14 gal. with the size of the units in 56 qt.; the number of units; the value of the two numbers.

**7.** Change $\frac{1}{2}$ to twelfths; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$.

**8.** Name three fractions equal in value to $\frac{1}{8}$; to $\frac{2}{3}$; to $\frac{3}{4}$.

**128.** It has been seen that *multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

**129.** A fraction is reduced to **higher terms** when the given numerator and denominator are expressed in larger numbers.

#### ORAL EXERCISE

**1.** Reduce to twelfths: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$.

**2.** Reduce to sixteenths: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.

**3.** Reduce to twentieths: $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{3}{5}$, $\frac{4}{5}$.

**4.** Reduce to twenty-fourths: $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{4}$, $\frac{7}{8}$.

**5.** Reduce to thirty-seconds: $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{3}{16}$, $\frac{1}{16}$, $\frac{9}{16}$, $\frac{7}{16}$.

**6.** Reduce to one-hundredths: $\frac{3}{4}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{4}{25}$, $\frac{9}{20}$, $\frac{1}{4}$, $\frac{7}{25}$.

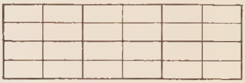
**7.** Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to fractions having the denominator 24.

## To Lowest Terms

### ORAL EXERCISE

**1.** $\frac{8}{24}$ equals how many thirds? $\frac{12}{24}$ equals how many halves?

**2.** Name the largest possible unit fraction. Why is this the largest possible unit fraction?



**3.** Change $\frac{6}{12}$ to the largest possible unit fraction; $\frac{8}{16}$; $\frac{25}{100}$; $\frac{50}{200}$; $\frac{125}{1000}$. Express $\frac{14}{16}$ in its simplest form. Reduce $\frac{25}{200}$ to its lowest terms.

**130.** A fraction is reduced to its **lowest terms** when the numerator and denominator are changed to numbers that are mutually prime.

**131. Example.** Reduce $\frac{96}{108}$ to its lowest terms.

Solution. 6 is a common factor of 96 and 108; dividing both terms by 6, the result is $\frac{16}{18}$. 2 is a common factor of 16 and 18; dividing both terms by 2, the result is $\frac{8}{9}$.

$$\frac{96}{108} = \frac{16}{18} = \frac{8}{9}$$

### ORAL EXERCISE

**1.** Reduce to fifteenths: $\frac{1}{3}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{4}{5}$.

**2.** Reduce to eighths: $\frac{9}{24}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{12}{32}$, $\frac{14}{16}$, $\frac{1}{4}$.

**3.** Reduce to fiftieths: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{24}{100}$, $\frac{7}{10}$, $\frac{8}{25}$, $\frac{16}{200}$.

**4.** Change to twentieths: $\frac{1}{2}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{9}{10}$, $\frac{3}{5}$.

**5.** Reduce to lowest terms: $\frac{6}{16}$, $\frac{8}{10}$, $\frac{8}{12}$, $\frac{14}{16}$, $\frac{9}{12}$, $\frac{6}{8}$.

### WRITTEN EXERCISE

**1.** Reduce to sixteenths: $\frac{110}{160}$, $\frac{180}{320}$, $\frac{7}{8}$, $\frac{25}{80}$, $\frac{3}{4}$, $\frac{160}{640}$.

**2.** Reduce to lowest terms: $\frac{720}{1728}$ cu. ft., $\frac{48}{160}$ A., $\frac{448}{2240}$ T.

**3.** Reduce to lowest terms: $\frac{100}{320}$ mi., £$\frac{165}{240}$, $\frac{4800}{5760}$ lb., $\frac{28}{80}$ mi.

**4.** Reduce to three-hundred-twentieths: $\frac{1}{8}$ mi., $\frac{1}{4}$ mi., $\frac{1}{16}$ mi.

**5.** Reduce to their simplest common fractional form: $\frac{1840}{2240}$ T., $\frac{1260}{2400}$ T., $\frac{92}{160}$ A., $\frac{124}{160}$ A., $\frac{360}{640}$ sq. mi., $\frac{492}{640}$ sq. mi., $\frac{248}{320}$ mi.

## INTEGERS AND MIXED NUMBERS TO IMPROPER FRACTIONS

### ORAL EXERCISE

**1.** How many quarts in 1 gal.? in 3 gal.?

**2.** How many sixths in 1? in 3? in 5? in 7?

**3.** How many fifths in 1? in $1\frac{1}{5}$? in $1\frac{3}{5}$? in $3\frac{1}{5}$?

**4.** Express as fourths: $6\frac{1}{4}$, $12\frac{3}{4}$, 13, 87, $61\frac{1}{4}$, $28\frac{1}{2}$.

**5.** Express as eighths: 15, 12, $10\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{5}{8}$, $1\frac{3}{8}$, $9\frac{7}{8}$.

**6.** Express as halves: 27, 14, $30\frac{1}{2}$, $171\frac{1}{2}$, $182\frac{1}{2}$, 249.

### WRITTEN EXERCISE

*Reduce to improper fractions:*

**1.** $83\frac{1}{3}$.
**2.** $166\frac{2}{3}$.
**3.** $333\frac{1}{3}$.
**4.** $666\frac{2}{3}$.
**5.** $180\frac{5}{32}$.
**6.** $212\frac{1}{64}$.
**7.** $265\frac{1}{48}$.
**8.** $319\frac{5}{16}$.
**9.** $146\frac{15}{32}$.
**10.** $3150\frac{5}{9}$.
**11.** $1625\frac{1}{6}$.
**12.** $2150\frac{5}{12}$.

## IMPROPER FRACTIONS TO INTEGERS OR MIXED NUMBERS

### ORAL EXERCISE

**1.** How many pecks in 240 qt.? $\frac{240}{8} = ?$ $\frac{129}{3} = ?$

**2.** Change to integers: $\frac{160}{4}$, $\frac{153}{9}$, $\frac{192}{16}$, $\frac{288}{12}$, $\frac{4860}{60}$, $\frac{1920}{32}$.

**3.** Express $28\frac{1}{4}$ as fourths; express $\frac{113}{4}$ as a mixed number.

**4.** Change to mixed numbers: $\frac{27}{4}$, $\frac{173}{3}$, $\frac{157}{8}$, $\frac{161}{4}$, $\frac{214}{3}$.

**5.** What is the value of: $\frac{288}{12}$ lb.? $\frac{128}{16}$ lb.? $\frac{168}{4}$ bu.? $\frac{244}{8}$ pk.? $\frac{366}{12}$ ft.? $\frac{4800}{160}$ A.? $\frac{960}{320}$ mi.? $\frac{250}{12}$ lb.? $\frac{720}{144}$ sq. ft.?

### WRITTEN EXERCISE

*Reduce to integers or mixed numbers:*

**1.** $\frac{856}{320}$ mi.
**2.** $\frac{1930}{160}$ A.
**3.** $\frac{9655}{2240}$ T.
**4.** $\frac{1728}{160}$ A.
**5.** $\frac{8275}{2000}$ T.
**6.** $\frac{6923}{2000}$ T.
**7.** $\frac{2456}{16}$ lb.
**8.** $\frac{9504}{1728}$ cu. ft.
**9.** $\frac{9200}{640}$ sq. mi.

**132.** When expressing **final results** reduce all proper fractions to their lowest terms and all improper fractions to integers or mixed numbers.

## To Least Common Denominator

### ORAL EXERCISE

**1.** How many pounds in 1 T. 500 lb.? 5 T. + 1000 lb. = ? lb. 5 T. 1000 lb. = ? T.

**2.** How must numbers be expressed before they can be added or subtracted?

**3.** $\frac{1}{2} = \frac{?}{8}$; $\frac{1}{2} + \frac{3}{8} = ?$ $\frac{1}{4} = \frac{?}{16}$; $\frac{1}{4} - \frac{1}{16} = \frac{?}{16}$; $\frac{1}{3} = \frac{?}{6}$; $\frac{1}{3} - \frac{1}{6} = ?$

**4.** What kind of fractions can be added or subtracted?

**5.** Express $\frac{3}{4}$ as sixteenths. Add $\frac{3}{4}$ and $\frac{5}{16}$; $\frac{1}{4}$ and $\frac{9}{16}$; $\frac{3}{4}$ and $\frac{7}{8}$.

**6.** Express $\frac{1}{2}$ as eighths. Subtract $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{1}{2}$ and $\frac{3}{16}$; $\frac{3}{4}$ and $\frac{1}{16}$.

**133.** Two or more fractions whose denominators are the same are said to have a **common denominator**; if this denominator is the smallest possible, the fractions are said to have a **least common denominator**. Two or more fractions having the same denominator are sometimes called **similar fractions**.

### ORAL EXERCISE

*Change to similar fractions:*

**1.** $\frac{1}{2}, \frac{1}{4}$.
**2.** $\frac{1}{2}, \frac{1}{3}$.
**3.** $\frac{1}{3}, \frac{1}{5}$.
**4.** $\frac{2}{3}, \frac{1}{2}$.
**5.** $\frac{3}{5}, \frac{4}{10}$.
**6.** $\frac{1}{5}, \frac{1}{4}$.
**7.** $\frac{2}{3}, \frac{3}{4}$.
**8.** $\frac{1}{2}, \frac{4}{5}$.
**9.** $\frac{3}{5}, \frac{2}{3}$.
**10.** $\frac{2}{5}, \frac{7}{8}$.
**11.** $\frac{2}{3}, \frac{7}{9}$.
**12.** $\frac{1}{2}, \frac{3}{16}$.
**13.** $\frac{2}{5}, \frac{7}{10}$.
**14.** $\frac{3}{4}, \frac{9}{10}$.
**15.** $\frac{3}{8}, \frac{7}{10}$.
**16.** $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$.
**17.** $\frac{1}{3}, \frac{1}{9}, \frac{4}{5}$.
**18.** $\frac{1}{4}, \frac{1}{8}, \frac{5}{32}$.
**19.** $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
**20.** $\frac{1}{2}, \frac{3}{8}, \frac{5}{16}$.

### WRITTEN EXERCISE

*Change to fractions having the least common denominator:*

**1.** $\frac{7}{8}, \frac{5}{32}, \frac{18}{64}$.
**2.** $\frac{3}{5}, \frac{9}{10}, \frac{1}{25}$.
**3.** $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{9}$.
**4.** $\frac{2}{5}, \frac{7}{9}, \frac{3}{10}, \frac{2}{3}$.
**5.** $\frac{5}{6}, \frac{7}{8}, \frac{5}{12}, \frac{7}{32}$.
**6.** $\frac{2}{3}, \frac{3}{5}, \frac{7}{36}, \frac{5}{48}$.
**7.** $\frac{1}{4}, \frac{7}{16}, \frac{9}{32}, \frac{17}{64}$.
**8.** $\frac{9}{10}, \frac{5}{24}, \frac{7}{12}, \frac{1}{8}$.
**9.** $\frac{1}{12}, \frac{5}{9}, \frac{2}{3}, \frac{1}{48}$.
**10.** $\frac{11}{16}, \frac{5}{32}, \frac{1}{4}, \frac{19}{64}$.
**11.** $\frac{15}{320}, \frac{3}{4}, \frac{17}{160}, \frac{5}{8}$.
**12.** $\frac{10}{640}, \frac{6}{16}, \frac{1}{32}, \frac{5}{32}$.

*Change the fractions to form for addition or subtraction:*

**13.** $31\frac{5}{16}, 7\frac{1}{10}$.
**14.** $134\frac{1}{64}, 112\frac{5}{20}$.
**15.** $6126\frac{5}{12}, 178\frac{5}{16}$.

## ADDITION

**134.** It has been seen that *only like numbers and parts of like units can be added.*

### ORAL EXERCISE

*State the sum of:*

**1.** $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.
**2.** $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{4}$.
**3.** $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$.
**4.** $\frac{2}{11}$, $\frac{5}{11}$, $\frac{6}{11}$.
**5.** $\frac{1}{5}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{7}{5}$.
**6.** $\frac{1}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$.
**7.** $2\frac{1}{2}$, $3\frac{1}{2}$, $12\frac{1}{2}$, $19\frac{1}{2}$.
**8.** $5\frac{1}{4}$, $12\frac{1}{4}$, $7\frac{1}{4}$, $10\frac{1}{4}$.
**9.** $7\frac{7}{8}$, $2\frac{5}{8}$, $8\frac{1}{8}$, $1\frac{1}{8}$, $2\frac{1}{8}$.
**10.** $2\frac{1}{3}$, $5\frac{2}{3}$, $8\frac{1}{3}$, $12\frac{1}{3}$, $10\frac{2}{3}$.
**11.** $1\frac{1}{6}$, $10\frac{5}{6}$, $15\frac{1}{6}$, $18\frac{1}{6}$, $12\frac{1}{6}$.
**12.** $5\frac{7}{16}$, $2\frac{9}{16}$, $1\frac{5}{16}$, $8\frac{3}{16}$, $3\frac{1}{16}$.

*By horizontal addition find the sum of:*

**13.** 2 pieces of gingham containing $41^1$ and $43^2$ yd.

In the dry-goods business fourths (quarters) are very common fractions. They are usually written without denominators by placing the numerators a little above the integers. Thus, $51^1$ equals $51\frac{1}{4}$, $54^2$ equals $54\frac{2}{4}$ ($54\frac{1}{2}$), and $52^3$ equals $52\frac{3}{4}$.

**14.** 4 pc. stripe containing $42^1$, $38^1$, $40^2$, and 49 yd.
**15.** 3 pc. fancy plaid containing $42^1$, $40^2$, and 41 yd.
**16.** 4 pc. duck containing $48^1$, $47^3$, $46^2$, and $40^2$ yd.
**17.** 2 pc. monument cotton containing $54^2$ and $55^2$ yd.
**18.** 4 pc. dress silk containing $32^1$, $34^2$, $35^3$, and $32^2$ yd.

**135. Examples.** **1.** Find the sum of $\frac{7}{8}$ and $\frac{2}{5}$.

SOLUTION. $\frac{7}{8}$ and $\frac{2}{5}$ are not similar fractions; hence, make them similar by reducing them to equivalent fractions having a least common denominator. $\frac{7}{8} = \frac{35}{40}$ and $\frac{2}{5} = \frac{16}{40}$. $\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$.

l. c. m. of 8 and 5 = 40

$$\frac{7}{8} = \frac{35}{40}; \quad \frac{2}{5} = \frac{16}{40}$$
$$\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$$

**2.** Find the sum of $56\frac{1}{3}$, $34\frac{1}{8}$, $52\frac{3}{4}$.

SOLUTION. By inspection determine the least common denominator of the given fractions; then make the fractions similar and add them, as shown in the margin. The result is $1\frac{5}{24}$, which added to the sum of the integers equals $143\frac{5}{24}$, the required result.

$$\begin{array}{ll} 56\frac{1}{3} & = \phantom{1}8 \\ 34\frac{1}{8} & = \phantom{1}3 \\ 52\frac{3}{4} & = \underline{18} \\ 143\frac{5}{24} & \phantom{=}\ \frac{29}{24} = 1\frac{5}{24}. \end{array}$$

### WRITTEN EXERCISE

*Find the sum of:*

**1.** $\frac{7}{16}$, $\frac{5}{8}$.

**2.** $\frac{9}{16}$, $\frac{17}{64}$.

**3.** $2\frac{1}{8}$, $17\frac{1}{4}$.

**4.** $12\frac{1}{2}$, $19\frac{5}{16}$.

**5.** $\frac{2}{3}$, $4\frac{1}{5}$, $19\frac{1}{4}$.

**6.** $2\frac{1}{2}$, $4\frac{3}{4}$, $25\frac{9}{16}$.

**7.** $12\frac{3}{5}$, $172\frac{7}{10}$.

**8.** $8\frac{1}{4}$, $\frac{3}{5}$, $\frac{19}{10}$, $27\frac{1}{8}$.

**9.** $52\frac{5}{8}$, $59\frac{1}{2}$, $57\frac{3}{4}$, $52\frac{1}{16}$.

**10.** $60\frac{5}{8}$, $18\frac{3}{4}$, $21\frac{5}{16}$, $142\frac{1}{16}$.

**11.** $20\frac{1}{4}$, $12\frac{1}{5}$, $18\frac{1}{3}$, $92\frac{1}{5}$, $75\frac{3}{7}$.

**12.** $140\frac{3}{4}$, $260\frac{1}{8}$, $145\frac{1}{2}$, $216\frac{7}{8}$, $390\frac{1}{3}$.

**13.** A carpet dealer sold at different times $125\frac{5}{8}$ yd., $272\frac{1}{4}$ yd., $169\frac{1}{2}$ yd., $186\frac{3}{4}$ yd., $241\frac{1}{4}$ yd., $265\frac{1}{8}$ yd., $296\frac{7}{8}$ yd., and $314\frac{5}{8}$ yd. of Axminster carpet, at \$2.65 per yard. If it cost him \$2.45 per yard, what was his gain?

**14.** A dry-goods merchant bought 50 pc. of dress silk at \$1 per yard. If the pieces contained $42^1$, $43^2$, $44^2$, $47^3$, $44^1$, $45^2$, $40^3$, $46^2$, $45^1$, 42, $47^1$, $48^2$, $40^3$, $40^1$, $40^2$, $40^3$, $50^2$, $40^3$, $47^2$, $48^3$, $40^3$, $45^1$, $40^2$, $45^2$, $44^2$, $47^3$, $46^2$, $41^1$, $51^3$, $42^3$, $53^2$, $57^2$, $53^1$, $51^1$, $48^3$, $47^2$, $40^1$, $45^2$, $45^2$, $40^3$, $40^1$, $45^3$, $47^2$, $48^1$, $51^1$, $52^2$, $57^2$, $61^3$, $60^2$, $50^1$ yd., respectively, and he sold the entire purchase at \$1.25 per yard, what was his gain?

## SHORT METHODS IN ADDITION

### ORAL EXERCISE

**1.** $\frac{1}{7}+\frac{1}{9}=\frac{16}{63}$. Observe that the numerator of the sum is equal to the sum of the denominators in the given fractions.

**2.** $\frac{1}{5}+\frac{1}{8}=?$ Give a short method for adding any two simple fractions whose numerators are 1.

**3.** $\frac{2}{7}+\frac{2}{5}=\frac{24}{35}$. Observe that the numerator of the sum is equal to the sum of the denominators multiplied by the numerator of either of the given fractions.

**4.** $\frac{2}{3}+\frac{2}{7}=?$ Give a short method for adding any two fractions whose numerators are alike.

**5.** Find the sum of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

SOLUTION. $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$; $\frac{7}{12}+\frac{1}{5}=\frac{47}{60}$, the required result.

### ORAL EXERCISE

*State the sum of:*

**1.** $\frac{1}{2}$, $\frac{1}{3}$.
**2.** $\frac{1}{4}$, $\frac{1}{5}$.
**3.** $\frac{1}{5}$, $\frac{1}{6}$.
**4.** $\frac{1}{7}$, $\frac{1}{8}$.
**5.** $\frac{1}{9}$, $\frac{1}{6}$.
**6.** $\frac{1}{3}$, $\frac{1}{5}$.
**7.** $\frac{1}{2}$, $\frac{1}{8}$.
**8.** $\frac{1}{4}$, $\frac{1}{6}$.
**9.** $\frac{1}{9}$, $\frac{1}{7}$.
**10.** $\frac{2}{7}$, $\frac{2}{9}$.
**11.** $\frac{2}{5}$, $\frac{2}{7}$.
**12.** $\frac{2}{5}$, $\frac{2}{3}$.
**13.** $\frac{1}{4}$, $\frac{1}{3}$.
**14.** $\frac{3}{8}$, $\frac{3}{7}$.
**15.** $\frac{4}{5}$, $\frac{4}{9}$.
**16.** $\frac{5}{8}$, $\frac{5}{9}$.
**17.** $\frac{3}{4}$, $\frac{3}{5}$.
**18.** $\frac{4}{7}$, $\frac{4}{5}$.
**19.** $\frac{7}{9}$, $\frac{7}{8}$.
**20.** $\frac{6}{7}$, $\frac{6}{10}$.
**21.** $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{3}$.
**22.** $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{2}$.
**23.** $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$.
**24.** $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{9}$.

**136.** The most common business fractions are usually small and of such a nature that they may be added with equally as much ease as integers. The following exercise will be found helpful to the student in learning to add these fractions in practically the same manner that he adds integers.

**137. Example.** Find the sum of $\frac{5}{16}$, $\frac{1}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$.

SOLUTION. By inspection determine that the least common denominator is 16. Then mentally reduce each fraction to 16ths and add as in whole numbers. Thus, 5, 7, 19, $\frac{27}{16}$, $1\frac{11}{16}$.

### ORAL EXERCISE

*Find the sum of:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{5}$ | $\frac{5}{6}$ | $\frac{1}{2}$ | $\frac{2}{9}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{5}{8}$ | $\frac{7}{9}$ |
| $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{2}{3}$ | $\frac{1}{8}$ | $\frac{3}{4}$ | $\frac{7}{9}$ |
| $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{10}$ | $\frac{1}{3}$ | $\frac{3}{8}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ |
| $\frac{1}{2}$ | $\frac{1}{9}$ | $\frac{3}{10}$ | $\frac{2}{3}$ | $\frac{5}{8}$ | $\frac{5}{6}$ | $\frac{3}{5}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{5}{6}$ |

| **11.** | **12.** | **13.** | **14.** | **15.** | **16.** | **17.** | **18.** | **19.** | **20.** |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{3}$ | $\frac{4}{5}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{8}$ | $\frac{7}{8}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{4}{5}$ | $\frac{1}{2}$ |
| $\frac{3}{5}$ | $\frac{3}{5}$ | $\frac{5}{6}$ | $\frac{3}{7}$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{3}{4}$ | $\frac{1}{5}$ | $\frac{1}{2}$ | $\frac{1}{5}$ |
| $\frac{1}{5}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{7}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{5}{6}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{2}$ |
| $\frac{4}{5}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{4}{5}$ | $\frac{5}{4}$ |
| $\frac{2}{3}$ | $\frac{7}{10}$ | $\frac{3}{8}$ | $\frac{3}{7}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{8}$ | $\frac{4}{5}$ | $\frac{3}{5}$ | $\frac{3}{5}$ |
| $\frac{1}{5}$ | $\frac{3}{10}$ | $\frac{1}{12}$ | $\frac{5}{7}$ | $\frac{1}{6}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{6}$ | $\frac{1}{5}$ | $\frac{7}{10}$ |
| $\frac{1}{3}$ | $\frac{9}{10}$ | $\frac{5}{12}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{5}{16}$ | $\frac{7}{8}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{3}{10}$ |
| $\frac{4}{5}$ | $\frac{7}{10}$ | $\frac{7}{12}$ | $\frac{1}{3}$ | $\frac{5}{6}$ | $\frac{11}{16}$ | $\frac{5}{12}$ | $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{10}$ |

Exercises similar to the foregoing should be continued until the student can name the successive results in the addition without hesitation.

**138.** The ordinary mixed numbers that come to an accountant should be arranged for addition practically the same as integers. In adding, the fractions should be combined first and then the integers.

**139. Example.** Find the sum of $2\frac{1}{2}$, $5\frac{1}{4}$, and $3\frac{7}{8}$.

$$\begin{array}{r} 2\frac{1}{2} \\ 5\frac{1}{4} \\ \underline{3\frac{7}{8}} \\ 11\frac{5}{8} \end{array}$$

Solution. By inspection determine that the least common denominator of the fractions is 8. Mentally find the sum of the fractions and the result is $1\frac{5}{8}$. Add this result to the integers and the entire sum is $11\frac{5}{8}$.

## ORAL EXERCISE

*State the sum of:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** |
|---|---|---|---|---|---|---|---|---|---|
| $2\frac{1}{2}$ | $3\frac{1}{3}$ | $3\frac{1}{4}$ | $8\frac{1}{5}$ | $14\frac{1}{2}$ | $5\frac{3}{4}$ | $4\frac{1}{5}$ | $2\frac{7}{8}$ | $3\frac{1}{3}$ | $14\frac{1}{7}$ |
| $3\frac{1}{4}$ | $2\frac{1}{6}$ | $5\frac{1}{3}$ | $7\frac{1}{2}$ | $17\frac{3}{4}$ | $13\frac{1}{2}$ | $7\frac{1}{3}$ | $16\frac{2}{5}$ | $17\frac{1}{8}$ | $16\frac{2}{9}$ |

| **11.** | **12.** | **13.** | **14.** | **15.** | **16.** | **17.** | **18.** | **19.** | **20.** |
|---|---|---|---|---|---|---|---|---|---|
| $9\frac{3}{4}$ | $5\frac{2}{3}$ | $1\frac{1}{6}$ | $1\frac{3}{7}$ | $8\frac{1}{5}$ | $4\frac{1}{4}$ | $5\frac{1}{3}$ | $4\frac{1}{9}$ | $4\frac{1}{5}$ | $4\frac{1}{2}$ |
| $10\frac{1}{2}$ | $4\frac{1}{4}$ | $2\frac{5}{6}$ | $6\frac{1}{2}$ | $3\frac{3}{5}$ | $2\frac{1}{3}$ | $2\frac{2}{3}$ | $5\frac{2}{3}$ | $2\frac{2}{3}$ | $1\frac{3}{8}$ |
| $13\frac{1}{4}$ | $11\frac{1}{2}$ | $7\frac{7}{12}$ | $5\frac{1}{2}$ | $2\frac{1}{10}$ | $4\frac{1}{5}$ | $4\frac{1}{9}$ | $6\frac{5}{6}$ | $6\frac{4}{5}$ | $7\frac{3}{4}$ |
| $10\frac{1}{4}$ | $12\frac{1}{12}$ | $8\frac{5}{12}$ | $13\frac{4}{7}$ | $4\frac{9}{10}$ | $3\frac{3}{4}$ | $6\frac{7}{9}$ | $2\frac{1}{3}$ | $3\frac{3}{10}$ | $12\frac{1}{4}$ |

| **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
|---|---|---|---|---|---|---|---|---|---|
| $4\frac{1}{2}$ | $1\frac{1}{2}$ | $5\frac{1}{6}$ | $3\frac{1}{4}$ | $8\frac{1}{4}$ | $4\frac{1}{2}$ | $5\frac{2}{3}$ | $4\frac{7}{9}$ | $4\frac{1}{2}$ | $4\frac{1}{8}$ |
| $5\frac{1}{4}$ | $5\frac{2}{3}$ | $2\frac{2}{3}$ | $1\frac{3}{7}$ | $6\frac{3}{4}$ | $2\frac{1}{8}$ | $1\frac{1}{6}$ | $1\frac{1}{9}$ | $2\frac{1}{5}$ | $6\frac{1}{2}$ |
| $3\frac{3}{4}$ | $5\frac{3}{4}$ | $2\frac{5}{6}$ | $2\frac{5}{7}$ | $7\frac{1}{5}$ | $4\frac{1}{5}$ | $9\frac{5}{6}$ | $7\frac{5}{6}$ | $6\frac{2}{3}$ | $9\frac{3}{8}$ |
| $6\frac{7}{8}$ | $5\frac{2}{7}$ | $7\frac{3}{4}$ | $6\frac{1}{2}$ | $2\frac{3}{5}$ | $3\frac{1}{4}$ | $4\frac{1}{9}$ | $5\frac{1}{3}$ | $6\frac{4}{5}$ | $7\frac{5}{8}$ |
| $3\frac{1}{2}$ | $1\frac{1}{2}$ | $1\frac{1}{4}$ | $7\frac{1}{2}$ | $3\frac{1}{5}$ | $3\frac{1}{2}$ | $7\frac{5}{9}$ | $2\frac{2}{3}$ | $9\frac{3}{5}$ | $8\frac{3}{4}$ |
| $4\frac{1}{4}$ | $1\frac{1}{3}$ | $2\frac{5}{6}$ | $2\frac{5}{7}$ | $5\frac{1}{5}$ | $3\frac{1}{8}$ | $3\frac{7}{9}$ | $7\frac{1}{6}$ | $4\frac{1}{5}$ | $2\frac{1}{2}$ |
| $3\frac{1}{8}$ | $4\frac{1}{3}$ | $8\frac{1}{12}$ | $3\frac{4}{7}$ | $2\frac{1}{4}$ | $3\frac{1}{5}$ | $2\frac{2}{3}$ | $9\frac{5}{6}$ | $3\frac{3}{10}$ | $6\frac{1}{4}$ |
| $5\frac{1}{8}$ | $2\frac{7}{12}$ | $8\frac{5}{12}$ | $13\frac{3}{7}$ | $5\frac{7}{10}$ | $3\frac{1}{4}$ | $12\frac{1}{3}$ | $15\frac{2}{3}$ | $9\frac{7}{10}$ | $12\frac{1}{2}$ |

Exercises similar to the above should be continued until the student can add with great facility. If the principles of grouping have not been well mastered, simple addition should be carefully reviewed.

### WRITTEN EXERCISE

*Copy or write from dictation and find the sum of:*

| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** |
|---|---|---|---|---|---|
| $1649\frac{1}{3}$ | $1672\frac{3}{4}$ | $1436\frac{1}{2}$ | $2110\frac{1}{3}$ | $6214\frac{1}{8}$ | $1214\frac{1}{3}$ |
| $4372\frac{1}{6}$ | $1485\frac{7}{8}$ | $1390\frac{3}{4}$ | $1640\frac{1}{9}$ | $1745\frac{1}{4}$ | $2167\frac{1}{4}$ |
| $8431\frac{2}{3}$ | $1635\frac{1}{4}$ | $2415\frac{1}{5}$ | $3680\frac{1}{2}$ | $3146\frac{3}{4}$ | $3159\frac{1}{6}$ |
| $5132\frac{7}{9}$ | $1264\frac{1}{2}$ | $1867\frac{1}{2}$ | $4590\frac{7}{9}$ | $1864\frac{1}{2}$ | $9275\frac{1}{4}$ |
| $1654\frac{1}{3}$ | $1269\frac{3}{4}$ | $1639\frac{1}{8}$ | $2169\frac{2}{9}$ | $2839\frac{5}{8}$ | $7215\frac{3}{4}$ |
| $1831\frac{5}{6}$ | $1748\frac{1}{8}$ | $4136\frac{3}{5}$ | $8432\frac{2}{3}$ | $6241\frac{7}{8}$ | $5261\frac{1}{2}$ |
| $1831\frac{3}{4}$ | $1936\frac{5}{8}$ | $1652\frac{1}{4}$ | $4041\frac{1}{2}$ | $4036\frac{3}{4}$ | $7215\frac{2}{3}$ |
| $1462\frac{1}{2}$ | $5413\frac{1}{8}$ | $3116\frac{1}{2}$ | $6542\frac{5}{6}$ | $8130\frac{5}{16}$ | $5144\frac{3}{4}$ |
| $1851\frac{1}{4}$ | $2114\frac{7}{16}$ | $1439\frac{1}{10}$ | $1862\frac{2}{3}$ | $2148\frac{7}{16}$ | $6257\frac{2}{3}$ |
| $1114\frac{7}{12}$ | $1116\frac{9}{16}$ | $2243\frac{7}{10}$ | $3246\frac{8}{9}$ | $1439\frac{9}{16}$ | $2186\frac{5}{12}$ |

| **7.** | **8.** | **9.** | **10.** | **11.** | **12.** |
|---|---|---|---|---|---|
| $9124\frac{1}{4}$ | $7249\frac{1}{5}$ | $1649\frac{1}{6}$ | $7529\frac{1}{5}$ | $7365\frac{1}{5}$ | $2814\frac{1}{2}$ |
| $2716\frac{1}{2}$ | $2724\frac{4}{5}$ | $2724\frac{1}{2}$ | $6214\frac{1}{2}$ | $2614\frac{1}{9}$ | $2910\frac{1}{4}$ |
| $2514\frac{1}{3}$ | $8692\frac{1}{2}$ | $8695\frac{1}{3}$ | $1825\frac{4}{5}$ | $1583\frac{1}{3}$ | $2817\frac{1}{8}$ |
| $2967\frac{1}{2}$ | $2476\frac{1}{4}$ | $1565\frac{1}{9}$ | $8614\frac{3}{5}$ | $1695\frac{1}{5}$ | $2714\frac{1}{8}$ |
| $2964\frac{1}{4}$ | $8695\frac{1}{2}$ | $2724\frac{7}{9}$ | $9215\frac{3}{5}$ | $1762\frac{1}{9}$ | $2913\frac{7}{8}$ |
| $6875\frac{1}{3}$ | $6214\frac{1}{4}$ | $8619\frac{1}{3}$ | $6719\frac{3}{4}$ | $1875\frac{5}{9}$ | $2874\frac{5}{8}$ |
| $8875\frac{3}{4}$ | $7241\frac{1}{2}$ | $2924\frac{2}{3}$ | $8516\frac{1}{4}$ | $1629\frac{7}{9}$ | $2619\frac{3}{8}$ |
| $2658\frac{1}{2}$ | $8614\frac{1}{2}$ | $6529\frac{1}{9}$ | $7528\frac{1}{8}$ | $7214\frac{2}{9}$ | $1472\frac{1}{8}$ |
| $8425\frac{2}{3}$ | $4725\frac{1}{4}$ | $8592\frac{1}{2}$ | $7216\frac{5}{8}$ | $2510\frac{1}{9}$ | $2615\frac{3}{4}$ |
| $8273\frac{3}{4}$ | $1649\frac{1}{4}$ | $2725\frac{1}{9}$ | $6729\frac{1}{8}$ | $2625\frac{2}{3}$ | $1813\frac{1}{4}$ |
| $1782\frac{3}{4}$ | $1286\frac{1}{2}$ | $8647\frac{2}{9}$ | $3514\frac{1}{4}$ | $8614\frac{1}{3}$ | $1962\frac{1}{2}$ |
| $8695\frac{1}{4}$ | $6248\frac{1}{4}$ | $8725\frac{5}{9}$ | $1686\frac{3}{4}$ | $2729\frac{1}{5}$ | $1862\frac{1}{2}$ |
| $2472\frac{1}{2}$ | $1286\frac{1}{2}$ | $6219\frac{7}{9}$ | $1725\frac{1}{8}$ | $2816\frac{1}{3}$ | $1759\frac{1}{2}$ |
| $6273\frac{1}{6}$ | $8537\frac{3}{4}$ | $8413\frac{1}{3}$ | $2538\frac{3}{8}$ | $2814\frac{1}{3}$ | $2864\frac{1}{4}$ |
| $9685\frac{5}{6}$ | $6982\frac{1}{4}$ | $7226\frac{5}{6}$ | $1758\frac{5}{8}$ | $2716\frac{2}{3}$ | $1624\frac{1}{4}$ |
| $9685\frac{11}{12}$ | $3685\frac{1}{5}$ | $1825\frac{1}{6}$ | $2752\frac{7}{8}$ | $1762\frac{1}{9}$ | $1729\frac{1}{2}$ |
| $1925\frac{5}{12}$ | $2614\frac{3}{5}$ | $4725\frac{1}{3}$ | $2114\frac{1}{5}$ | $1875\frac{1}{9}$ | $1805\frac{3}{4}$ |
| $4212\frac{1}{12}$ | $8796\frac{4}{5}$ | $2816\frac{2}{3}$ | $2216\frac{1}{4}$ | $2614\frac{2}{3}$ | $1721\frac{7}{8}$ |
| $2729\frac{1}{12}$ | $1592\frac{3}{4}$ | $2519\frac{1}{3}$ | $1872\frac{1}{5}$ | $2075\frac{1}{3}$ | $1465\frac{3}{8}$ |

## SUBTRACTION

### ORAL EXERCISE

**1.** 172 A. – 154 A. = ? $\frac{5}{7} - \frac{1}{7} = ?$ 1 bu. – 3 pk. = ?

**2.** Find the difference between $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{3}{7}$ and $\frac{2}{5}$.

**140.** It is clear that *only like numbers and parts of like units can be subtracted.*

**141. Examples.** **1.** Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

Solution. The given fractions must be reduced to equivalent fractions having a least common denominator. The least common denominator is 24. $\frac{7}{8} = \frac{21}{24}$ and $\frac{5}{12} = \frac{10}{24}$. $\frac{21}{24} - \frac{10}{24} = \frac{11}{24}$, the required result.

**2.** From $21\frac{1}{3}$ take $17\frac{1}{2}$.

Solution. Change the given fractions to similar fractions as in example 1. $\frac{3}{6}$ cannot be subtracted from $\frac{2}{6}$, hence 1 is taken from 21 and mentally united to $\frac{2}{6}$, making $\frac{8}{6}$. $\frac{3}{6}$ from $\frac{8}{6}$ leaves $\frac{5}{6}$, and 17 from 20 leaves 3. The required result is therefore $3\frac{5}{6}$.

### ORAL EXERCISE

*Find the value of:*

**1.** $2\frac{3}{4} - \frac{1}{3}$.
**2.** $2\frac{5}{7} - \frac{1}{4}$.
**3.** $3\frac{7}{9} - \frac{4}{5}$.
**4.** $7\frac{5}{8} - 1\frac{3}{8}$.
**5.** $4\frac{5}{7} - 1\frac{2}{3}$.
**6.** $6\frac{7}{8} - 4\frac{5}{16}$.
**7.** $7\frac{1}{8} - 3\frac{5}{16}$.
**8.** $12\frac{1}{4} - 6\frac{1}{8}$.
**9.** $30 - 11\frac{7}{8}$.
**10.** $45 - 16\frac{3}{4}$.
**11.** $11\frac{1}{2} - 6\frac{2}{3}$.
**12.** $70\frac{3}{4} - 20\frac{1}{2}$.

The following is a recent clipping from a daily paper. It shows the prices of wheat on the Chicago market. The first line of prices is for wheat to be delivered in July, and the second line for wheat to be delivered in September.

Chicago Wheat Quotations

| DELIVERY | PREVIOUS CLOSING | OPENING | HIGHEST | LOWEST | CLOSING |
|---|---|---|---|---|---|
| July | $91\frac{1}{4}$¢ | $90\frac{7}{8}$¢ | $92\frac{7}{8}$¢ | $90\frac{3}{4}$¢ | $92\frac{1}{4}$¢ |
| September | $87\frac{5}{8}$¢ | $87\frac{1}{4}$¢ | $88\frac{1}{4}$¢ | 87¢ | $87\frac{1}{2}$¢ |

**13.** What was the difference between the highest and the lowest price of July wheat? of September wheat?

**14.** What was the difference between the opening and the closing price of September wheat? of July wheat?

**15.** What was the difference between the opening price and the previous closing (yesterday's closing) price of July wheat? of September wheat?

**16.** A bought 1000 bu. July wheat at the lowest price and sold the same at the closing price. What was his gain?

SUGGESTION. $1\frac{1}{2}$¢ = \$0.015; 1000 times \$0.015 = \$?

**17.** B bought 1000 bu. September wheat at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the closing price, what would have been his gain?

**18.** C bought 25,000 bu. July wheat at the opening price and sold it at the highest price. What was his gain?

### WRITTEN EXERCISE

*Find the value of:*

**1.** $39 - 11\frac{5}{6}$.
**2.** $85 - 21\frac{3}{4}$.
**3.** $168 - 45\frac{5}{6}$.
**4.** $264\frac{9}{16} - 131\frac{1}{3}$.
**5.** $165\frac{1}{2} - 41\frac{1}{32}$.
**6.** $245\frac{3}{4} - 17\frac{3}{16}$.
**7.** $177\frac{2}{3} - 17\frac{3}{16}$.
**8.** $2150 - 121\frac{15}{16}$.
**9.** $\frac{5}{8} - \frac{1}{4} - \frac{1}{3}$.
**10.** $\frac{7}{8} - \frac{9}{16} - \frac{1}{8}$.
**11.** $2\frac{1}{2} + 1\frac{5}{8} - 1\frac{7}{12}$.
**12.** $25\frac{1}{2} - 8\frac{3}{4} - 15\frac{1}{2}$.

**142.** When the numerators of any two fractions are alike, the subtraction may be performed as in the following examples.

**143. Examples.** **1.** From $\frac{1}{7}$ take $\frac{1}{9}$. **2.** From $\frac{3}{5}$ take $\frac{3}{8}$.

SOLUTIONS. **1.** $9 - 7 = 2$, the new numerator. $9 \times 7 = 63$, the new denominator. Therefore, the required result is $\frac{2}{63}$. **2.** $\overline{8 - 5} \times 3 = 9$, the new numerator. $8 \times 5 = 40$, the new denominator. Therefore, $\frac{9}{40}$ is the required result.

### ORAL EXERCISE

*State the value of:*

**1.** $\frac{1}{2} - \frac{1}{3}$.
**2.** $\frac{1}{2} - \frac{1}{4}$.
**3.** $\frac{1}{2} - \frac{1}{6}$.
**4.** $\frac{1}{3} - \frac{1}{7}$.
**5.** $\frac{1}{3} - \frac{1}{8}$.
**6.** $\frac{1}{3} - \frac{1}{9}$.
**7.** $\frac{1}{5} - \frac{1}{7}$.
**8.** $\frac{1}{5} - \frac{1}{8}$.
**9.** $\frac{1}{5} - \frac{1}{9}$.
**10.** $\frac{1}{4} - \frac{1}{5}$.
**11.** $\frac{1}{4} - \frac{1}{6}$.
**12.** $\frac{1}{4} - \frac{1}{7}$.
**13.** $\frac{1}{4} - \frac{1}{8}$.
**14.** $\frac{1}{6} - \frac{1}{7}$.
**15.** $\frac{1}{6} - \frac{1}{8}$.
**16.** $\frac{2}{3} - \frac{2}{7}$.
**17.** $\frac{3}{4} - \frac{3}{5}$.
**18.** $\frac{3}{4} - \frac{3}{7}$.
**19.** $\frac{4}{5} - \frac{4}{7}$.
**20.** $\frac{5}{6} - \frac{5}{7}$.
**21.** $\frac{5}{6} - \frac{5}{9}$.
**22.** $\frac{5}{6} - \frac{5}{8}$.
**23.** $\frac{7}{8} - \frac{7}{9}$.
**24.** $\frac{2}{7} - \frac{2}{9}$.
**25.** $12\frac{1}{2} - 6\frac{1}{3}$.
**26.** $13\frac{1}{4} - 2\frac{1}{5}$.
**27.** $14\frac{1}{5} - 7\frac{1}{6}$.
**28.** $16\frac{3}{5} - 12\frac{3}{8}$.

## MULTIPLICATION

### ORAL EXERCISE

**1.** 12 times 2 A. are how many acres? 12 times 2 fifths ($\frac{2}{5}$) are how many fifths? $\frac{24}{5} = ?$

**2.** 32 mi. divided by 4 equals how many miles? $\frac{1}{4}$ of 32 mi. equals how many miles? Multiplying by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, etc., is the same as dividing by what integer?

**3.** If 5 men can dig 125 bu. of potatoes in 1 da., how many bushels can 3 men dig in the same time? $\frac{3}{5}$ of 125 bu. equals how many bushels?

**144. Example.** Multiply $\frac{3}{8}$ by 248.

SOLUTIONS. (*a*) 248 times 3 eighths = 744 eighths = $\frac{744}{8}$ = 93; but,

(*b*) If the multiplication is indicated as in the margin, the work may be shortened by cancellation.

(*a*)

$$\frac{3}{8} \times 248 = \frac{744}{8} = 93$$

(*b*)

$$\frac{\overset{31}{\cancel{248}} \text{ times } 3}{\cancel{8}} = 93$$

**145.** Therefore, to find the product of an integer and a fraction, *find the product of the integer and the numerator, and divide it by the denominator.*

Before actually multiplying, indicate the multiplication and cancel if possible.

### ORAL EXERCISE

**1.** If 1 yd. of cloth costs \$0.87$\frac{1}{2}$ (\$$\frac{7}{8}$), what will 16 yd. cost? 48 yd.? 128 yd.? 72 yd.?

**2.** When oats cost \$0.33$\frac{1}{3}$¢ (\$$\frac{1}{3}$) a bushel, how much must be paid for 29 bu.? for 36 bu.? for 129 bu.?

**3.** A boy earns \$0.75 (\$$\frac{3}{4}$) a day. How much will he earn in 18 da.? in 40 da.? in 84 da.? in 128 da.? in 160 da.?

**4.** When property rents for \$720 a year, what is the rent for $\frac{1}{2}$ yr.? for $\frac{1}{4}$ yr.? for $\frac{1}{3}$ yr.? for $\frac{1}{12}$ yr.? for $\frac{1}{6}$ yr.?

**5.** A ship is worth \$48,000. What is $\frac{1}{8}$ of the ship worth? $\frac{7}{16}$ of the ship? $\frac{5}{6}$ of the ship? $\frac{7}{8}$ of the ship? $\frac{7}{12}$ of the ship?

### WRITTEN EXERCISE

*Find the product of:*

**1.** $98 \times \frac{5}{8}$.
**2.** $80 \times \frac{2}{3}$.
**3.** $50 \times \frac{7}{24}$.
**4.** $97 \times \frac{7}{10}$.
**5.** $92 \times \frac{5}{16}$.
**6.** $188 \times \frac{1}{64}$.
**7.** $\frac{7}{8}$ of 95.
**8.** $\frac{5}{7}$ of 25.
**9.** $\frac{4}{5}$ of 88.
**10.** $\frac{9}{16}$ of 51.
**11.** $\frac{5}{24}$ of 99.
**12.** $\frac{7}{16}$ of 77.
**13.** $784 \times \frac{5}{7}$.
**14.** $459 \times \frac{1}{9}$.
**15.** $400 \times \frac{1}{16}$.
**16.** $510 \times \frac{7}{10}$.
**17.** $990 \times \frac{1}{64}$.
**18.** $800 \times \frac{15}{16}$.
**19.** $\frac{5}{6}$ of 2420.
**20.** $\frac{7}{8}$ of 2500.
**21.** $\frac{2}{3}$ of 3240.
**22.** $\frac{5}{8}$ of 5117.
**23.** $\frac{7}{16}$ of 7254.
**24.** $\frac{7}{16}$ of 1024.

**146. Example.** Multiply 25 by $4\frac{3}{8}$.

Solution. $\frac{3}{8}$ of $25 = \frac{75}{8}$ or $9\frac{3}{8}$. Write $\frac{3}{8}$ as shown in the margin, and carry 9 to the product of the integers. $4 \times 25 + 9 = 109$. Therefore, 25 multiplied by $4\frac{3}{8} = 109\frac{3}{8}$.

$$\begin{array}{r} 25 \\ \underline{4\frac{3}{8}} \\ 109\frac{3}{8} \end{array}$$

**147.** Therefore, to find the product of a mixed number and a whole number, *multiply the integer and the fraction separately and find the sum of the products.*

### ORAL EXERCISE

*Find the cost of:*

**1.** $15\frac{3}{4}$ lb. of fish at 9¢.
**2.** $7\frac{3}{8}$ yd. of cloth at $3.
**3.** 16 lb. of beef at $12\frac{1}{2}$¢.
**4.** $16\frac{1}{2}$ lb. of sugar at 5¢.
**5.** 12 yd. of cloth at $11\frac{1}{4}$¢.
**6.** $6\frac{3}{4}$ bu. turnips at 32¢.
**7.** $12\frac{1}{2}$ bu. of oats at 39¢.
**8.** $10\frac{1}{2}$ yd. of calico at 4¢.
**9.** $16\frac{1}{2}$ yd. of ribbon at 20¢.
**10.** $8\frac{1}{2}$ gal. of molasses at 40¢.

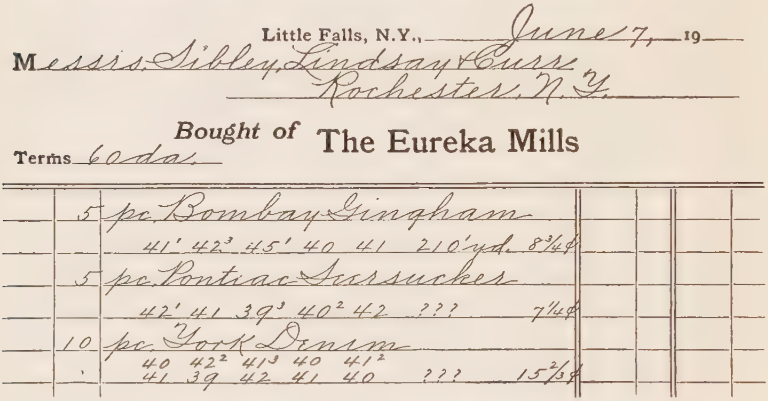
### WRITTEN EXERCISE

**1.** A merchant bought 24 pc. of English serge, containing 52, $47^2$, $50^1$, $48^3$, 49, $51^3$, 47, $48^2$, $45^3$, $49^1$, $52^2$, $50^2$, $51^3$, 50, 52, $53^1$, $52^3$, $47^3$, $48^1$, $51^2$, $51^3$, $48^2$, 49, and 53 yd., at $\$1.12\frac{1}{2}$ per yard, and sold it all at $1.35 per yard. What did he gain?

**2.** I bought 25 pc. taffeta silk, containing $42^1$, $40^2$, 43, $44^3$, 45, $41^2$, 43, $46^1$, $47^2$, 44, $45^2$, $49^1$, $47^1$, $45^1$, 46, 44, $43^3$, 40, $41^3$, 46, 47, $40^2$, $45^1$, 42, and $47^3$ yd., at $87\frac{1}{2}$¢ per yard, and sold the first 15 pc. at $1.05 and the remainder at $1.10. What did I gain?

**3.** A merchant bought 25 pc. of striped denim containing $41^1$, $41^1$, $42^2$, $43^2$, $42^1$, $44^2$, $43^1$, $40^2$, $42^1$, $45^3$, $42^1$, $40^2$, $41^2$, $47^3$, $45^1$, $41^1$, $43^2$, $47^2$, $44^3$, $42^3$, $43^2$, $39^1$, $42^1$, $48^2$, and 47 yd., at 11¢ per yard. If he sold the first 11 pc. at 15¢ per yard and the remainder at 17¢ per yard, what was his gain?

**4.** Copy and find the amount of the following bill:



Little Falls, N.Y., June 7, 19

Messrs. Sibley, Lindsay & Curr,
Rochester, N. Y.

Terms 60 da. **Bought of The Eureka Mills**

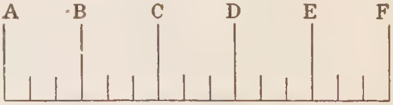
| | | | | | | |
|---|---|---|---|---|---|---|
| | 5 | pc. Bombay Gingham | | | | |
| | | $41^1$ $42^3$ $45^1$ 40 41 $210^1$ yd. 8¾¢ | | | | |
| | 5 | pc. Pontiac Seersucker | | | | |
| | | $42^1$ 41 $39^3$ $40^2$ 42 ??? 7¼¢ | | | | |
| | 10 | pc. York Denim | | | | |
| | | 40 $42^2$ $41^3$ 40 $41^2$<br>41 39 42 41 40 ??? 15⅔¢ | | | | |

**148.** The expressions $\frac{1}{2}$ of $\frac{3}{5}$ and $\frac{1}{2} \times \frac{3}{5}$ have the same meaning; hence the sign of multiplication may be read *of*, or *multiplied by*, when it immediately follows a fraction.

**149. Examples.** **1.** Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

Solution. To multiply $\frac{3}{5}$ by $\frac{2}{3}$ is to find $\frac{2}{3}$ of $\frac{3}{5}$.

Let the line *AF* in the accompanying diagram represent a unit divided into 5 equal parts.



Then *AD* will represent $\frac{3}{5}$. Subdivide each of the five equal parts into 3 equal parts and the line *AF* will represent a unit divided into 15 equal parts, each of which is $\frac{1}{15}$ of the whole. It is then clear that $\frac{1}{3}$ of $\frac{1}{5}$ equals $\frac{1}{15}$. Since $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{3}{5}$ is $\frac{3}{15}$. But $\frac{2}{3}$ of $\frac{3}{5}$ is 2 times $\frac{1}{3}$ of $\frac{3}{5}$; therefore $\frac{2}{3}$ of $\frac{3}{5}$ equals $\frac{6}{15}$.

**2.** Find the product of $2\frac{1}{2}$, $\frac{4}{5}$, and $\frac{7}{15}$.

Solution. Reduce the mixed number $2\frac{1}{2}$ to an improper fraction and obtain $\frac{5}{2}$. Cancel, and there remains in the numerators 2 times 7, and in the denominators 15, from which obtain the fraction $\frac{14}{15}$.

$$\frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{5}} \times \frac{7}{15} = \frac{14}{15}$$

**150.** Hence, to multiply a fraction by a fraction:

*Reduce the mixed numbers and integers to improper fractions and cancel all factors common to the numerators and denominators.*

*Find the product of the remaining numerators for the required numerator, and the product of the remaining denominators for the required denominator.*

### ORAL EXERCISE

**1.** How many yards in $\frac{2}{3}$ rd.? feet in $\frac{3}{5}$ rd.?

**2.** When barley is worth $25\frac{1}{2}$¢ per bushel, what is the value of $\frac{1}{2}$ bu.? of $\frac{3}{4}$ bu.?

**3.** A book, the retail price of which was $5, was sold at wholesale for $\frac{4}{5}$ of the retail price, with $\frac{1}{10}$ off from that for cash. Find the selling price of 10 books.

### WRITTEN EXERCISE

*Reduce to their simplest form:*

**1.** $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{3}{7}$.

**2.** $\frac{3}{4}$ of $\frac{8}{9}$ of $2\frac{1}{3}$.

**3.** $7\frac{1}{5} \times 25 \times \frac{5}{8}$.

**4.** $3\frac{5}{8} \times 4\frac{1}{2} \times 20$.

**5.** $50 \times \frac{7}{10} \times 7\frac{3}{5}$.

**6.** $1\frac{2}{3} \times 4\frac{1}{2} \times 8\frac{4}{5}$.

**7.** A saves $9.75 per week and B $\frac{2}{3}$ as much. How much more will A have than B at the end of a year?

**8.** A merchant bought a piece of cloth containing $43\frac{1}{2}$ yd. at $1.50 per yard. He sold $\frac{3}{4}$ of it at $1.62$\frac{1}{2}$ a yard, and the remainder at $1.37$\frac{1}{2}$ a yard. Did he gain or lose, and how much?

The following is a recent clipping from a daily paper. It shows the prices of corn on the New York market.

New York Corn Quotations

| Delivery | Previous Closing | Opening | Highest | Lowest | Closing |
|---|---|---|---|---|---|
| July | $66\frac{1}{2}$¢ | $65\frac{7}{8}$¢ | $66\frac{1}{2}$¢ | $64\frac{7}{8}$¢ | $65\frac{3}{8}$¢ |
| September | $65\frac{1}{4}$¢ | $64\frac{7}{8}$¢ | $65\frac{1}{8}$¢ | $64\frac{1}{4}$¢ | $64\frac{5}{8}$¢ |

**9.** D bought 25,000 bu. September corn at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

**10.** E bought 12,500 bu. July corn at the lowest price and sold it at the closing price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

**11.** A gold dollar weighs 25.8 troy grains. For every 90 parts of pure gold there are ten parts of alloy. How many grains of each kind in a gold dollar? in a 5-dollar gold piece?

**12.** A 5-cent piece weighs 77.16 troy grains. For every part of nickel there are three parts of copper. How many grains of each kind in a 5-cent piece?

**13.** The second general coinage act (1834) of the United States made one silver dollar weigh approximately as much as sixteen gold dollars, and this ratio of **sixteen to one** has been maintained up to the present time. What is the approximate weight of a silver dollar? If silver coins are $\frac{9}{10}$ pure, approximately how much pure silver in 10 silver dollars?

## SHORT METHODS IN MULTIPLICATION

**151.** When mixed numbers are large, they may be multiplied as shown in the following example.

**152. Example.** Multiply $255\frac{1}{3}$ by $24\frac{2}{5}$.

SOLUTION. Multiply the fractions together and obtain $\frac{2}{15}$, which write as shown in the margin. Multiply the integer in the multiplicand by the fraction in the multiplier and obtain 102. Multiply the fraction in the multiplicand by the integer in the multiplier and obtain 8. Multiply the integers together and add the partial products. The result is $6230\frac{2}{15}$.

$$\begin{array}{rl}
255\frac{1}{3} & \\
\underline{24\frac{2}{5}} & \\
\frac{2}{15} & = \frac{2}{5} \text{ of } \frac{1}{3} \\
102\phantom{\frac{2}{15}} & = \frac{2}{5} \text{ of } 255 \\
8\phantom{\frac{2}{15}} & = 24 \text{ times } \frac{1}{3} \\
\left.\begin{array}{r} 1020 \\ \underline{510} \end{array}\right\} & = 24 \text{ times } 255 \\
6230\frac{2}{15} & = 24\frac{2}{5} \text{ times } 255\frac{1}{3}
\end{array}$$

### WRITTEN EXERCISE

*Multiply:*

**1.** $975\frac{1}{4}$ by $18\frac{1}{2}$.
**2.** $876\frac{3}{4}$ by $21\frac{7}{8}$.
**3.** $720\frac{7}{8}$ by $21\frac{3}{4}$.
**4.** $445\frac{1}{2}$ by $46\frac{3}{4}$.
**5.** $512\frac{7}{16}$ by $16\frac{1}{2}$.
**6.** $450\frac{9}{16}$ by $20\frac{3}{4}$.

### SQUARING NUMBERS ENDING IN $\frac{1}{2}$ OR .5

**153. Examples.** **1.** Multiply $9\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to either the integer in the multiplicand or multiplier. Therefore, add 1 to the integer in the multiplicand and multiply by the multiplier. $9 \times 10 = 90$. Then, $9\frac{1}{2} \times 9\frac{1}{2} = 90\frac{1}{4}$.

$$\begin{array}{r} 9\frac{1}{2} \\ \underline{9\frac{1}{2}} \\ 90\frac{1}{4} \end{array}$$

**2.** Find the cost of 8.5 T. of coal at \$8.50 per ton.

SOLUTION. The principles embodied in this example are practically the same as those in problem 1. $.5 \times .5 = .25$, $8 \times 9 = 72$. Therefore, 8.5 tons of coal at \$8.50 per ton will cost \$72.25.

$$\begin{array}{r} 8.5 \\ \underline{8.5} \\ 72.25 \end{array}$$

**3.** Find the cost of 75 A. of land at \$75 per acre.

SOLUTION. This problem is similar to example 2, the only difference being in the matter of the decimal point. Since the decimal point has no particular bearing upon the steps in the process of multiplying, proceed to find the product as in example 2. $5 \times 5 = 25$, which write as shown in the margin. $7 \times 8 = 56$, which write to complete the product. 75 acres of land at \$75 an acre will therefore cost \$5625.

$$\begin{array}{r} 75 \\ \underline{75} \\ 5625 \end{array}$$

#### ORAL EXERCISE

*Multiply:*

**1.** $1\frac{1}{2}$ by $1\frac{1}{2}$.
**2.** $2\frac{1}{2}$ by $2\frac{1}{2}$.
**3.** $3\frac{1}{3}$ by $3\frac{1}{2}$.
**4.** $4\frac{1}{4}$ by $4\frac{1}{2}$.
**5.** $5\frac{1}{2}$ by $5\frac{1}{5}$.
**6.** $6\frac{1}{2}$ by $6\frac{1}{2}$.
**7.** 7.5 by 7.5.
**8.** 8.5 by 8.5.
**9.** 9.5 by 9.5.
**10.** 10.5 by 10.5.
**11.** $13\frac{1}{2}$ by $13\frac{1}{2}$.
**12.** $14\frac{1}{2}$ by $14\frac{1}{2}$.
**13.** $15\frac{1}{2}$ by $15\frac{1}{2}$.
**14.** 11.5 by 11.5.
**15.** 12.5 by 12.5.
**16.** $16\frac{1}{2}$ by $16\frac{1}{2}$.
**17.** $17\frac{1}{2}$ by $17\frac{1}{2}$.
**18.** $18\frac{1}{2}$ by $18\frac{1}{2}$.
**19.** 195 by 195.
**20.** 205 by 205.

#### WRITTEN EXERCISE

*In the following problems make all the extensions mentally.*

**1.** Find the total cost of:

85 lb. of tea at 85¢.
75 gal. sirup at 75¢.
45 gal. sirup at 45¢.
$2\frac{1}{2}$ bu. beans at \$2.50.
35 gal. molasses at 35¢.
65 cs. horseradish at 65¢.
$4\frac{1}{2}$ cs. baking powder at \$4.50.
55 lb. tea at 55¢.
75 bbl. flour at \$7.50.
650 bbl. oatmeal at \$6.50.
25 doz. cans olives at \$2.50.
95 cs. salad dressing at 95¢.
750 lb. cream codfish at $7\frac{1}{2}$¢.
$3\frac{1}{2}$ cs. baking powder at \$3.50.

### MULTIPLICATION OF ANY NUMBERS ENDING IN $\frac{1}{2}$ OR .5

**154. Examples.** **1.** Multiply $7\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to $\frac{1}{2}$ of $\overline{6+7}$, or $6\frac{1}{2}$, which added to $\frac{1}{2}$ of $\frac{1}{2}$ equals $6\frac{3}{4}$. Write $\frac{3}{4}$ as shown in the margin, and carry 6. $\overline{6 \times 7 + 6} = 48$. Therefore, $7\frac{1}{2} \times 6\frac{1}{2} = 48\frac{3}{4}$.

$$\begin{array}{r} 6\frac{1}{2} \\ \underline{7\frac{1}{2}} \\ 48\frac{3}{4} \end{array}$$

**2.** Multiply $7\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\overline{7+9} = 8$, with no remainder. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin, and carry 8. $\overline{7 \times 9 + 8} = 71$. Therefore, $7\frac{1}{2} \times 9\frac{1}{2} = 71\frac{1}{4}$.

$$\begin{array}{r} 7\frac{1}{2} \\ \underline{9\frac{1}{2}} \\ 71\frac{1}{4} \end{array}$$

Observe that : (1) in finding $\frac{1}{2}$ of any number (dividing a number by 2) there is either nothing remaining or 1 remaining; (2) in finding $\frac{1}{2}$ of an even number there can be no remainder, and in finding $\frac{1}{2}$ of an odd number there is always a remainder 1. Hence, to multiply numbers ending in $\frac{1}{2}$ or .5:

*Mentally determine the sum of the integers in the multiplicand and multiplier. If it is an even number, write $\frac{1}{4}$ (.25 or 25) in the product. If it is an odd number, write $\frac{3}{4}$ (.75 or 75) in the product. Multiply the integers and to the product add $\frac{1}{2}$ of their sum.*

#### ORAL EXERCISE

*Multiply:*

**1.** $3\frac{1}{2}$ by $7\frac{1}{2}$.
**2.** $4\frac{1}{4}$ by $5\frac{1}{2}$.
**3.** $16\frac{1}{2}$ by $4\frac{1}{2}$.
**4.** $17\frac{1}{2}$ by $2\frac{1}{2}$.
**5.** $14\frac{1}{2}$ by $6\frac{1}{2}$.
**6.** $21\frac{1}{2}$ by $9\frac{1}{2}$.
**7.** 3.5 by 8.5.
**8.** 7.5 by 6.5.
**9.** 5.5 by 8.5.

#### WRITTEN EXERCISE

*Make the extensions in each of the following problems mentally.*

**1.** Find the total cost of :

6.5 T. coal at $8.50.
2.5 T. hay at $17.50.
15.5 cd. wood at $3.50.
8.5 T. coal at $9.50.
16.5 T. hay at $11.50.
14.5 cd. wood at $5.50.

**2.** Find the total cost of :

45 bu. beans at $2.50.
35 bbl. flour at $6.50.
45 bbl. flour at $8.50.
350 bu. wheat at $1.05.
350 bu. beans at $2.50.
85 bbl. oatmeal at $7.50.

## DIVISION

### ORAL EXERCISE

**1.** 8 A. ÷ 4 = ? 8 ninths ($\frac{8}{9}$) ÷ 4 ?

**2.** If 2 lb. of coffee costs \$0.66$\frac{2}{3}$ (\$$\frac{2}{3}$), what will 1 lb. cost? Divide $\frac{2}{3}$ by 2. What is the effect of dividing the numerator of a fraction?

**3.** $\frac{4}{5} \div 2 = ?$ $\frac{1}{2}$ of $\frac{4}{5} = ?$

**4.** Because $\frac{4}{5} \div 2 = \frac{1}{2}$ of $\frac{4}{5}$, therefore, $\frac{1}{4} \div 5 = \frac{1}{5}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{5}$. $\frac{1}{4} \times \frac{1}{5} = ?$

**5.** What is the quotient of $\frac{1}{3} \div 5$? of $\frac{1}{2} \div 8$? of $\frac{1}{5} \div 2$?

**6.** Because $\frac{1}{3} \div 5 = \frac{1}{5}$ of $\frac{1}{3}$, therefore $\frac{2}{3} \div 5 = 2$ times $\frac{1}{3}$ of $\frac{1}{5}$. That is, $\frac{2}{3} \div 5 = \frac{1}{5}$ of $\frac{2}{3}$, or $\frac{2}{3} \times \frac{1}{5}$. $\frac{2}{3} \times \frac{1}{5} = ?$

**7.** How much is $\frac{3}{4} \div 5$? $\frac{5}{6} \div 3$? $7\frac{1}{2}$ ($\frac{15}{2}$) ÷ 8? $3\frac{1}{2} \div 6$?

**8.** What is the effect of multiplying the denominator of a fraction?

**155.** In the above exercise it is clear that

*Dividing the numerator of a fraction by an integer divides the whole fraction;* and,

*Multiplying the denominator of a fraction by an integer divides the whole fraction.*

### ORAL EXERCISE

*Find the quotient of:*

**1.** $\frac{8}{9} \div 4$.
**2.** $\frac{8}{13} \div 2$.
**3.** $\frac{15}{9} \div 5$.
**4.** $\frac{3}{4} \div 12$.
**5.** $\frac{5}{8} \div 12$.
**6.** $\frac{9}{10} \div 3$.
**7.** $\frac{7}{12} \div 4$.
**8.** $\frac{7}{16} \div 9$.
**9.** $\frac{5}{16} \div 7$.
**10.** $\frac{3}{5} \div 9$.
**11.** $\frac{1}{5} \div 6$.
**12.** $\frac{1}{8} \div 5$.
**13.** $\frac{1}{5} \div 19$.
**14.** $\frac{3}{16} \div 5$.
**15.** $\frac{7}{16} \div 5$.

**156. Examples.** 1. Divide $28\frac{7}{8}$ by 7.

Solution. First divide the integers and the result is 4; then divide the fraction by 7 and the result is $\frac{1}{8}$. Therefore, $28\frac{7}{8} \div 7 = 4\frac{1}{8}$.

$$\begin{array}{r} 4\frac{1}{8} \\ 7\overline{)28\frac{7}{8}} \end{array}$$

**2.** Divide $26\frac{1}{2}$ by 8.

Solution. Divide 26 by 8 and the result is 3 with a remainder 2. Join the remainder, 2, with the fraction, $\frac{1}{2}$, making $2\frac{1}{2}$. Reduce $2\frac{1}{2}$ to an improper fraction and the result is $\frac{5}{2}$. $\frac{5}{2} \div 8 = \frac{5}{16}$. Therefore, $26\frac{1}{2} \div 8 = 3\frac{5}{16}$.

$$\begin{array}{r} 3\frac{5}{16} \\ 8\overline{)26\frac{1}{2}} \end{array}$$

### ORAL EXERCISE

*Divide:*

1. $16\frac{1}{2}$ by 4.
2. $18\frac{1}{5}$ by 9.
3. $25\frac{3}{4}$ by 2.
4. $17\frac{1}{2}$ by 8.
5. $32\frac{3}{5}$ by 4.
6. $27\frac{1}{2}$ by 7.
7. $19\frac{7}{8}$ by 9.
8. $20\frac{2}{3}$ by 10.
9. $21\frac{1}{2}$ by 8.
10. $24\frac{6}{7}$ by 6.
11. $45\frac{5}{9}$ by 5.
12. $40\frac{2}{3}$ by 10.
13. $8\frac{1}{2}$ by 5.
14. $14\frac{3}{4}$ by 7.
15. $11\frac{1}{2}$ by 9.
16. $26\frac{1}{2}$ by 10.

### ORAL EXERCISE

**1.** How many eighths in one? $1 \div \frac{1}{8} = ?$

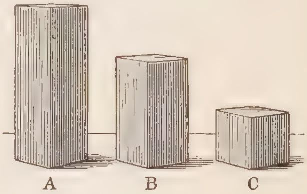
**2.** What is the value of: $1 \div \frac{1}{10}$? $3 \div \frac{1}{5}$? $17 \div \frac{1}{8}$? $125 \div \frac{1}{12}$? $250 \div \frac{1}{4}$?

**3.** Read aloud the following, supplying the missing word: *To divide an integer by a unit fraction, multiply the integer by the —— of the fraction.*

**4.** What is the value of $25 \div \frac{1}{3}$? $2.5 \div \frac{1}{2}$? $7.5 \div \frac{1}{4}$? $25.5 \div \frac{1}{12}$? $54 \div \frac{1}{4}$? $48 \div \frac{1}{5}$? $29 \div \frac{1}{3}$? $2\frac{1}{2} \div \frac{1}{6}$?

**5.** If $B$, in the accompanying diagram, is 1, what is $C$? How many blocks like $C$ in $B$? $1 \div \frac{1}{2} = ?$

**6.** If $A$ is 1, what is $B$? $A$ is how many times $B$? That is, $A \div B = ?$ $1 \div \frac{2}{3} = ?$



**7.** If $1 \div \frac{2}{3} = \frac{3}{2}$ $(1\frac{1}{2})$, then $2 \div \frac{2}{3} = ?$

**8.** What is the value of $4 \div \frac{2}{3}$? $5 \div \frac{3}{4}$? $12 \div \frac{2}{5}$? $15 \div \frac{2}{3}$?

**9.** Read aloud the following, supplying the missing words: If $A$ is 1, $B$ is ——, and $C$ is ——. If $B$ is contained in $A$ $\frac{3}{2}$ $(1\frac{1}{2})$ times, it is contained in $C$ $\frac{1}{3}$ of $\frac{3}{2}$ times or —— times. That is, $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} =$ ——.

**10.** What is the value of $\frac{1}{2} \div \frac{1}{4}$? $\frac{2}{3} \div \frac{2}{5}$? $\frac{2}{7} \div \frac{2}{3}$? $\frac{3}{4} \div \frac{5}{6}$?

**157.** The **reciprocal of a fraction** is 1 divided by that fraction.

Thus, the reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3}$, or $\frac{3}{2}$. That is, *the reciprocal of a fraction is the fraction inverted.*

**158. Reciprocal numbers**, as we use the terms in arithmetic, are numbers whose product is 1.

Thus, 4 and $\frac{1}{4}$, $\frac{4}{5}$ and $\frac{5}{4}$, $\frac{1}{6}$ and 6, $\frac{2}{3}$ and $\frac{3}{2}$, are reciprocal numbers, because their product is equal to 1.

**159.** It has been seen that the brief method for dividing a fraction or an integer by a fraction is to *multiply the dividend by the reciprocal of the divisor.*

The principles of cancellation should be used whenever possible. Integers and mixed numbers should be reduced to improper fractions before applying the rule.

### WRITTEN EXERCISE

*Divide:*

**1.** $\frac{7}{8}$ by $\frac{2}{3}$.
**2.** $7\frac{1}{2}$ by $\frac{1}{8}$.
**3.** $95$ by $\frac{5}{8}$.
**4.** $88$ by $\frac{3}{4}$.
**5.** $16$ by $\frac{3}{5}$.
**6.** $15\frac{1}{2}$ by $\frac{1}{5}$.
**7.** $\frac{3}{4}$ by $\frac{2}{3}$.
**8.** $4\frac{5}{7}$ by $\frac{2}{3}$.
**9.** $\frac{9}{10}$ by $\frac{7}{8}$.
**10.** $6\frac{2}{3}$ by $1\frac{1}{2}$.
**11.** $160$ by $4\frac{1}{2}$.
**12.** $250$ by $3\frac{2}{3}$.
**13.** $\frac{7}{8}$ by $\frac{4}{5}$.
**14.** $\frac{3}{5}$ by $\frac{2}{7}$.
**15.** $169$ by $4\frac{7}{8}$.
**16.** $640$ by $5\frac{3}{8}$.
**17.** $625$ by $83\frac{1}{3}$.
**18.** $920\frac{3}{5}$ by $73$.

**160. Examples.** **1.** Divide $2190$ by $48\frac{2}{3}$.

SOLUTION. Multiplying both dividend and divisor by the same number does not affect the quotient; hence, multiply the dividend and divisor by 3 and obtain for the new dividend and divisor 6570 and 146, respectively. Divide the same as in simple numbers and obtain the result 45. Or,

Reduce both the dividend and divisor to thirds, obtaining $\frac{6570}{3}$ and $\frac{146}{3}$. Reject the common denominators and divide as in whole numbers.

$$\begin{array}{r} 48\frac{2}{3})2190 \\ 3 \qquad 3 \\ \hline 146)\ 6570(45 \\ 584 \\ \hline 730 \\ 730 \\ \hline \end{array}$$

**2.** Divide $65\frac{1}{2}$ by $12\frac{1}{3}$.

SOLUTION. Multiply both dividend and divisor by 6, the least common denominator of the fractions, and divide as in simple numbers. The result is $5\frac{23}{74}$. Or,

Reduce both the dividend and divisor to sixths, obtaining as a result $\frac{74}{6}$ and $\frac{393}{6}$. Reject the common denominator and divide as in simple numbers.

$$\begin{array}{r} 12\frac{1}{3})\,65\frac{1}{2} \\ 6 \qquad 6 \\ \hline 74)\,393\,(5\frac{23}{74} \\ 370 \\ \hline 23 \end{array}$$

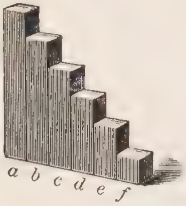
### WRITTEN EXERCISE

*Divide:*

**1.** $270\frac{1}{2}$ by $12\frac{1}{2}$.
**2.** $508\frac{1}{4}$ by $30\frac{2}{3}$.
**3.** $1431\frac{1}{5}$ by $20\frac{1}{2}$.
**4.** $962\frac{1}{2}$ by $31\frac{1}{4}$.
**5.** $650\frac{3}{4}$ by $26\frac{1}{2}$.
**6.** $1680\frac{1}{8}$ by $45\frac{1}{2}$.
**7.** $7552$ by $78\frac{2}{3}$.
**8.** $470\frac{3}{4}$ by $17\frac{1}{8}$.
**9.** $1054\frac{1}{2}$ by $168\frac{1}{3}$.

## FRACTIONAL RELATIONS

### ORAL EXERCISE



**1.** If $f$ in the accompanying diagram is 1, what is $e$? $d$? $c$? $b$? $a$?

**2.** What part of $e$ is $f$? of $d$? of $c$? of $b$? of $a$? What part of 6 is 1? of 5? of 4? of 3? of 2?

**3.** What part of $a$ is $e$? $d$? $c$? $b$? What part of 6 is 2? 3? 4? 5?

**4.** What part of $d$ is $f$? What part of $b$ is $e$? What part of $\frac{1}{2}$ $(\frac{3}{6})$ is $\frac{1}{6}$? What part of $\frac{5}{6}$ is $\frac{1}{3}$ $(\frac{2}{6})$?

**5.** What part of 7 bu. is 1 bu.? What part of 7 eighths $(\frac{7}{8})$ is 1 eighth $(\frac{1}{8})$?

**6.** What part of $\frac{3}{5}$ is $\frac{2}{5}$?

Solution. $\frac{3}{5}$ and $\frac{2}{5}$ are similar fractions; hence they may be compared in the same manner as concrete integral numbers. 2 is $\frac{2}{3}$ of 3; therefore, $\frac{2}{5}$ is $\frac{2}{3}$ of $\frac{3}{5}$; or,

$$\frac{2}{5} \text{ is } \frac{\frac{2}{5}}{\frac{3}{5}} \text{ of } \frac{3}{5}. \quad \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{5} \times \frac{5}{3} = \frac{2}{3}.$$

**7.** $\frac{3}{4}$ is what part of $1\frac{3}{4}$ $(\frac{7}{4})$? of $2\frac{3}{4}$? of $5\frac{1}{4}$?

**8.** $\frac{1}{8}$ is what part of $\frac{1}{2}$?

Solution. $\frac{1}{2} = \frac{4}{8}$. $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{4}{8}$, therefore, $\frac{1}{8} = \frac{1}{4}$ of $\frac{1}{2}$; or,

$$\frac{1}{8} \text{ is } \frac{\frac{1}{8}}{\frac{1}{2}} \text{ of } \frac{1}{2}. \quad \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}.$$

**161.** To find what fraction one number is of another, *take the number denoting a part for the numerator of the fraction, and the number denoting the whole for the denominator.*

### ORAL EXERCISE

**1.** If a piece of work can be performed in 12 da., what part of it can be performed in 5 da.? in 7 da.?

**2.** If A can do a piece of work in 15 da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $7\frac{1}{2}$ da.?

**3.** If B can do a piece of work in $7\frac{1}{2}$ da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $5\frac{1}{2}$ da.? in $6\frac{1}{2}$ da.?

**4.** What part of 100 is $33\frac{1}{3}$? $12\frac{1}{2}$? $66\frac{2}{3}$? $8\frac{1}{3}$? 25? 75? 125? $16\frac{2}{3}$? $83\frac{1}{3}$? $62\frac{1}{2}$? $22\frac{2}{9}$? $9\frac{1}{11}$? $56\frac{1}{4}$? $6\frac{2}{3}$?

**5.** What part of \$1 is $33\frac{1}{3}$¢? $66\frac{2}{3}$¢? 25¢? 75¢? $16\frac{2}{3}$¢? $8\frac{1}{3}$¢? $6\frac{2}{3}$¢? $3\frac{1}{3}$¢? $6\frac{1}{4}$¢? $62\frac{1}{2}$¢? $87\frac{1}{2}$¢? $37\frac{1}{2}$¢? $14\frac{2}{7}$¢?

**6.** What part of 1000 is 125? $166\frac{2}{3}$? $666\frac{2}{3}$? 625? $333\frac{1}{3}$?

**7.** What part of \$10 is $\$3.33\frac{1}{3}$? \$1.25? $\$1.66\frac{2}{3}$? $\$8.33\frac{1}{3}$? \$2.50? \$6.25? $\$6.66\frac{2}{3}$?

### WRITTEN EXERCISE

**1.** A man asked for a horse $\frac{2}{5}$ more than it cost, but finally reduced the price $\frac{1}{10}$. He gained \$26. What was the cost of the horse? the price asked? the selling price?

**2.** A had $\frac{1}{3}$ of his money invested in bonds, $\frac{5}{24}$ in bank stock, and the remainder, \$1980, on deposit in the First National Bank. How much was invested in bonds? in bank stock?

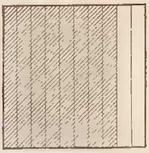
**3.** A man left his estate to his four sons. To the first son he gave $\frac{1}{3}$ of the estate; to the second, $\frac{1}{2}$ of the remainder; to the third, $\frac{1}{4}$ of the estate; to the fourth son, \$1556. What was the value of the estate?

**4.** A merchant reduced the marked price of a machine $\frac{1}{6}$, and then sold it so that he gained $\frac{1}{4}$ of the first cost. If he gained \$8, what was the first cost of the machine, and the marked price before any reduction was made?

**5.** A man placed a house and lot in the hands of a real estate agent to be sold at such a price that he, the owner, might realize \$5985, after paying the agent $\frac{1}{20}$ of the selling price of the property. For how much was the property sold?

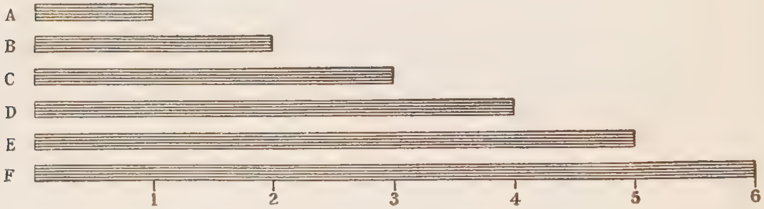
**6.** A farmer had three bins containing wheat, rye, and oats respectively. The quantity of oats was three times that of the wheat, and the rye was one half of the quantity of the oats. If the value of the oats at 35¢ per bushel was \$1155, how many bushels of each kind of grain did the farmer have? If the wheat was worth 95¢ per bushel, and the rye $67\frac{1}{2}$¢ per bushel, what was the value of the entire lot of grain?

### WRITTEN EXERCISE



**1.** The square in the margin represents the total population of the state of New York (state census of 1910), and the shaded area represents the urban (city) population. If the rural (country) population is 1,800,000, what is the entire population of the state? the urban population?

**2.** In a recent year the population of Massachusetts was in round numbers 3,360,000, and there were fourteen persons living in the cities of the state to each person living in the country. Represent this graphically as in problem 1, and find the city population and the country population for the state.



**3.** Suppose that $C$ in the diagram represents the population of the United States in 1870, $A$ the population in 1830, and $F$ the population in 1900. If the population in 1870 was 38,400,000 (round numbers), what was the population (round numbers) in 1900? In 1830?

**4.** Suppose that $F$ in the diagram represents the population of the United States in 1900, and $C$ the proportion of this population living in cities in 1900. What proportion of the population lived in cities in 1900? Suppose that $F$ represents the population in 1860 and $A$ the proportion of this population living in cities. Assuming that the city population in 1860 was 5,240,554, find the total population for the same year.

**5.** The total population of New Jersey (state census of 1910) is 2,537,167, and the rural population, 629,957. Represent this graphically and find the urban population.

## CONVERSION OF FRACTIONS

### ORAL EXERCISE

**1.** What is the denominator of the decimal .6? of .75?

**2.** What is the numerator of .4? of .04? of .004? of .0004?

**3.** Write as a common fraction .7; .23; .079; .0013; .00123.

**162.** A decimal may be written as a common fraction.

**163. Examples.** **1.** Reduce .0625 to a common fraction.

SOLUTION. .0625 means $\frac{625}{10000}$; but $\frac{625}{10000}$ may be expressed in simpler form. Dividing both terms of the fraction by 625, the result is $\frac{1}{16}$.

$$\frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

### WRITTEN EXERCISE

*Reduce to a common fraction or to a mixed number:*

**1.** 0.375.
**2.** 0.0625.
**3.** 0.0016.
**4.** 0.5625.
**5.** 0.9375.
**6.** $1.66\frac{2}{3}$.
**7.** 0.4375.
**8.** 0.125.
**9.** 0.0335.
**10.** $0.0056\frac{1}{4}$.
**11.** 181.875.
**12.** 171.245.
**13.** 260.675.
**14.** 126.1875.
**15.** 175.0625.
**16.** 172.0075.

**164.** A common fraction may be written as a decimal.

**165. Example.** Reduce $\frac{3}{8}$ to a decimal.

SOLUTION. $\frac{3}{8}$ equals $\frac{1}{8}$ of 3 units. 3 units equals 3000 thousandths. $\frac{1}{8}$ of 3000 thousands equals 375 thousandths (.375).

$$\begin{array}{r} .375 \\ 8\overline{)3.000} \end{array}$$

### ORAL EXERCISE

**1.** Reduce to equivalent decimals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{3}{16}$, $\frac{1}{9}$, $\frac{1}{11}$.

**2.** Reduce to common fractions: .5, .25, .50, .75, $.33\frac{1}{3}$, $.66\frac{2}{3}$, $.16\frac{2}{3}$, $.12\frac{1}{2}$, .6, .4, .60, .40, .2, $.83\frac{1}{3}$, .20, $.08\frac{1}{3}$, .375, .125, $.37\frac{1}{2}$, $.87\frac{1}{7}$, .875, .0625, $.11\frac{1}{9}$, $.09\frac{1}{11}$.

### WRITTEN EXERCISE

*Reduce to equivalent decimals:*

**1.** $\frac{7}{8}$.
**2.** $\frac{5}{16}$.
**3.** $\frac{9}{16}$.
**4.** $\frac{7}{16}$.
**5.** $\frac{9}{320}$.
**6.** $\frac{11}{16}$.
**7.** $\frac{17}{480}$.
**8.** $\frac{7}{25}$.
**9.** $\frac{5}{6400}$.
**10.** $5\frac{7}{12}$.
**11.** $21\frac{5}{8}$.
**12.** $165\frac{17}{20}$.

## THE SOLUTION OF PROBLEMS

**166.** The steps in the solution of a problem are: (1) reading the problem to find what is given and what is required; (2) determining from what is given how to find what is required; (3) outlining a process of computation and then performing it; (4) checking results.

**167.** A problem should be thoroughly understood before any attempt is made to solve it; and when the relation of what is given to what is required has been discovered, the process of computation should be briefly indicated and then performed as briefly and rapidly as possible.

**168.** To insure accuracy the work should always be checked in some manner. If the answer to the problem is estimated in advance, it will prove an excellent check against absurd results.

Thus, 42 doz. boys' hose at $48 a dozen is equal to approximately 40 × $50; $9\frac{2}{3}\%$ of 1290 bu. is equal to approximately $\frac{1}{10}$ of 1290 bu.; etc.

**169. Example.** A tailor used 30 yd. of flannel in making 18 waistcoats; at that rate how many yards will he require in making 45 waistcoats?

Solution

1. The quantity needed in making 18 waistcoats is given and the quantity needed in making 45 waistcoats is required.

2. One waistcoat requires $\frac{30}{18}$ yd.; 45 waistcoats will require 45 times $\frac{30}{18}$ yd.

3. $\frac{\overset{15}{\cancel{45}} \text{ times } \overset{5}{\cancel{30}}}{\underset{3}{\cancel{18}}} = 75$; that is 75 yd. of flannel are required in making 45 waistcoats.

4. $\frac{30}{18}$ yd. $= \frac{5}{3}$ yd.; $\frac{75}{45}$ yd. $= \frac{5}{3}$ yd.; therefore the work is probably correct.

**170.** If reasons for conclusions, processes, and results are given, they should be brief and accurate. It is also a mistake to try to use the language of the book or the instructor. Such artificial work stifles thought and conceals the condition of the learner.

The subject of analysis should not be unduly emphasized. A correct solution may generally be accepted as evidence that the correct analysis has been made.

### ORAL EXERCISE

In the following problems first find each result as required, and then give a brief, accurate explanation of the steps taken in the solution. Do not use pen or pencil.

**1.** If 2 T. cost \$8, what will 5 T. cost?

SUGGESTION. \$20; since 2 T. cost \$8, 5 T., which are $2\frac{1}{2}$ times 2 T., will cost $2\frac{1}{2}$ times \$8, or \$20.

**2.** 24 is $\frac{6}{7}$ of what number? $\frac{3}{8}$ of what number? $\frac{2}{13}$ of what number?

**3.** 220 is $\frac{1}{2}$ less than what number? 450 is $\frac{1}{3}$ less than what number?

**4.** A, having spent $\frac{1}{5}$ of his money, finds he has \$84 left. How much had he at first?

**5.** \$124 is $\frac{1}{3}$ more than what sum of money? \$300 is $\frac{1}{4}$ more than what sum of money?

**6.** A man sold $\frac{5}{16}$ of an acre of land for \$35. At that rate what is his entire farm of 100 acres worth?

**7.** A man bought a stock of goods and sold it at $\frac{1}{4}$ above cost. If he received \$275, what was the cost of the goods?

**8.** B bought a stock of goods which he sold at $\frac{1}{5}$ below cost. If he received for the sale of the goods \$240, what was the cost and what was his loss?

**9.** $\frac{9}{16}$ of the students in a high school are girls and the remainder are boys. If the number of boys is 350, how many scholars in the school?

**10.** A bought a quantity of wheat which he sold at $\frac{1}{2}$ above cost. If he received \$300 for the wheat, what did it cost him and what was his gain?

**11.** A bought a quantity of dry goods and sold them so as to realize $\frac{1}{3}$ more than the cost. If the selling price was \$720, what was the cost and what was the gain?

**12.** D bought a stock of carpeting which he was obliged to sell at $\frac{1}{6}$ below cost. If he received \$750 for the sale of the carpeting, what was the cost of same, and what was his loss?

### WRITTEN EXERCISE

In the following problems give both analysis and computation.

**1.** If $\frac{1}{2}$ lb. of tea cost 21¢, what will $9\frac{1}{2}$ lb. cost?

| COMPUTATION | ANALYSIS |
|---|---|
| $9\frac{1}{2}=\frac{19}{2}$<br>$19 \times 21¢ = \$3.99$ | $9\frac{1}{2}=\frac{19}{2}$; $9\frac{1}{2}$ is therefore 19 times $\frac{1}{2}$. If $\frac{1}{2}$ lb. cost 21¢, $9\frac{1}{2}$ lb. will cost 19 times 21¢, or \$3.99. |

**2.** If $\frac{1}{2}$ of a pound of tea cost 42¢, what will $35\frac{1}{2}$ lb. cost?

**3.** If a drain can be dug in 17 da. by 45 men, how many men will it take to dig $\frac{1}{5}$ of it in 3 da.?

**4.** In what time will 3 boys at $\$0.62\frac{1}{2}$ per day earn as much as 4 men at \$2.25 each per day will earn in $45\frac{3}{4}$ da.?

**5.** A spends \$72 per week or $\frac{3}{4}$ of his income; B saves \$48 per week or $\frac{3}{8}$ of his income. How long will it take A to save as much as B saves in five weeks?

**6.** If 115 bu. of wheat are required to make 23 bbl. of flour, how many bushels will be required to make 50 bbl. of flour? 117 bbl. of flour? 259 bbl. of flour?

### ORAL REVIEW EXERCISE

**1.** $.05 \times 6 \times 0 \times 2\frac{1}{2} = ?$

**2.** \$0.75 is what part of \$3?

**3.** What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{10}$?

**4.** Find the value of $.45 + (.25 \times 5) - .04$.

**5.** 60 is $\frac{3}{5}$ of what number? $\frac{3}{4}$? $\frac{3}{7}$? $\frac{4}{5}$? $\frac{5}{6}$?

**6.** At 25¢ a yard, what will $2\frac{1}{2}$ yd. of cloth cost?

**7.** $\frac{6}{7}$ is $\frac{1}{2}$ of what number? $\frac{3}{5}$ is $\frac{3}{4}$ of what number?

**8.** If $\frac{3}{4}$ of an acre of land costs \$75, what will 50 A. cost?

**9.** If $\frac{2}{5}$ of a number is 84, what is 5 times the same number?

**10.** The dividend is $4\frac{1}{2}$ and the quotient is $6\frac{3}{4}$; what is the divisor?

**11.** If 6 bu. of apples cost \$15, what will 80 bu. cost at the same rate?

**12.** At \$460 per half mile, what will be the cost of grading 6 mi. of road?

**13.** How much will 4 carpenters earn in 10 da. at the rate of $2.25 each per day?

**14.** At $4.50 per cord, what will be the cost of $4\frac{1}{2}$ cd. of wood? of $6\frac{1}{2}$ cd.? of $12\frac{1}{2}$ cd.? of $7\frac{1}{2}$ cd.?

**15.** A bought a horse for $96 and sold it for $\frac{7}{8}$ of its cost. What part of the cost was the loss sustained?

**16.** A bought $4\frac{1}{4}$ yd. of velvet at $5.20 per yard and gave in payment a $50 bill. How much change should he receive?

**17.** I sold 5 A. of land for $375 and sustained a loss equal to $\frac{1}{6}$ of the original cost of the land. What did the land cost per acre?

**18.** D and E agree to mow a field for $36. If D can do as much in 2 da. as E can do in 3, how should the money be divided?

**19.** N sold a watch to O and received $\frac{1}{3}$ more than it cost him. If O paid $64 for the watch, what did it cost N? What per cent did N gain?

**20.** A earns $125 per month. Of this sum he spends $75 and saves the remainder. What part of his monthly earnings does he save? What per cent?

### WRITTEN REVIEW EXERCISE

**1.** Find the cost of 1100 eggs at $23\frac{3}{4}$¢ per dozen.

**2.** Counting 2000 lb. to a ton, find the cost of $5\frac{7}{8}$ T. of steel at $1\frac{5}{16}$¢ per pound.

**3.** When flour is sold at $6.02 per barrel of 196 lb., what should be paid for $55\frac{1}{2}$ lb.?

**4.** I bought 300 bbl. of flour at $5.75 per barrel. At what price must I sell it per barrel in order to gain $150?

**5.** The cost of 200 bu. of wheat was $204.50 and the selling price $212.35. What was the gain per bushel?

**6.** A can do a piece of work in $5\frac{1}{2}$ da. and B in $7\frac{1}{2}$ da. If they join in the completion of the work, how long will it take them?

**7.** How much will 7 men earn in 6 da., working 10 hr. per day, at 25¢ per hour?

**8.** At $2.50 per day of 8 hr., how much should a man receive for $11\frac{1}{2}$ hours' work?

**9.** A boy works $4\frac{1}{2}$ da. at the rate of $5.75 per week of 6 da. How much does he earn?

**10.** W, in $\frac{1}{3}$ of a day, earns $1.25, and Y, in $\frac{1}{2}$ of a day, earns $\$0.87\frac{1}{2}$. How much will the two together earn in $40\frac{1}{2}$ da.?

**11.** A and B together can do a piece of work in 10 da. If A can complete the work alone in 16 da., how long will it take B to do it?

**12.** Nov. 1, in a recent year, was on Tuesday. How much did B earn during November if he was employed every working day at the rate of $3.75 per day?

**13.** In one year the cotton produced in the United States approximated 14,000,000 bales of 500 lb. each. If Texas produced $\frac{1}{4}$ of the crop, Georgia $\frac{1}{6}$, South Carolina $\frac{1}{8}$, and Alabama $\frac{1}{9}$, what was the value of the cotton produced in each state, at $10\frac{1}{2}$¢ per pound?

**14.** In one year the value of the cotton crop (including the seed) in Texas was $188,000,000. The value of the cotton seed was $25,000,000. The value of the cotton was what fraction of the total value? The value of the seed was what fraction of the total value? (Express decimally, using three places.)

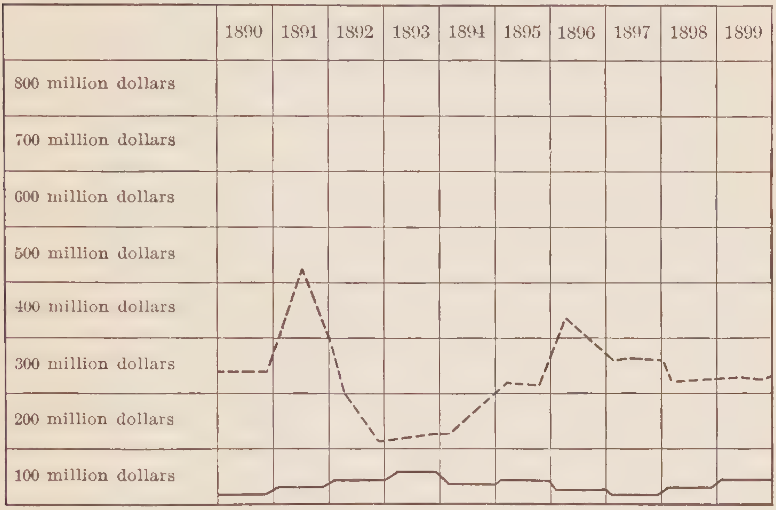
**15.** In one day 14,310 beef cattle were received at the Union Stock Yards, in Chicago. If $\frac{2}{3}$ of the number were of an average weight of 1056 lb. and sold for $6.80 per hundred pounds, and if the remainder were of an average weight of 1192 lb. and sold at $6.95 per hundred pounds, what was the total value of the cattle?

**16.** In one year the approximate production of sugar in the United States (including Hawaii and Porto Rico) was as follows: beet sugar, 1,240,000,000 lb.; cane sugar, 1,108,000 short tons. If the beet sugar was worth $1\frac{9}{10}$¢ per pound, and the cane sugar $1\frac{3}{10}$¢ per pound, what was the total value of the crop? (The price is based on the export value of refined sugar.)

## GRAPHIC OUTLINE

A comparison of the money value of the wheat crop, and the fire losses paid by insurance companies, in the United States, 1890 to 1899 inclusive.

- - - - - - - - - - - value of wheat crop. ———————— value of fire losses.



### WRITTEN EXERCISE

**1.** The figures below give the value of the wheat crop, and the fire losses paid by fire insurance companies, in the United States, for the years 1890 to 1899 inclusive. (See Graphic Outline.)

| | Farm Val. of Wheat | Fire Losses |
|---|---|---|
| 1890 | $334,773,678 | $108,993,792 |
| 1891 | 513,473,711 | 143,764,967 |
| 1892 | 322,111,881 | 151,516,098 |
| 1893 | 213,171,381 | 167,544,370 |
| 1894 | 225,902,025 | 140,006,484 |
| 1895 | 237,938,998 | 142,110,233 |
| 1896 | 310,602,593 | 118,737,420 |
| 1897 | 428,547,121 | 116,254,575 |
| 1898 | 392,770,320 | 130,593,505 |
| 1899 | 319,545,259 | 153,597,830 |

**2.** Make a graphic outline comparing the wheat crop, and the fire losses paid by the fire insurance companies, in the United States, for the years 1880 to 1889 inclusive.

| | FARM VAL. OF WHEAT | FIRE LOSSES |
|---|---|---|
| 1880 | $474,201,850 | $74,643,400 |
| 1881 | 456,880,427 | 81,280,900 |
| 1882 | 445,602,123 | 84,505,024 |
| 1883 | 383,649,272 | 100,149,228 |
| 1884 | 330,862,260 | 110,008,611 |
| 1885 | 275,320,390 | 102,813,796 |
| 1886 | 314,226,020 | 104,924,750 |
| 1887 | 310,612,960 | 120,283,055 |
| 1888 | 385,248,030 | 110,885,665 |
| 1889 | 342,491,707 | 123,046,833 |

**3.** Make a graphic outline comparing the wheat crop, the cotton crop, and the fire losses paid by the fire insurance companies, in the United States, for the years 1900 to 1909 inclusive.

| | FARM VAL. OF WHEAT | FARM VAL. OF COTTON | FIRE LOSSES |
|---|---|---|---|
| 1900 | $323,515,177 | $515,828,431 | $160,929,805 |
| 1901 | 467,350,156 | 439,166,710 | 165,817,810 |
| 1902 | 422,224,117 | 501,897,135 | 161,087,040 |
| 1903 | 443,024,826 | 660,549,230 | 145,302,155 |
| 1904 | 510,489,874 | 652,031,626 | 229,198,050 |
| 1905 | 518,372,727 | 632,298,332 | 165,221,650 |
| 1906 | 490,332,760 | 721,647,237 | 518,611,800 |
| 1907 | 554,437,000 | 700,956,011 | 215,084,709 |
| 1908 | 616,826,000 | 681,230,956 | 217,885,850 |
| 1909 | 730,046,000 | 812,089,833 | 188,705,150 |

Use a dotted line to represent the cotton crop.

The figures representing the fire losses do not include the cost of maintaining fire departments, nor the losses sustained by the interruption of business.

The United States exceeds all other countries in losses by fire. A large per cent of these losses are due to carelessness.

### ORAL REVIEW EXERCISE

**1.** $\frac{3}{4}$ of 36 is what part of 81?

**2.** Multiply 126 by 101; 92 by 102.

**3.** Divide $4\frac{1}{2}$ by $2\frac{3}{4}$; $3\frac{2}{3}$ by $2\frac{1}{2}$.

**4.** Find the cost of each of the following:

- *a.* 35 bu. of seed at 35¢ per bushel.
- *b.* 65 A. of land at $65 per acre.
- *c.* 45 yd. of cloth at 45¢ per yard.

**5.** Divide $\frac{2}{3}$ by $\frac{4}{5}$; $\frac{8}{9}$ by $\frac{4}{7}$; $\frac{5}{6}$ by $\frac{2}{5}$.

**6.** Multiply 36 by $\frac{3}{4}$; 49 by $\frac{6}{7}$; 55 by $\frac{5}{11}$.

**7.** What is the square of 15? of 1.5? of $1\frac{1}{2}$?

**8.** $64 \times \frac{1}{2} = ?$ $64 \div \frac{1}{2} = ?$ $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = ?$

**9.** How many yards of cloth can be bought for $25 at $12\frac{1}{2}$¢ per yard?

**10.** If it costs $7.50 to harvest $6\frac{1}{2}$ A. of corn, what will it cost to harvest 65 A.?

**11.** An agent received $7.20 for collecting a debt, and the merchant received $232.80. What was the total debt?

**12.** C and D received $21.75 for work done jointly. If C does only half as much work as D, how should the money be divided?

**13.** If a lot of articles were bought at the rate of 3 for 2¢ and sold at the rate of 2 for 3¢, how many must be sold to gain $5?

**14.** A and B received $34 for work done jointly. If A can do as much work in 8 da. as B can do in 9 da., how should the money be divided?

**15.** Name the results quickly:

*a.* A can do a piece of work in 4 da., and B in 5 da. If they work together, in how many days will they finish the task?

*b.* F can do a piece of work in $3\frac{1}{3}$ da., and G in 5 da. If they work together, in how many days will they finish the task?

*c.* C can do a piece of work in 2 da., D in 3 da., and E in 4 da. If they work together, in how many days will they finish the task?

*d.* H and J together can do a piece of work in 20 da. If H alone can do the work in 30 da., in how many days can J alone do the work?

### WRITTEN EXERCISE

### PROBLEMS OF THE FARM

**1.** If 15 sheep consume 5785 lb. of dry fodder in a year, what is the cost per sheep if the fodder is worth $8.50 per ton?

**2.** If eggs are worth 24 ¢ per dozen, what is the difference in the value of two hens, in a year, if one lays 180 eggs and the other lays 96 eggs?

**3.** An apple tree produced $9\frac{1}{4}$ bu. of apples, $6\frac{1}{2}$ bu. of which graded as "firsts" and the remainder as "seconds." What fractional part of the yield were firsts, and what fractional part were seconds?

**4.** The apple tree referred to in Ex. 3 was sprayed the year following, and that year it produced $10\frac{1}{2}$ bu. of which $9\frac{1}{4}$ bu. were firsts and the remainder seconds. What fractional part of the yield were firsts? What part were seconds?

**5.** If the apples referred to in Exs. 3 and 4 were sold at $1.20 per bushel for firsts and 70 ¢ a bushel for seconds, what was the value of the spraying?

**6.** It is estimated that a quail in one year eats 28 ¢ worth of grain and saves $1.68 worth of grain by destroying insects and weeds. What is the value of a pair of quails to the farmer annually, not counting the value of the brood?

**7.** An undrained field produced 24 bu. of grain per acre, and after being drained it produced 33 bu. per acre. What was the fractional increase? What was the value of the increase if the grain sold for $55\frac{1}{2}$ ¢ per bushel?

**8.** A flock of hens averaged 78 eggs each per year. What would be the value to the farmer of introducing a better breed of hens that would produce 120 eggs each per year, if he kept a flock of 40 hens, and received 24 ¢ per dozen for the eggs?

**9.** If 6 A. of unfertilized land produced 275 bu. of corn, and if fertilized, it would have produced 350 bu., what would the farmer have gained by fertilizing the land if the corn was sold for 68 ¢ per bushel, and the fertilizer cost $24 per ton, and 400 lb. were used on each acre?

### WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

**1.** A, B, and C hire a pasture for \$81. A puts in 6 cows for 4 mo.; B, 6 cows for 6 mo.; and C, 6 cows for 5 mo. What sum should each pay?

**2.** A man owned $\frac{6}{7}$ of a tract of land; he sold $\frac{2}{3}$ of his share for \$14,504.46. At that rate, what was the value of his original share? What was the whole tract worth?

**3.** The owner of a house received a net yearly income from rental of \$408.90, after paying the following: insurance, \$64.20; taxes, \$74.50; repairs, \$28.40. What was the monthly rental?

**4.** A man drew $\frac{1}{3}$ of his money from the bank and then paid bills of the following amounts: \$12.50, \$18.25, and \$7.50; he then had left in cash \$11.75. What sum had he in the bank before drawing the check?

**5.** A man placed a mortgage on his house and lot for \$2967. The lot cost \$2720; the improvements, \$260.50; and the dwelling, \$5920.50. The mortgage was what fractional part of the total value of the property?

**6.** At the end of a season a dealer sold a machine for \$64, after reducing the marked price $\frac{1}{5}$. If he still gained $\frac{1}{7}$ of the cost, what was the first cost? The marked price was what fraction above the cost price?

**7.** From dictation, write results for the following: $\frac{1}{4}$ of $25\frac{1}{2}$; $\frac{1}{6}$ of $26\frac{1}{2}$; $\frac{1}{5}$ of $36\frac{1}{4}$; $\frac{1}{7}$ of $17\frac{1}{2}$; $\frac{1}{3}$ of $42\frac{2}{3}$; $\frac{1}{9}$ of $28\frac{1}{2}$; $\frac{1}{11}$ of $22\frac{1}{3}$; $\frac{1}{9}$ of $64\frac{1}{2}$; $\frac{1}{8}$ of $50\frac{1}{4}$; $\frac{1}{10}$ of $35\frac{1}{5}$; $\frac{1}{12}$ of $50\frac{1}{2}$.

**8.** A merchant closed his business under the following conditions: resources, \$22,455.20; liabilities, \$33,682.80. What fractional part of his debts can he pay? If he owes James S. Brown \$202.50, how much will Brown receive in settlement?

**9.** A, B, and C are partners in a mercantile business in which A has invested \$9180; B, \$6120; and C, \$3060. At the end of 1 yr. they divided a gain of \$3060.90. If each partner received of the gain according to his fractional part of the investment, how much did each receive?

# CHAPTER X

## ALIQUOT PARTS

**171.** An **aliquot part** of a number is a part that is contained in the number an integral number of times.

Thus, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

### ORAL EXERCISE

**1.** How many cents in $\$\frac{1}{2}$? in $\$\frac{1}{4}$? in $\$\frac{1}{8}$? in $\$\frac{1}{6}$?

**2.** What aliquot part of \$1 is 25¢? 50¢? $6\frac{1}{4}$¢? $12\frac{1}{2}$¢?

**3.** Read aloud the following, supplying the missing terms: $16 \times 50¢ = 16 \times \$\frac{1}{2} = \frac{1}{2}$ of \$16; $16 \times 25¢ = 16 \times \$\frac{1}{4} = \frac{1}{4}$ of \$16; $16 \times 12\frac{1}{2}¢ = 16 \times \$$ —— = —— of \$16; $16 \times 6\frac{1}{4}¢ = 16 \times \$$ —— = —— of \$16.

**4.** Give a short method for finding the cost when the quantity is given and the price is 50¢; 25¢; $12\frac{1}{2}$¢; $6\frac{1}{4}$¢.

**5.** What is the cost of 160 yd. of dress goods at \$1? at 50¢? at 25¢? at $12\frac{1}{2}$¢? at $6\frac{1}{4}$¢?

**6.** How many cents in $\$\frac{1}{3}$? in $\$\frac{1}{6}$? in $\$\frac{1}{12}$? in $\$\frac{1}{15}$? in $\$\frac{1}{7}$? in $\$\frac{1}{5}$? in $\$\frac{1}{10}$?

**7.** What aliquot part of \$1 is $33\frac{1}{3}$¢? $16\frac{2}{3}$¢? $8\frac{1}{3}$¢? $6\frac{2}{3}$¢? $14\frac{2}{7}$¢? 20¢? 10¢?

**8.** Read aloud the following, supplying the missing terms: $140 \times 14\frac{2}{7}¢ = 140 \times \$\frac{1}{7} = \frac{1}{7}$ of \$140; $90 \times 6\frac{2}{3}¢ = 90 \times \$$ —— = —— of \$90; $90 \times 20¢ = 90 \times \$$ —— = —— of \$90.

**9.** Read aloud the following, supplying the missing terms: $240 \times 33\frac{1}{3}¢ = 240 \times \$$ —— $= \frac{1}{3}$ of \$240; $240 \times 16\frac{2}{3} = 240 \times \$\frac{1}{6} =$ —— of \$240; $240 \times 12\frac{1}{2}¢ = 240 \times \$$ —— = —— of \$240.

**10.** Give a short method for finding the cost when the quantity is given and the price is $33\frac{1}{3}$¢; $16\frac{2}{3}$¢; $8\frac{1}{3}$¢; $6\frac{2}{3}$¢; $14\frac{2}{7}$¢.

**11.** Find the cost of 960 yd. of cloth at $33\frac{1}{3}$¢; at $16\frac{2}{3}$¢; at $8\frac{1}{3}$¢.

### ORAL EXERCISE

*State the cost of:*

**1.** 240 lb. tea at 50¢; at $33\frac{1}{3}$¢; at 25¢.
**2.** 360 lb. coffee at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $12\frac{1}{2}$¢.
**3.** 720 gal. cider at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
**4.** 2400 doz. eggs at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢.
**5.** 2400 yd. prints at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢.
**6.** 960 yd. cotton at $6\frac{1}{4}$¢; at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
**7.** 2040 yd. plaids at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $16\frac{2}{3}$¢.
**8.** 480 lb. lard at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 10¢.
**9.** 3600 lb. raisins at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢; at $33\frac{1}{3}$¢.
**10.** 480 yd. lining at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at 10¢; at $12\frac{1}{2}$¢; at $6\frac{2}{3}$¢.
**11.** 4200 yd. silesia at 10¢; at 20¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at $14\frac{2}{7}$¢.
**12.** 1500 yd. plaids at $1; at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢.
**13.** 420 yd. stripe at 10¢; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at $16\frac{2}{3}$¢; at 25¢.
**14.** 120 yd. gingham at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
**15.** 1240 yd. wash silk at 25¢; at 50¢; at $33\frac{1}{3}$¢; at 20¢.

**16.** At the rate of 3 for 50¢, what will 27 handkerchiefs cost?

**17.** At $33\frac{1}{3}$¢ per half dozen, what will 12 doz. handkerchiefs cost? 17 doz.? 25 doz.? $7\frac{1}{2}$ doz.? $4\frac{1}{2}$ doz.?

**18.** A merchant bought cloth at $33\frac{1}{3}$¢ per yard and sold it at 50¢ per yard. What was his gain on 1680 yd.?

### ORAL EXERCISE

**1.** What is the cost of $12\frac{1}{2}$ yd. of silk at 96¢ per yard?

SUGGESTION. The cost of $12\frac{1}{2}$ yd. at 96¢ = the cost of 96 yd. at $12\frac{1}{2}$¢. *Interchanging the multiplicand and multiplier considered as abstract numbers does not affect the product.*

**2.** Find the cost of 25 yd. of silk at $1.72 per yard.

SUGGESTION. The cost of 25 yd. at $1.72 (172¢) = the cost of 172 yd. at 25¢.

**3.** Find the cost of:

*a.* 25 yd. at 16¢.
*b.* $12\frac{1}{2}$ yd. at 48¢.
*c.* $6\frac{1}{4}$ lb. at 32¢.
*d.* $12\frac{1}{2}$ lb. at 80¢.
*e.* 25 yd. at 84¢.
*f.* $12\frac{1}{2}$ yd. at $1.75.

### Table of Aliquot Parts

| Nos. | $\frac{1}{2}$'s | $\frac{1}{4}$'s | $\frac{1}{8}$'s | $\frac{1}{16}$'s | $\frac{1}{3}$'s | $\frac{1}{6}$'s | $\frac{1}{12}$'s | $\frac{1}{15}$'s | $\frac{1}{5}$'s | $\frac{1}{10}$'s |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .50 | .25 | $.12\frac{1}{2}$ | $.06\frac{1}{4}$ | $.33\frac{1}{3}$ | $.16\frac{2}{3}$ | $.08\frac{1}{3}$ | $.06\frac{2}{3}$ | .20 | .10 |
| 10 | 5. | $2\frac{1}{2}$ | $1\frac{1}{4}$ | $.62\frac{1}{2}$ | $3\frac{1}{3}$ | $1\frac{2}{3}$ | $.83\frac{1}{3}$ | $.66\frac{2}{3}$ | 2. | 1. |
| 100 | 50. | 25. | $12\frac{1}{2}$ | $6\frac{1}{4}$ | $33\frac{1}{3}$ | $16\frac{2}{3}$ | $8\frac{1}{3}$ | $6\frac{2}{3}$ | 20. | 10. |
| 1000 | 500. | 250. | 125. | $62\frac{1}{2}$ | $333\frac{1}{3}$ | $166\frac{2}{3}$ | $83\frac{1}{3}$ | $66\frac{2}{3}$ | 200. | 100. |

### WRITTEN EXERCISE

*In the three problems following make all the extensions mentally.*

1. Without copying, find quickly the total cost of:

84 lb. tea at 50¢.
75 lb. tea at $33\frac{1}{3}$¢.
72 lb. coffee at 25¢.
84 lb. coffee at $33\frac{1}{3}$¢.
25 lb. coffee at 28¢.
88 lb. candy at $12\frac{1}{2}$¢.
24 lb. tapioca at $6\frac{1}{4}$¢.
$6\frac{1}{4}$ lb. tea at 64¢.
25 lb. cocoa at 52¢.
$12\frac{1}{2}$ lb. cocoa at 48¢.
360 lb. codfish at $6\frac{2}{3}$¢.
66 lb. crackers at $8\frac{1}{3}$¢.
25 lb. chocolate at 36¢.
25 cs. horseradish at 64¢.

2. Without copying, find quickly the total cost of:

25 yd. silk at 84¢.
$12\frac{1}{2}$ yd. silk at 96¢.
750 pc. lace at $6\frac{2}{3}$¢.
112 yd. ticking at $6\frac{1}{4}$¢.
210 yd. plaids at $33\frac{1}{3}$¢.
128 gro. buttons at $12\frac{1}{2}$¢.
68 yd. lansdowne at 50¢.
77 yd. duck at $14\frac{2}{7}$¢.
$6\frac{1}{4}$ gro. buttons at 32¢.
155 yd. cheviot at 20¢.
96 yd. gingham at $8\frac{1}{3}$¢.
84 yd. shirting at $12\frac{1}{2}$¢.
25 doz. spools thread at 25¢.
168 yd. striped denim at $8\frac{1}{3}$ ¢.

3. Without copying, find quickly the total cost of:

25 bu. corn at 64¢.
25 bu. corn at \$0.72.
$12\frac{1}{2}$ bu. oats at \$0.36.
25 bu. beans at \$2.80.
$12\frac{1}{2}$ bu. wheat at \$1.04.
$12\frac{1}{2}$ bu. millet at \$1.24.
25 bu. clover seed at \$3.60.
50 bu. clover seed at \$3.75.
25 bu. corn at \$0.84.
25 bu. corn at \$0.44.
25 bu. oats at \$0.35.
$12\frac{1}{2}$ bu. rye at \$1.04.
$6\frac{1}{4}$ bu. wheat at \$1.20.
$6\frac{1}{4}$ bu. wheat at \$1.12.
25 bu. timothy seed at \$2.40.
50 bu. timothy seed at \$2.75.

### ORAL EXERCISE

**1.** Multiply by 10: 4; 15; .07; 8¢; \$1.12; \$24.60; \$12.125.

**2.** Multiply by 100: 3; 17; .09; 12¢; \$1.64; \$21.17.

**3.** Multiply by 1000: 7; 29; .19; 15¢; \$1.75; \$23.72.

**4.** What aliquot part of \$10 is \$2.50? Find the cost of 16 articles at \$10 each; at \$2.50 each.

**5.** Find the cost of 84 bu. of wheat at \$1.25.

SOLUTION. \$1.25 is $\frac{1}{8}$ of \$10. 84 bu. at \$10 = \$840; $\frac{1}{8}$ of \$840 = \$105.

**6.** Formulate a short method for finding the cost when the quantity is given and the price is \$1.25.

SOLUTION. \$1.25 is $\frac{1}{8}$ of \$10; hence, *multiply the quantity by 10 and take* $\frac{1}{8}$ *of the product.*

**7.** Formulate a short method for finding the cost when the quantity is given and the price is \$2.50; $\$3.33\frac{1}{3}$; $\$1.66\frac{2}{3}$.

**8.** Find the cost of 168 yd. of cloth at \$1.25; at \$2.50; at $\$3.33\frac{1}{3}$; at $\$1.66\frac{2}{3}$.

**9.** What aliquot part of \$100 is \$25? \$12.50? \$6.25?

**10.** Find the cost of 72 chairs at \$25 each.

SOLUTION. 72 chairs at \$100 = \$7200; but the price is \$25, which is $\frac{1}{4}$ of \$100; therefore, $\frac{1}{4}$ of \$7200, or \$1800, is the required cost.

**11.** Give a short method for multiplying any number by 25; by $12\frac{1}{2}$; by $6\frac{1}{4}$; by $33\frac{1}{3}$; by $8\frac{1}{3}$.

**12.** Find the cost of 25 T. coal at \$7.20; of $6\frac{1}{4}$ T.; of $12\frac{1}{2}$ T.

**13.** What aliquot part of 1000 is 250? 500? 125? $62\frac{1}{2}$? $333\frac{1}{3}$? $166\frac{2}{3}$? 200? 100? $83\frac{1}{3}$? $66\frac{2}{3}$?

**14.** Formulate a short method for multiplying a number by 250.

SOLUTION. Since $250 = \frac{1000}{4}$, *multiply by 1000 and take* $\frac{1}{4}$ *of the product.*

**15.** Give a short method for finding the cost when the quantity is given and the price is \$125; $\$166\frac{2}{3}$.

**16.** Multiply 84 by 50; by 25; by $12\frac{1}{2}$; by $16\frac{2}{3}$; by $33\frac{1}{3}$.

**17.** Multiply 160 by $2\frac{1}{2}$; by $1\frac{1}{4}$; by $12\frac{1}{2}$; by 125; by $62\frac{1}{2}$.

**18.** Multiply 240 by $3\frac{1}{3}$; by $1\frac{2}{3}$; by $33\frac{1}{3}$; by $16\frac{2}{3}$; by $333\frac{1}{3}$.

**19.** Find the cost of 250 sofa beds at $32 each.

Solution. The cost of 250 beds at $32 = the cost of 32 beds at $250. The cost of 32 beds at $1000 = $32,000; but the price is $250, which is ¼ of $1000; therefore, ¼ of $32,000, or $8000, is the required cost.

**20.** Find the cost of 720 couches at $12.50 each.

**21.** Find the cost of 440 lb. sugar at 2½¢.

Solution. 2½¢ is ¼ of 10¢. The cost of 440 lb. at 10¢ = $44; but the price is 2½¢, therefore, ¼ of $44, or $11 = the required cost.

**22.** Formulate a short method for finding the cost when the quantity is given and the price is 1¼¢.

Solution. 1¼¢ = ⅛ of 10¢; hence, *point off one place in the quantity and take ⅛ of the result.*

**23.** Give a short method for finding the cost when the quantity is given and the price is 2½¢; 3⅓¢; 1⅔¢.

**24.** Find the cost of 160 lb. at 2½¢; at 1¼¢; at 3⅓¢; at 1⅔¢. Also of 240 lb. at each of these prices.

**25.** Find the cost of 2400 lb. at 2½¢; at 1¼¢; at 3⅓¢; at 1⅔¢. Also of 360 lb. at each of these prices.

### ORAL EXERCISE

*By inspection find the cost of:*

**1.** 25 lb. tea at 54¢.
**2.** 25 lb. tea at 33⅓¢.
**3.** 125 lb. tea at 64¢.
**4.** 6¼ A. land at $112.
**5.** 25 T. coal at $8.40.
**6.** 25 T. coal at $5.20.
**7.** 18 T. coal at $6.25.
**8.** 164 A. land at $25.
**9.** 72 T. coal at $6.25.
**10.** 250 yd. silk at 88¢.
**11.** 250 yd. silk at 96¢.
**12.** 25 pc. lace at $6.60.
**13.** 250 yd. silk at $1.12.
**14.** 192 A. land at $12.50.
**15.** 165 gro. buttons at 33⅓¢.
**16.** 1¼ yd. silk at 88¢.
**17.** 64 pc. lace at $1.25.
**18.** 125 yd. silk at $1.12.
**19.** 1250 bbl. beef at $24.
**20.** 78 yd. velvet at $2.50.
**21.** 2½ bu. potatoes at 96¢.
**22.** 640 bu. apples at 87½¢.
**23.** 840 yd. prints at 16⅔¢.
**24.** 12½ bu. potatoes at 64¢.
**25.** 84 bookcases at $12.50.
**26.** 810 bbl. pork at $12.50.
**27.** 125 yd. crepon at $3.60.
**28.** 12½ yd. cheviot at $1.04.
**29.** 24 oak sideboards at $125.
**30.** 12½ yd. gunner's duck at 48¢.

### WRITTEN EXERCISE

*In the following problems make all the extensions mentally. See how many of the problems can be done in 10 minutes.*

**1.** Without copying, find the total cost of:

425 lb. at 10¢.
310 lb. at 20¢.
100 lb. at 14¢.
1000 lb. at 27¢.
1000 lb. at $4\frac{1}{2}$¢.
1250 lb. at 44¢.
2500 lb. at 64¢.
1600 lb. at 25¢.
1893 lb. at $3\frac{1}{3}$¢.
2500 lb. at 14¢.
1400 lb. at 25¢.
1250 lb. at 88¢.
24 lb. at $1\frac{1}{4}$¢.
48 lb. at $2\frac{1}{2}$¢.
$2\frac{1}{2}$ lb. at 96¢.
125 lb. at 24¢.
192 lb. at $3\frac{1}{3}$¢.
88 lb. at $12\frac{1}{2}$¢.

**2.** Without copying, find the total cost of:

88 yd. at $1\frac{1}{4}$¢.
72 yd. at $3\frac{1}{3}$¢.
104 yd. at $2\frac{1}{2}$¢.
480 yd. at $6\frac{1}{4}$¢.
360 yd. at $8\frac{1}{3}$¢.
121 yd. at 11¢.
174 yd. at 10¢.
123 yd. at 11¢.
127 yd. at 11¢.
246 yd. at 25¢.
$171^2$ yd. at 10¢.
$178^3$ yd. at 10¢.
24 yd. at 12¢.
78 yd. at $3\frac{1}{3}$¢.
165 yd. at 20¢.
114 yd. at $6\frac{2}{3}$¢.
1280 yd. at $6\frac{1}{4}$¢.
192 yd. at $33\frac{1}{3}$¢.

**3.** Copy and find the total cost of:

450 lb. at $1\frac{1}{2}$¢.
820 lb. at $1\frac{1}{2}$¢.
1200 lb. at $4\frac{1}{4}$¢.
1400 lb. at $6\frac{1}{2}$¢.
7961 lb. at 50¢.
1293 lb. at 30¢.
1480 lb. at 40¢.
249 lb. at 25¢.
240 lb. at $3\frac{1}{3}$¢.
200 lb. at $3\frac{1}{2}$¢.
450 lb. at $6\frac{2}{3}$¢.
$79\frac{1}{2}$ lb. at 40¢.
$78\frac{1}{2}$ lb. at 50¢.
750 lb. at $33\frac{1}{3}$¢.
$6\frac{1}{4}$ lb. at 88¢.
92 lb. at $2\frac{1}{2}$¢.
$12\frac{1}{2}$ lb. at 24¢.
18 lb. at $4\frac{1}{2}$¢.
125 lb. at 18¢.
648 lb. at $6\frac{1}{4}$¢.
1900 lb. at $4\frac{1}{4}$¢.

**4.** Copy and find the total cost of:

750 gal. at $8\frac{1}{3}$¢.
488 gal. at $6\frac{2}{3}$¢.
640 gal. at $6\frac{1}{4}$¢.
194 gal. at 50¢.
176 gal. at 25¢.
280 gal. at $12\frac{1}{2}$¢.
720 gal. at $33\frac{1}{3}$¢.
366 gal. at $16\frac{2}{3}$¢.
99 gal. at 30¢.
60 gal. at $6\frac{2}{3}$¢.
50 gal. at 76¢.
25 gal. at 74¢.
$12\frac{1}{2}$ gal. at 88¢.
79 gal. at $33\frac{1}{3}$¢.
20 gal. at $1.79.
$6\frac{1}{4}$ gal. at $1.96.
360 gal. at 5¢.
625 gal. at 64¢.
810 gal. at $1\frac{1}{2}$¢.
920 gal. at $2\frac{1}{2}$¢.
165 gal. at $6\frac{2}{3}$¢.
240 gal. at $62\frac{1}{2}$¢.
666 gal. at $66\frac{2}{3}$¢.
1680 gal. at $16\frac{2}{3}$¢.

### ORAL EXERCISE

**1.** How much less than $1 is 75¢? what fractional part of $1 less?

**2.** Find the cost of 144 pc. of lace at 75¢ per piece.

Solution. At $1 per piece the cost would be $144; but the cost is not $1 but $\frac{1}{4}$ less than $1. Deducting $\frac{1}{4}$ of $144, the result is $108, the required cost.

**3.** Find the cost of 124 bookcases at $7.50.

Solution. $7.50 is $\frac{1}{4}$ less than $10. $1240 less $\frac{1}{4}$ of itself = $930, the required result.

**4.** Formulate a rule for multiplying a number by .75; by $7\frac{1}{2}$; by 75; by 750.

**5.** How much more than $1 is $1.12$\frac{1}{2}$? What fractional part of $1 more?

**6.** Find the cost of 84 yd. of silk at $1.16$\frac{2}{3}$ per yard.

Solution. At $1 per yard, the cost would be $84; but $1.16$\frac{2}{3}$ is $\frac{1}{6}$ more than $1. Adding $\frac{1}{6}$ of $84 to itself, the result is $98, the required cost.

**7.** Formulate a short method for finding the cost when the quantity is given and the price is $1.12$\frac{1}{2}$; $1.16$\frac{2}{3}$; $1.33$\frac{1}{3}$; $11.25; $112.50.

**8.** How much less than $1 is 87$\frac{1}{2}$¢? what fractional part of $1 less? Formulate a short method for multiplying a number by 87$\frac{1}{2}$.

**9.** Formulate a short method for multiplying a number by .83$\frac{1}{3}$; by 1.25.

**10.** Compare the cost of 87$\frac{1}{2}$ yd. at 64¢ with the cost of 64 yd. at 87$\frac{1}{2}$¢.

### ORAL EXERCISE

*State the cost of:*

**1.** 24 yd. at 75¢.
**2.** 75 yd. at 24¢.
**3.** 192 yd. at 87$\frac{1}{2}$¢.
**4.** 240 yd. at 83$\frac{1}{3}$¢.
**5.** 87$\frac{1}{2}$ yd. at $2.48.
**6.** 176 yd. at $1.12$\frac{1}{2}$.
**7.** 87$\frac{1}{2}$ yd. at $2.88.
**8.** 25 yd. at 4¢.
**9.** 28 yd. at 75¢.
**10.** 27 yd. at 75¢.
**11.** 75 yd. at 84¢.
**12.** 75 yd. at 16¢.
**13.** 270 yd. at 11$\frac{1}{9}$¢.
**14.** 144 yd. at 11$\frac{1}{9}$¢.
**15.** 11$\frac{1}{9}$ yd. at 18¢.
**16.** 1125 yd. at 64¢.
**17.** 1125 yd. at 32¢.
**18.** 1125 yd. at 48¢.

## WRITTEN REVIEW EXERCISE

**1.** Find the total of the costs called for in problems 1–15 in the oral exercise at the top of page 139.

**2.** Find the total cost of the items in the oral exercise at the bottom of page 142; of the items in the oral exercise at the bottom of page 144.

**3.** Find the total cost of:

84 yd. at 7¢.
112½ yd. at 5¢.
112½ yd. at 6¢.
98 yd. at 9¢.
79 yd. at 11¢.
17 yd. at 16¢.
72 yd. at 75¢.
87½ yd. at 88¢.
320 yd. at 11¢.

**4.** Find the total cost of:

71 yd. at 22¢.
31 yd. at 44¢.
82 yd. at 88¢.
71 yd. at 72¢.
85 yd. at 30¢.
17 yd. at 25¢.
12½ yd. at 39¢.
250 yd. at 64¢.
30 yd. at 7½¢.
24 yd. at 8⅓¢.
56 yd. at 83⅓¢.
124 yd. at $1.12½.

**5.** Find the total cost of:

192 lb. at 3⅓¢.
384 lb. at 6¼¢.
378 lb. at 6¼¢.
149 lb. at 6¼¢.
167 lb. at 12½¢.
184 lb. at 37½¢.
2164 lb. at 2½¢.
1369 lb. at 2½¢.
1151¾ lb. at 10¢.
1721½ lb. at 15¢.
2911½ lb. at 33⅓¢.
2706 lb. at 33⅓¢.

**6.** Copy and find the amount of the following bills, less 3 %:

*a.*

Rochester, N.Y., Aug. 2, 19

MR. C. G. GARLIC
North Rose, N.Y.

*To* SMITH, PERKINS & CO., *Dr.*

Terms: cash, less 3 %.

| | | | | | |
|---|---|---|---|---|---|
| | 330 lb. Granulated Sugar | 6¼¢ | | | |
| | 32 " Butter | 22¢ | | | |
| | 64 " Cheese | 16⅔¢ | | | |
| | 75 " Young Hyson Tea | 24¢ | | | |
| | 155 " Dried Apples | 8¢ | | | |
| | 300 " Brown Sugar | 3½¢ | | | |
| | 60 " Oolong Tea | 51¢ | | | |
| | 125 " Rio Coffee | 28¢ | | | |
| | 250 " Mocha Coffee | 24¢ | | | |

*b.*

Buffalo, N.Y., Aug. 5, 19

Mr. George A. Collier
Savannah, N.Y.

*Bought of* George H. Buell & Co.

Terms: cash, less 3 %.

| | | | | |
|---|---|---|---|---|
| | 72 pr. Boys' Hose | 12½¢ | | |
| | 18 doz. Linen Handkerchiefs | 2.50 | | |
| | 18 " Lace Handkerchiefs | 3.33⅓ | | |
| | 78 yd. Silk Velvet | 3.33⅓ | | |
| | 75 pc. Black Ribbon | 28¢ | | |
| | 347 yd. Pontiac Seersucker | 6¼¢ | | |
| | 186 " Washington Cambric | 12½¢ | | |

### ORAL EXERCISE

**1.** At 33⅓¢ per pound, how many pounds of coffee can be bought for $12?

Solution. .33⅓ = $⅓; 3 pounds can be bought for $1; then, 12 × 3 lb. = 36 lb., the required result.

**2.** When the cost is given and the price is 25¢, how may the quantity be found?

Solution. When the price is 25¢, the quantity is 4 times the cost; hence, *multiply the cost by 4.*

**3.** Give a short method for finding the quantity when the cost is given and the price is 20¢; 33⅓¢; 12½¢; 6¼¢; 6⅔¢; 16⅔¢.

**4.** Formulate a short method for dividing any number by 125.

Solution. 125 is ⅛ of 1000; then the quotient by 125 will be 8 times the quotient by 1000. Therefore, *divide by 1000 and multiply the result by 8.* Or, $\frac{1}{125} = \frac{8}{1000}$. Therefore, *multiply by 8 and move the decimal point three places to the left.*

**5.** Give a short method for dividing by 6¼.

Solution. 6¼ = $\frac{1}{16}$ of 100; then the quotient by 6¼ will be 16 times the quotient by 100. Therefore, *move the decimal point two places to the left and multiply the result by 16.* Or, $\frac{1}{6\frac{1}{4}} = \frac{16}{100}$. Therefore, *multiply by 16 and move the decimal point two places to the left.*

**6.** Give a short method for dividing a number by $12\frac{1}{2}$; by $16\frac{2}{3}$; by $33\frac{1}{3}$; by $6\frac{1}{4}$; by $66\frac{2}{3}$; by $333\frac{1}{3}$; by $166\frac{2}{3}$.

**7.** Formulate a short method for dividing a number by .75.

SOLUTION. .75 increased by $\frac{1}{3}$ of itself = 1. When the divisor is 1 the quotient is the same as the dividend. Hence, to divide a number by .75 *increase the number by $\frac{1}{3}$ of itself.*

**8.** At 75¢ per bushel, how many bushels of wheat can be bought for \$144? for \$192? for \$240? for \$780? for \$1260? for \$360? for \$1350? for \$810?

**9.** At \$7.50 per dozen, how many dozen men's gloves can be bought for \$1440?

SOLUTION. \$7.50 + $\frac{1}{3}$ of itself = \$10. To divide by 10 is to point off one place to the left. \$ 1440 + $\frac{1}{3}$ of itself = \$1920 ; \$1920 ÷ \$10 = 192, the number of pairs of gloves.

**10.** State a short method for dividing a number by $7\frac{1}{2}$; by 75; by 750.

### ORAL EXERCISE

*Find the quantity:*

| | COST | PRICE PER YARD | | COST | PRICE PER POUND |
|---|---|---|---|---|---|
| **1.** | \$65 | $33\frac{1}{3}$¢ | **7.** | \$75 | $6\frac{2}{3}$¢ |
| **2.** | \$250 | 25¢ | **8.** | \$12 | $1\frac{2}{3}$¢ |
| **3.** | \$120 | $6\frac{1}{4}$¢ | **9.** | \$25 | $1\frac{1}{4}$¢ |
| **4.** | \$215 | $2\frac{1}{2}$¢ | **10.** | \$38 | $1\frac{1}{2}$¢ |
| **5.** | \$126 | $12\frac{1}{2}$¢ | **11.** | \$125 | \$1.25 |
| **6.** | \$125 | 20¢ | **12.** | \$420 | $12\frac{1}{2}$¢ |

### WRITTEN EXERCISE

*Find the quantity:*

| | COST | PRICE PER YARD | | COST | PRICE PER BUSHEL |
|---|---|---|---|---|---|
| **1.** | \$570.00 | 75¢ | **6.** | \$1721.00 | $33\frac{1}{3}$¢ |
| **2.** | \$612.00 | 75¢ | **7.** | \$1842.50 | 25¢ |
| **3.** | \$274.50 | $7\frac{1}{2}$¢ | **8.** | \$1785.00 | $87\frac{1}{2}$¢ |
| **4.** | \$281.50 | $12\frac{1}{2}$¢ | **9.** | \$2142.00 | $33\frac{1}{3}$¢ |
| **5.** | \$864.50 | $12\frac{1}{2}$¢ | **10.** | \$2720.50 | $16\frac{2}{3}$¢ |

### REVIEW EXERCISE

This exercise may be used in a number of different ways, some of which are suggested below.

1. One student may make the oral extension, using the first quantity, 60 yd., by each price in column 1; a second student may use the same quantity and make the extension by each price in column 2, and so on for the ten lists.

2. Each student in the class may take the same quantity and make the extension by each price in column 1, and foot the extensions. Compare results. Such an exercise should occupy one minute. This work may be continued for ten or fifteen minutes daily, as the instructor desires, a different quantity being used for each minute.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 50¢ | 25¢ | $66\frac{2}{3}$¢ | 60¢ | 25¢ | $6\frac{1}{4}$¢ | 50¢ | 75¢ | \$1.50 | \$$1.33\frac{1}{3}$ |
| 20¢ | $62\frac{1}{2}$¢ | $6\frac{2}{3}$¢ | $87\frac{1}{2}$¢ | 20¢ | $16\frac{2}{3}$¢ | 90¢ | $8\frac{1}{3}$¢ | \$1.25 | \$$1.66\frac{2}{3}$ |
| $12\frac{1}{2}$¢ | $33\frac{1}{3}$¢ | 75¢ | $37\frac{1}{2}$¢ | 60¢ | 30¢ | 10¢ | $37\frac{1}{2}$¢ | \$$1.12\frac{1}{2}$ | \$1.20 |
| $16\frac{2}{3}$¢ | 30¢ | $8\frac{1}{3}$¢ | $6\frac{1}{4}$¢ | $12\frac{1}{2}$¢ | $66\frac{2}{3}$¢ | $62\frac{1}{2}$¢ | 80¢ | \$1.10 | \$$1.16\frac{2}{3}$ |
| 90¢ | 10¢ | 40¢ | 80¢ | $33\frac{1}{3}$¢ | $6\frac{2}{3}$¢ | 40¢ | $87\frac{1}{2}$¢ | \$1.75 | \$1.30 |

**1.** Find the cost of:

| | | | | |
|---|---|---|---|---|
| *a.* 60 yd. | *b.* 72 yd. | *c.* 90 yd. | *d.* 75 yd. | *e.* 48 yd. |
| 80 yd. | 50 yd. | 20 yd. | 84 yd. | 96 yd. |
| 40 yd. | 44 yd. | 25 yd. | 54 yd. | 64 yd. |

**2.** Find the cost of:

| | | | | |
|---|---|---|---|---|
| *a.* 24 yd. | *b.* 78 yd. | *c.* 12 yd. | *d.* 92 yd. | *e.* 15 yd. |
| 16 yd. | 69 yd. | 36 yd. | 21 yd. | 18 yd. |
| 32 yd. | 81 yd. | 42 yd. | 46 yd. | 10 yd. |

**3.** Find the cost of:

| | | | | |
|---|---|---|---|---|
| *a.* 100 yd. | *b.* 108 yd. | *c.* 160 yd. | *d.* 200 yd. | *e.* 320 yd. |
| 120 yd. | 135 yd. | 180 yd. | 240 yd. | 400 yd. |
| 150 yd. | 144 yd. | 128 yd. | 300 yd. | 360 yd. |

## WRITTEN REVIEW EXERCISE

| NAME | QUANTITY | PRICES | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Boucle Stripe | yd. | $0.10½ | $0.11 | $0.10 | $0.11½ | $0.12 | $0.12½ |
| Dress Silks | yd. | 1.20 | 1.25 | 1.33⅓ | 1.37½ | 1.40 | 1.50 |
| English Serge | yd. | 1.33⅓ | 1.30 | 1.35 | 1.25 | 1.37½ | 1.45 |
| Fancy Gingham | yd. | .06¼ | .06 | .06½ | .07 | .07½ | .07¼ |
| Fancy Plaids | yd. | .31½ | .32 | .33⅓ | .35 | .34 | .37½ |
| Gunner's Duck | yd. | .14 | .15 | .14½ | .16 | .17 | .17½ |
| Percale Shirting | yd. | .07 | .07½ | .08 | .08½ | .09 | .09½ |
| Scotch Cheviot | yd. | .39 | .40 | .37½ | .45 | .44 | .48 |
| Taffeta Silk | yd. | .87½ | .85 | .90 | .88 | .92½ | .95 |
| Wash Silk | yd. | .30 | .37½ | .40 | .35 | .42 | .41½ |

*Prepare each of the following invoices in correct form, omit the heading, and find the value by each price list.*

**1.** 22 yd. Boucle Stripe
27 yd. English Serge
56 yd. Percale Shirting
48 yd. Wash Silk

**2.** 72 yd. Dress Silk
104 yd. Scotch Cheviot
64 yd. Taffeta Silk
60 yd. Gunner's Duck

**3.** 36 yd. Fancy Gingham
88 yd. Scotch Cheviot
92 yd. Wash Silk
120 yd. Boucle Stripe
80 yd. Gunner's Duck

**4.** 96 yd. Fancy Plaids
36 yd. English Serge
100 yd. Taffeta Silk
70 yd. Percale Shirting
90 yd. Wash Silk

**5.** 72 yd. Boucle Stripe
56 yd. Wash Silk
50 yd. Fancy Plaids
80 yd. Dress Silk
70 yd. Gunner's Duck

**6.** 84 yd. Taffeta Silk
66 yd. Fancy Gingham
84 yd. Scotch Cheviot
66 yd. English Serge
45 yd. Percale Shirting

# CHAPTER XI

## BILLS AND ACCOUNTS

### BILLS

**172.** A detailed statement of goods sold, or of goods bought to be sold, is called either a **bill** or an **invoice.** A detailed statement of goods bought to be used or consumed, such as office furniture, stationery, and fuel, or a statement of services rendered, or of a work performed, is called a **bill.**

Thus, a physician's statement of services rendered, or a transportation company's bill for work performed, and the charges for the same, is called a *bill;* but a statement of a quantity of silk bought or sold by a dry-goods merchant in the course of trade is called either a *bill* or an *invoice.*

**173.** The models following show a variety of current practices in billing. They will therefore be found helpful as studies.

### 1. GROCERIES

**Boston, Mass.,** Oct. 15, **19**

Messrs. SMITH, PERKINS & CO.
Rochester, N.Y.

**Bought of E. E. GRAY COMPANY**

**Terms** 30 da. **Telephone,** Main 167

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | bbl. Rolled Oats | $6.25 | 18 | 75 | | |
| 10 | " Gold Medal Flour | 6.50 | 65 | 00 | | |
| 5 | bx. Wool Soap | 3.10 | 15 | 50 | 99 | 25 |

This is one of the simplest bill forms; it is the form that is common in a great many lines of business.

### 2. Groceries

Boston, Mass., Nov. 12, 19

Messrs. E. O. Sherman & Co.
Charlestown, Mass.

Bought of S. S. PIERCE COMPANY

Terms 30 da.; 3% 10 da.

| | | | | |
|---|---|---|---|---|
| 10 Red Label Hams | 146 lb. | | $0.23 | $33.58 |
| 20 mats Java Coffee | 1500 | " | .25 | 375.00 |
| 12 6-lb. tins Mustard | 72 | " | .36 | 25.92 |
| 15 6-lb. tins Cocoa | 90 | " | .34 | 30.60 |
| | | | | $465.10 |

Goods bought by the mat, chest, case, etc., are frequently billed by the pound. The above bill shows the form in such cases.

### 3. Hardware

The following bill is sometimes used in the hardware business. The first number after the name of the article is the quantity; the number above the horizontal line following, the price; and the number below the line, the grade. Thus the first item in the bill shows that 12 doz. porcelain knobs in all were sold, of which 6 doz. were No. 8 at $1.25 and 6 doz. No. 16 at $1.33⅓.

New York, Apr. 6, 19

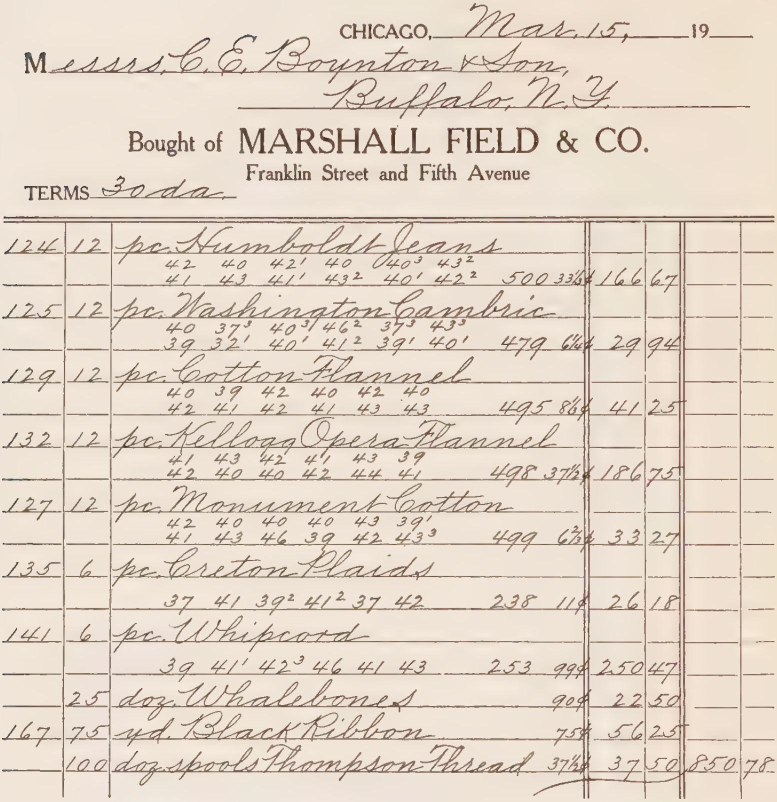
Messrs. L. M. Shipman & Son
Springfield, Mass.

Bought of The Eureka Hardware Company

Terms 3% 10 da.; net 30 da.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 12 | doz. Porcelain Knobs 6 $\frac{1^{25}}{\#8}$ 6 $\frac{1^{33\frac{1}{3}}}{\#16}$ | 15 | 50 | | |
| | 15 | " Steelyards 3 $\frac{8^{50}}{\#17}$ 12 $\frac{9^{75}}{\#64}$ | 142 | 50 | | |
| | 25 | " Jack Planes 5 $\frac{6^{25}}{\#14}$ 5 $\frac{6^{00}}{\#21}$ 15 $\frac{6^{66\frac{2}{3}}}{\#28}$ | 161 | 25 | 319 | 25 |

## 4. Wholesale Dry Goods



CHICAGO, Mar. 15, 19

Messrs. C. E. Boynton & Son,
Buffalo, N. Y.

Bought of **MARSHALL FIELD & CO.**
Franklin Street and Fifth Avenue

TERMS 30 da.

| | | | | | | |
|---|---|---|---|---|---|---|
| 124 | 12 | pc. Humboldt Jeans | | | | |
| | | 42 40 $42^1$ 40 $40^3$ $43^2$<br>41 43 $41^1$ $43^2$ $40^1$ $42^2$ 500 33⅓¢ | 166 | 67 | | |
| 125 | 12 | pc. Washington Cambric | | | | |
| | | 40 $37^3$ $40^3$ $46^2$ $37^3$ $43^3$<br>39 $32^1$ $40^1$ $41^2$ $39^1$ $40^1$ 479 6¼¢ | 29 | 94 | | |
| 129 | 12 | pc. Cotton Flannel | | | | |
| | | 40 39 42 40 42 40<br>42 41 42 41 43 43 495 8⅓¢ | 41 | 25 | | |
| 132 | 12 | pc. Kellogg Opera Flannel | | | | |
| | | 41 43 42 41 43 39<br>42 40 40 42 44 41 498 37½¢ | 186 | 75 | | |
| 127 | 12 | pc. Monument Cotton | | | | |
| | | 42 40 40 40 43 $39^1$<br>41 43 46 39 42 $43^3$ 499 6⅔¢ | 33 | 27 | | |
| 135 | 6 | pc. Creton Plaids | | | | |
| | | 37 41 $39^2$ $41^2$ 37 42 238 11¢ | 26 | 18 | | |
| 141 | 6 | pc. Whipcord | | | | |
| | | 39 $41^1$ $42^3$ 46 41 43 253 99¢ | 250 | 47 | | |
| | 25 | doz. Whalebones 90¢ | 22 | 50 | | |
| 167 | 75 | yd. Black Ribbon 75¢ | 56 | 25 | | |
| | 100 | doz. spools Thompson Thread 37½¢ | 37 | 50 | 850 | 78 |

In the wholesale dry-goods business the price is generally for a yard, and the number of yards to the piece varies in some kinds of cloth. The first item in the above bill is followed by a series of numbers, 41, 42, etc.; these represent the number of yards in each of the 12 pc. Immediately following these numbers is recorded the total number of yards in the 12 pc. The total number of yards should be found by horizontal addition.

## 5. Manufacturer's

The following is a bill for neckwear. The different styles are distinguished by the marks at the left of the quantity. This form is common among manufacturers, jobbers, and wholesalers. Bills on which trade discounts (see page 188) are allowed are arranged as shown in this bill.

New York, Oct. 10, 19

Messrs. J. E. Whiting & Co.
Boston, Mass.

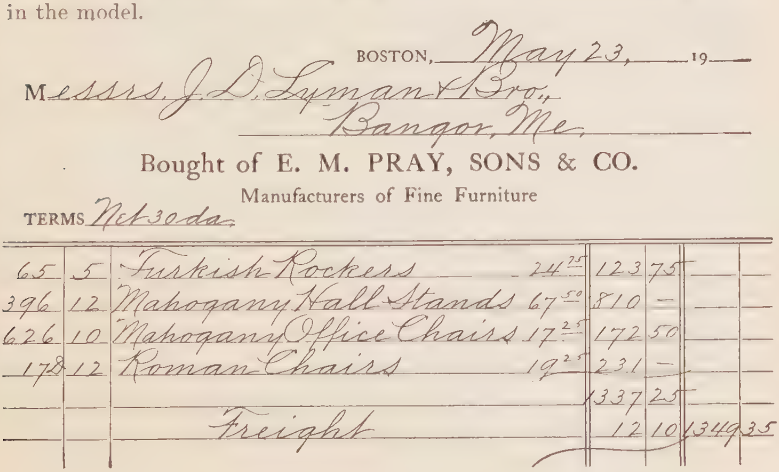
**Bought of Johnson Bros., Sons & Co.**

Terms Net 30 da.

| | | | | | |
|---|---|---|---|---|---|
| 721 | 1½ | doz. Neckwear | $4.50 | 6 75 | |
| 1026 | ½ | | 27.00 | 13 50 | |
| 1025 | 1½ | | 27.50 | 41 25 | |
| 1020 | ¾ | | 9.00 | 6 75 | |
| 923 | 2½ | | 18.00 | 45 00 | |
| 1015 | 1¾ | | 24.00 | 42 00 | |
| | | | | 155 25 | |
| | | Less 2% | | 3 11 | 152 14 |

## 6. Furniture

In the following bill the goods were sold delivered on the cars (f. o. b.) Boston, but the shippers prepaid the freight to Bangor. The freight is a part of the selling price and is added to the amount of the bill, as shown in the model.



BOSTON, May 23, 19

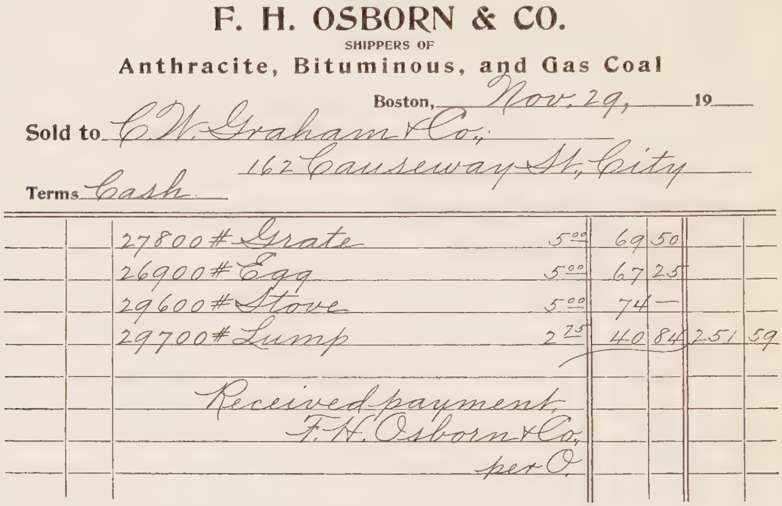
Messrs. J. D. Lyman & Bro.,
Bangor, Me.

Bought of E. M. PRAY, SONS & CO.
Manufacturers of Fine Furniture

TERMS Net 30 da.

| | | | | | |
|---|---|---|---|---|---|
| 65 | 5 | Turkish Rockers | 24.75 | 123 75 | |
| 396 | 12 | Mahogany Hall Stands | 67.50 | 810 — | |
| 626 | 10 | Mahogany Office Chairs | 17.25 | 172 50 | |
| 178 | 12 | Roman Chairs | 19.25 | 231 — | |
| | | | | 1337 25 | |
| | | Freight | | 12 10 | 1349 35 |

## 7. Wholesale Coal



**F. H. OSBORN & CO.**

SHIPPERS OF

**Anthracite, Bituminous, and Gas Coal**

Boston, Nov. 29, 19

Sold to C. W. Graham & Co.,

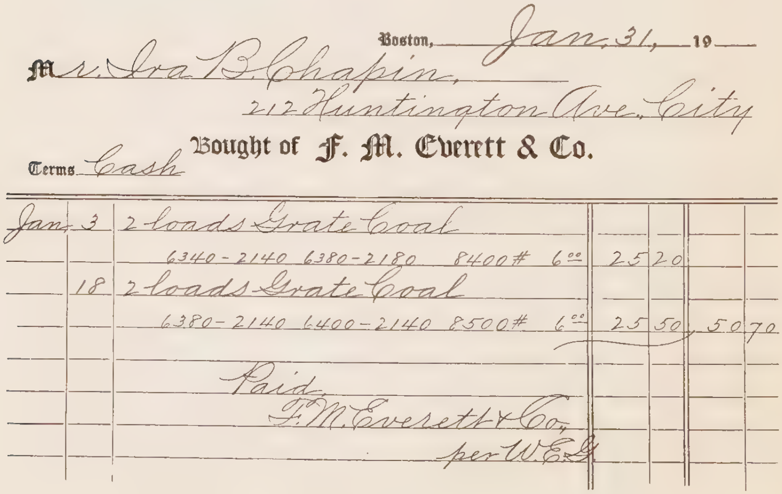
162 Causeway St., City

Terms Cash

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 27800# Grate | 5.00 | 69 | 50 | | |
| | | 26900# Egg | 5.00 | 67 | 25 | | |
| | | 29600# Stove | 5.00 | 74 | — | | |
| | | 29700# Lump | 2.75 | 40 | 84 | 251 | 59 |
| | | | | | | | |
| | | Received payment, | | | | | |
| | | F. H. Osborn & Co., | | | | | |
| | | per O. | | | | | |

The above is the form of bill generally used for wholesale transactions in coal. It is called a **receipted bill,** and shows that the coal has been paid for.

## 8. Retail Coal



Boston, Jan. 31, 19

Mr. Ira B. Chapin,

212 Huntington Ave., City

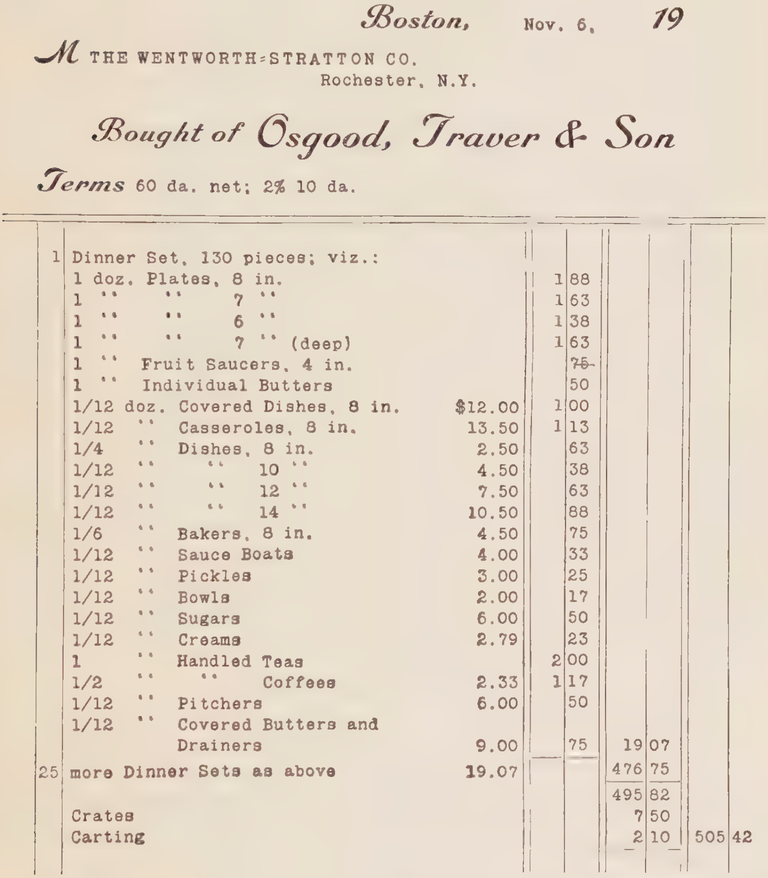
Bought of **F. M. Everett & Co.**

Terms Cash

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Jan. | 3 | 2 loads Grate Coal | | | | | |
| | | 6340-2140 6380-2180 8400# | 6.00 | 25 | 20 | | |
| | 18 | 2 loads Grate Coal | | | | | |
| | | 6380-2140 6400-2140 8500# | 6.00 | 25 | 50 | 50 | 70 |
| | | | | | | | |
| | | Paid, | | | | | |
| | | F. M. Everett & Co., | | | | | |
| | | per W. E. G. | | | | | |

The foregoing bill shows a form sometimes used by retailers. The numbers at the left of the hyphen are the gross weights, and the numbers at the right the tares of the different loads.

## 9. China and Glassware



*Boston,* Nov. 6, *19*

*M* THE WENTWORTH-STRATTON CO.
Rochester, N.Y.

*Bought of* ***Osgood, Traver & Son***

*Terms* 60 da. net; 2% 10 da.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | Dinner Set, 130 pieces; viz.: | | | | | | | |
| | 1 doz. Plates, 8 in. | | 1 | 88 | | | | |
| | 1 " " 7 " | | 1 | 63 | | | | |
| | 1 " " 6 " | | 1 | 38 | | | | |
| | 1 " " 7 " (deep) | | 1 | 63 | | | | |
| | 1 " Fruit Saucers, 4 in. | | | 75 | | | | |
| | 1 " Individual Butters | | | 50 | | | | |
| | 1/12 doz. Covered Dishes, 8 in. | $12.00 | 1 | 00 | | | | |
| | 1/12 " Casseroles, 8 in. | 13.50 | 1 | 13 | | | | |
| | 1/4 " Dishes, 8 in. | 2.50 | | 63 | | | | |
| | 1/12 " " 10 " | 4.50 | | 38 | | | | |
| | 1/12 " " 12 " | 7.50 | | 63 | | | | |
| | 1/12 " " 14 " | 10.50 | | 88 | | | | |
| | 1/6 " Bakers, 8 in. | 4.50 | | 75 | | | | |
| | 1/12 " Sauce Boats | 4.00 | | 33 | | | | |
| | 1/12 " Pickles | 3.00 | | 25 | | | | |
| | 1/12 " Bowls | 2.00 | | 17 | | | | |
| | 1/12 " Sugars | 6.00 | | 50 | | | | |
| | 1/12 " Creams | 2.79 | | 23 | | | | |
| | 1 " Handled Teas | | 2 | 00 | | | | |
| | 1/2 " " Coffees | 2.33 | 1 | 17 | | | | |
| | 1/12 " Pitchers | 6.00 | | 50 | | | | |
| | 1/12 " Covered Butters and Drainers | 9.00 | | 75 | 19 | 07 | | |
| 25 | more Dinner Sets as above | 19.07 | | | 476 | 75 | | |
| | | | | | 495 | 82 | | |
| | Crates | | | | 7 | 50 | | |
| | Carting | | | | 2 | 10 | 505 | 42 |

The above form is common in the china and glassware business. In this case a charge is made for the crates used in packing and the prices do not include delivery. The cost of the crate and the cost for carting are therefore made a part of the bill.

### 10. Lumber

**The H. M. Bickford Co.**

Boston, Mass., Oct. 8, 19

Sold to L. A. Hammond & Co.
Paterson, N.J.

Terms Fgt. net cash; bal. in 5 da. less 1½%

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 23,289 | ft. | ½ × 2½ | #1 | N. C. | Ceiling | $18.50 | $430.85 |
| 3,520 | " | " | 2 | " " | " | 17.00 | 59.84 |
| 10,307 | " | ⅜ × 2½ | 1 | " " | " | 13.50 | 139.14 |
| 1,690 | " | " | 2 | " " | " | 12.50 | 21.13 |
| | | | | | | | $650.96 |
| | Less freight (45,200 lb. at 24¢) | | | | | | 108.48 |
| | | | | | | | $542.48 |

Lumber is generally sold by the thousand feet. In the above bill the goods were sold free on board cars (f. o. b.) Paterson, N.J., but the shippers have not prepaid the freight. They find that these charges are $108.48 and deduct this amount from the total of the bill. In the wholesale lumber business the prices quoted usually include the cost of delivery, and when the freight charges are not known at the time of making the shipment, they are paid by the consignees and deducted from the amount of the bill on the arrival of the goods. The freight bill is then sent to the shippers for credit.

#### WRITTEN EXERCISE

**1.** Study the model bill, page 150. Increase the price of each article 25¢ and then copy and find the amount of the bill.

**2.** Study the first model bill, page 151, and then copy and find the amount of it at the following prices: hams, 27¢; coffee, 23¢; mustard, 31¢; cocoa, 39¢.

**3.** Study the second model bill, page 151, and then copy and find the amount of it at the following prices: porcelain knobs #8, $1.12½; #16, $1.25; steelyards #64, $11; #17, $8.33⅓; jack-planes #14, $6; #21, $6.25; #48, $6.75.

**4.** Apr. 15, you bought of S. S. Pierce Co., Boston, Mass., for cash: 25 gal. finest New Orleans molasses at 48¢; 15 gal. fancy sugar-house sirup at 49¢; 75 lb. raw mixed coffee at 29¢; 25 lb. raw Pan-American coffee at 19¢; 5 cartons Fowle's entire-wheat flour at $39\frac{1}{2}$¢; $\frac{1}{2}$ bbl. Franklin Mills flour at \$6.75; $\frac{1}{8}$ bbl. pastry flour at \$5.25. Write the bill.

**5.** Mar. 19, Frank M. Richmond & Co., New York City, sold to Charles M. Thompson, Poughkeepsie, N.Y., 12 doz. porcelain knobs: 3 doz. #71 at \$6.35, 9 doz. #74 at \$6.75; 12 doz. shingle hatchets: 6 doz. #16 at \$9.75, 6 doz. #34 at \$12.50; 7 doz. steel squares: 3 doz. #91 at \$35, 4 doz. #73 at \$33. Terms: 30 da. Write the bill.

**6.** Study the model bill on page 152. Increase the prices of the articles marked 124 and 132 five cents each and the remainder of the articles one cent each; then copy and find the amount of the bill.

**7.** Nov. 15, J. B. Ford & Co., Albany, N.Y., bought of the Clinton Mills, Little Falls, N.Y., 10 pc. percale shirting containing 42, 48, $52^1$, 58, 62, 38, 49, 51, 54, and $46^1$ yd., at $7\frac{1}{2}$¢; 10 pc. fine wool cheviot containing $58^1$, 42, 49, 51, $44^2$, 46, 48, $41^2$, 39, and 42 yd., at \$$1.12\frac{1}{2}$; 5 pc. cashmere containing $49^3$, $40^1$, $48^3$, $49^1$, and 49 yd. at \$$1.37\frac{1}{2}$. Terms: 60 da., or 3% discount for cash within 10 da. Write the bill.

**8.** Study the first model bill on page 153. Increase the prices of styles 1026, 1025, 1020, and 923, 25¢ each and diminish the prices of the other styles 25¢ each; then copy and find the amount of the bill. Omit the discount.

**9.** Sept. 24, Geo. W. Fairchild, Buffalo, N.Y., bought of E. M. Lawrence & Co., New York City, silk ribbon as follows: 12 pc. #1142 at \$2.25; 5 pc. #1321 at \$1.25; 25 pc. #171 at \$$4.37\frac{1}{2}$; 8 pc. #1927 at \$1.75; 36 pc. #2114 at \$$1.66\frac{2}{3}$; 15 pc. #1371 at \$$1.33\frac{1}{2}$; 15 pc. #624 at \$$4.37\frac{1}{2}$; 12 pc. #909 at \$$1.87\frac{1}{2}$; 25 pc. #1008 at \$$3.33\frac{1}{3}$; 25 pc. #1246 at \$4.75; 18 pc. #2119 at \$$1.12\frac{1}{2}$. Terms: 30 da., or 2% discount for cash in 10 da. Write the bill.

**10.** Study the second model bill on page 153. Increase the price of the articles marked 65 and 396, 25¢ each, and diminish the price of the other articles $12\frac{1}{2}$¢ each; then copy and find the amount of the bill. Freight added, $14.70.

**11.** July 20, The Hayden Furniture Co., Rochester, N.Y., bought of John H. Pray & Son, Boston, Mass., 25 #31 card tables at $11; 24 #94 china closets at $27.50; 15 #16 dining sets at $85; 25 #3060 fancy rockers at $9.25; 15 #35 music cabinets at $2.75; 25 #26 mahogany office chairs at $12.50; 12 #89 oak sideboards at $125. Terms: 30 da. The prices are free on board Boston, and the shipper prepaid the freight, $34.50. Write the bill.

**12.** Study the first model bill on page 154. Increase the price of the stove coal 25¢ per ton and the price of each of the other kinds $12\frac{1}{2}$¢ per ton; then copy and find the amount of the bill. Receipt the bill for F. H. Osborn & Co.

**13.** May 19, C. E. Williams & Co., Cleveland, O., bought of Fairbanks & Co., Scranton, Pa.: 3 car loads of stove coal weighing 20,500, 26,400, and 25,600 lb., respectively, at $4.75 per ton (2000 lb.); 1 car load grate coal weighing 21,900 lb. at $4.25 per ton; 1 car load cannel coal weighing 22,500 lb. at $7.75 per ton. Terms: 30 da., or 3% discount for cash in 10 da. Write the bill.

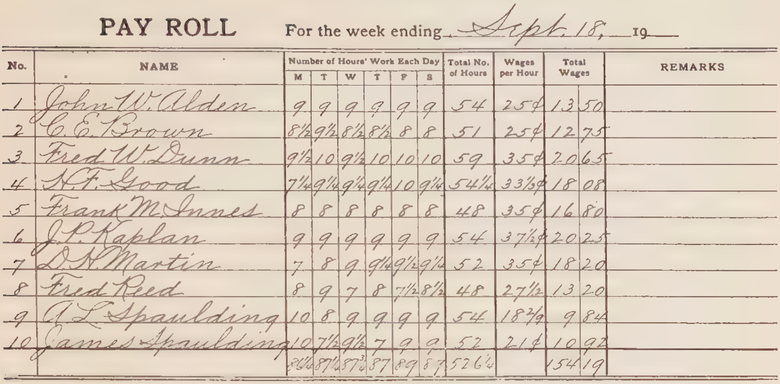
**14.** Study the second model bill, page 154, then copy and find the amount of it at $6.25 per ton for each sale.

**15.** Copy the bill in problem 14 in accordance with the model shown on page 154. Make the price of the coal $6.66\frac{2}{3}$.

**16.** Study the model bill on page 155. Increase each price given five cents and then copy and find the amount of the bill. Cost of crates used in packing, $6.40; carting, $2.80.

**17.** July 15, Henry Nelson & Co., Portland, Me., bought of Jones, Stratton & Co., New York City, 5 doz. plates, 8 in., at $1.50; 35 doz. plates, 7 in., at $1.35; 15 doz. plates, 6 in., at $1.10; 10 doz. plates, 5 in., at 90¢; 65 doz. handled teas at $1.85. Terms: 30 da. Cost of crate used in packing, $2; cartage, 75¢. Write the bill.

## PAY ROLLS



**PAY ROLL** For the week ending Sept. 18, 19

| No. | NAME | Number of Hours' Work Each Day M | T | W | T | F | S | Total No. of Hours | Wages per Hour | Total Wages | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | John W. Alden | 9 | 9 | 9 | 9 | 9 | 9 | 54 | 25¢ | 13 50 | |
| 2 | C. E. Brown | 8½ | 9½ | 8½ | 8½ | 8 | 8 | 51 | 25¢ | 12 75 | |
| 3 | Fred W. Dunn | 9½ | 10 | 9½ | 10 | 10 | 10 | 59 | 35¢ | 20 65 | |
| 4 | H. F. Good | 7¼ | 9¼ | 9¼ | 9¼ | 10 | 9¼ | 54¼ | 33⅓¢ | 18 08 | |
| 5 | Frank McInnes | 8 | 8 | 8 | 8 | 8 | 8 | 48 | 35¢ | 16 80 | |
| 6 | J. P. Kaplan | 9 | 9 | 9 | 9 | 9 | 9 | 54 | 37½¢ | 20 25 | |
| 7 | D. H. Martin | 7 | 8 | 9 | 9¼ | 9½ | 9¼ | 52 | 35¢ | 18 20 | |
| 8 | Fred Reed | 8 | 9 | 7 | 8 | 7½ | 8½ | 48 | 27½ | 13 20 | |
| 9 | A. L. Spaulding | 10 | 8 | 9 | 9 | 9 | 9 | 54 | 18²⁄₉ | 9 84 | |
| 10 | James Spaulding | 10 | 7½ | 9½ | 7 | 9 | 9 | 52 | 21¢ | 10 92 | |
| | | 86¼ | 87¼ | 87¾ | 87 | 89 | 87 | 526¼ | | 154 19 | |

This form is most common among manufacturing establishments, but it is also used by printers, contractors, and builders.

Checks are sometimes used in paying off employees, but most large concerns find the envelope system the most convenient and satisfactory. To pay off employees by the envelope system it is necessary for the bookkeeper to find first the amount of money required and then the bills and fractional currency that are necessary to pay each employee. The amount required is the total of the pay roll, and the bills and fractional currency desired may be found as shown in the following illustration. This illustration, called **a change memorandum**, shows the method of finding just the denominations wanted for the pay roll at the top of the page. A change memorandum may be proved correct as shown in the **pay-roll memorandum** at the top of page 160.

| No. | Bills | | | | | Coins | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $20 | $10 | $5 | $2 | $1 | 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |
| 1 | | 1 | | 1 | 1 | 1 | | | | |
| 2 | | 1 | | 1 | | 1 | 1 | | | |
| 3 | 1 | | | | | 1 | | 1 | 1 | |
| 4 | | 1 | 1 | 1 | 1 | | | | 1 | 3 |
| 5 | | 1 | 1 | | 1 | 1 | 1 | | 1 | |
| 6 | 1 | | | | | | 1 | | | |
| 7 | | 1 | 1 | 1 | 1 | | | 2 | | |
| 8 | | 1 | | 1 | 1 | | | 2 | | |
| 9 | | | 1 | 2 | | 1 | 1 | | 1 | 4 |
| 10 | | 1 | | | | 1 | 1 | 1 | 1 | 2 |
| | 2 | 7 | 4 | 7 | 5 | 6 | 5 | 6 | 5 | 9 |

When the amount of the pay roll and the necessary bills and fractional currency have been determined, a check payable to the order of *Pay Roll* is written. A pay-roll memorandum similar to the accompanying form is then attached to the check and both are sent to the bank. The pay-roll memorandum should foot the same as the pay-roll book, and is therefore a check upon the correctness of the change memorandum.

In a large pay roll the adept bookkeeper frequently *estimates* the kind of change required. This is done by scanning the pay roll first to find the number of pennies required, then the number of nickels, etc. The experienced bookkeeper can make a very accurate estimate.

*FIRST NATIONAL BANK*
*Westfield, Mass.*
*PAY-ROLL MEMORANDUM*
*NELSON & CO.*
*require the following:*

| | | | |
|---|---|---|---|
| Pennies | 9 | | 09 |
| Nickels | 5 | | 25 |
| Dimes | 6 | | 60 |
| Quarters | 5 | 1 | 25 |
| Halves | 6 | 3 | 00 |
| Dollars | 5 | 5 | 00 |
| 2's | 7 | 14 | 00 |
| 5's | 4 | 20 | 00 |
| 10's | 7 | 70 | 00 |
| 20's | 2 | 40 | 00 |
| | | 154 | 19 |

PAY ROLL For the week ending Nov. 6, 19__

| No. | NAME | Time in hours | | | | | | Total time | Rate per hour | Amount advanced | | Amount due | | Bills and silver necessary | | | | | | | | | | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | M | T | W | T | F | S | | | | | | | $20 | $10 | $5 | $2 | $1 | 50c | 25c | 10c | 5c | 1c | |
| 1 | Ralph Roe | 9 | 9 | 9 | 9 | 9 | 9 | 54 | $44\frac{4}{9}$¢ | | | 24 | — | 1 | | | 2 | | | | | | | |
| 2 | Emma Tyng | 9 | 9 | 9 | 9 | 9 | 9 | 54 | $44\frac{4}{9}$¢ | | | 24 | — | 1 | | | 2 | | | | | | | |
| 3 | John Walsh | 8 | 9 | 8 | 9 | 9 | 9 | 52 | 35¢ | | | 18 | 20 | | 1 | 1 | 1 | 1 | | | 2 | | | |
| 4 | Edward Paine | 8½ | 8½ | 9 | 9 | 9 | 9 | 53 | 35¢ | | | 18 | 55 | | 1 | 1 | 1 | 1 | 1 | | | 1 | | |
| 5 | Dora Small | 9½ | 9 | 8½ | 9 | 8½ | 9½ | 54 | $22\frac{2}{9}$¢ | | | 12 | — | | 1 | | 1 | | | | | | | |
| 6 | Ellen Dale | 9 | 8½ | 8½ | 8½ | 9 | 9½ | 53 | $33\frac{1}{3}$ | 5 | — | 12 | 67 | | 1 | | 1 | | 1 | | 1 | 1 | 2 | |
| 7 | J. B. Cox | 9 | 9 | 9 | 9 | 9 | 9 | 54 | 35¢ | | | 18 | 90 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| 8 | Charles Frey | 8 | 9 | 9 | 9 | 9 | 9 | 53 | 25¢ | | | 13 | 25 | | 1 | | 1 | 1 | | 1 | | | | |
| 9 | Benj. A. Green | 9 | 9 | 9 | 9 | 9 | 9 | 54 | $18\frac{2}{9}$¢ | | | 9 | 84 | | | 1 | 2 | | 1 | 1 | | 1 | 4 | |
| 10 | F. M. Sperry | 8 | 7½ | 12 | 12 | 9 | 9 | 57½ | $22\frac{2}{9}$¢ | | | 12 | 78 | | 1 | | 1 | | 1 | 1 | | | 3 | |
| 11 | Ada Neal | 10 | 10 | 10 | 9 | 8 | 8 | 55 | $14\frac{7}{9}$¢ | | | 8 | 13 | | | 1 | 1 | 1 | | | 1 | | 3 | |
| 12 | Ora Snell | 9 | 9 | 9 | 8 | 8 | 8 | 51 | 35¢ | | | 17 | 85 | | 1 | 1 | 1 | | 1 | 1 | 1 | | | |
| 13 | E. H. Spring | 9 | 9 | 8 | 9 | 9 | 9 | 53 | 30¢ | | | 15 | 90 | | 1 | 1 | | | 1 | 1 | 1 | 1 | | |
| 14 | Sam'l Dobson | 9 | 9 | 9 | 9½ | 9½ | 8 | 54 | $22\frac{2}{9}$¢ | | | 12 | — | | 1 | | 1 | | | | | | | |
| | | 124 | 124½ | 127 | 128 | 124 | 124 | 751½ | | 5 | — | 218 | 07 | 2 | 10 | 7 | 16 | 5 | 7 | 6 | 7 | 5 | 12 | |
| | | | | | | | | | | | | | | 40 | 100 | 35 | 32 | 5 | 3.50 | 1.50 | 70¢ | 25¢ | 12¢ = 218.07 | |

### WRITTEN EXERCISE

1. Study the model pay roll, page 159, and find the amount of it at the following wages per hour: #1, 18¢; #2, $21\frac{2}{9}$¢; #3, 25¢; #4, 35¢; #5, $33\frac{1}{3}$¢; #6, 35¢; #7, $37\frac{1}{2}$¢; #8, 35¢; #9, $27\frac{1}{2}$¢; #10, $18\frac{2}{9}$¢. Make a change memorandum.

**2.** Study the model pay roll on page 160, and then find the amount of it at the following wages per hour: #1, 50¢; #2, 45¢; #3, $33\frac{1}{3}$¢; #4, 35¢; #5, $27\frac{1}{2}$¢; #6, $37\frac{1}{2}$¢; #7, 25¢; #8, $33\frac{1}{3}$¢; #9, $44\frac{4}{9}$¢; #10, $22\frac{2}{9}$¢; #11, $22\frac{2}{9}$¢; #12, $14\frac{2}{7}$¢; #13, $12\frac{1}{2}$¢; #14, 30¢.

**3.** Make a pay roll memorandum from problem 2.

### WRITTEN REVIEW EXERCISE

**1.** Find the amount of each of the following bills:

*New York*, May 31, *19*

MESSRS. GRAY, SALISBURY & CO.
Rochester, N.Y.

*Bought of J. E. PAGE, SONS & CO.*

*Terms:* net, 60 da.; 2% 10 da.

| CASE | PIECES | DESCRIPTION OF ARTICLES | YDS. | PRICE | ITEMS | AMOUNT |
|---|---|---|---|---|---|---|
| #364 | 10 | Velveteen | | | | |
| | | $42^1$ 40 40 46 $38^1$ | | | | |
| | | 40 42 42 41 39 | | 25¢ | | |
| #359 | 12 | Corduroy | | | | |
| | | 36 $38^1$ $39^2$ 42 $41^2$ $39^2$ | | | | |
| | | 37 37 41 45 41 $40^1$ | | $66\frac{2}{3}$¢ | | |
| #371 | 15 | Gray Homespun | | | | |
| | | 39 38 35 42 41 | | | | |
| | | 45 39 41 34 37 | | | | |
| | | 41 40 41 38 $42^3$ | | $83\frac{1}{3}$¢ | | |
| #360 | 6 | Storm Serge | | | | |
| | | 40 $42^1$ 43 42 39 $42^1$ | | 44¢ | | |
| #373 | 24 | Fine English Serge | | | | |
| | | 42 38 42 42 $40^2$ $42^1$ | | | | |
| | | 40 39 40 41 $40^1$ 43 | | | | |
| | | 42 42 $38^2$ 38 41 42 | | | | |
| | | 43 44 41 40 $37^1$ 37 | | $1.37\frac{1}{2}$ | | |
| #381 | 24 | Groveland Flannel | | | | |
| | | 32 40 39 42 41 45 | | | | |
| | | 45 46 35 41 38 41 | | | | |
| | | 37 42 43 40 37 42 | | | | |
| | | 37 40 42 41 44 41 | | $33\frac{1}{3}$ | | |

**2.** Make out a bill for the following order. Bill the English breakfast tea at 41¢; Finest oolong tea at 65¢; Young Hyson tea at $97\frac{1}{2}$¢; Choice Japan tea at 59¢; Orinda kaughphy at $1.90; raw Java coffee at $30\frac{1}{2}$¢; gluten flour at 30¢ a carton and $7.75 per barrel. Assume that half a chest of tea weighs 75 lb., and a mat of coffee 70 lb.

E. M. BARBER & SON
RETAIL GROCERS

Springfield, Mass., Aug. 13, 19

S. S. Pierce Company
Boston, Mass.

Gentlemen:

Please ship us via B. & A. R.R., the following goods:

3 hf. cht. English Breakfast Tea
3 " " Finest Oolong Tea
5 " " Young Hyson Tea
25 lb. Choice Japan Tea
5 5-lb. cans Orinda Kaughphy
7 mats Raw Java Coffee
5 hf. bbl. Gluten Flour
25 5-lb. cartons Gluten Flour

Respectfully yours

*E. M. Barber & Son*

**3.** Boston, Mass., Apr. 16, E. O. Burrill, Philadelphia, Pa., bought of Jones, Talcott & Co., on account, 30 da., 25 Turkish rugs $4\frac{1}{2} \times 7$ at $10.25; 750 yd. matting at 55¢; 225 yd. linoleum at $27\frac{1}{2}$¢; 25 Turkish rugs $8\frac{1}{2} \times 12$ at $21.75; 25 Persian rugs $6 \times 9$ at $12.25; 12 Persian rugs $7 \times 11$ at $16.25; 10 rolls, each containing 150 yards, Brussels carpeting at $2.25; 275 yd. Moquette carpeting at $1.75. Find the amount of the bill.

**TIME SLIP**

Friday, 4/26, 19—

|  | IN | OUT | IN | OUT |
|---|---|---|---|---|
| 1 | 751 | 1201 | 1256 | 502 |
| 2 | 747 | 1202 | 1247 | 503 |
| 3 | 744 | 1204 | 1254 | 504 |
| 4 | 751 | 1205 | 1255 | 501 |
| 5 | 753 | 1206 | 1259 | 505 |
| 6 | 751 | 1202 | 1259 | 503 |
| 7 | 859 | 1201 | 1254 | 502 |
| 8 | 756 | 1202 | 1250 | 501 |
| 9 | 757 | 1201 | 1259 | 502 |

**TIME SLIP**

Saturday, 4/27, 19—

|  | IN | OUT | IN | OUT |
|---|---|---|---|---|
| 1 | 753 | 1200 | 1258 | 502 |
| 2 | 757 | 1204 | 1259 | 501 |
| 3 | 753 | 1200 | 1256 | 504 |
| 4 | 755 | 1202 | 1254 | 502 |
| 5 | 749 | 1201 | 1257 | 504 |
| 6 | 828 | 1203 | 1255 | 503 |
| 7 | 755 | 1202 | 1253 | 501 |
| 8 | 857 | 1201 | 1254 | 502 |
| 9 | 758 | 1202 | 1258 | 504 |

The above slips show a record of time for 9 employees for 2 da. in a large printing establishment. The records are made by a large mechanical timekeeper, and at convenient periods are copied in the pay-roll book. Fractions are recorded to the nearest $\frac{1}{6}$ of an hour. The above record is based on an eight-hour day, from 8 to 12 A.M., and 1 to 5 P.M. If a record is made before the hour for beginning work, either in the morning or at noon, it is not counted in the regular time, but if an employee is late, his record for time begins at the nearest multiple of ten after the record is made. For instance, if an employee rang in at 8:42 A.M., his time for the morning would begin at 8:50. In the above slip for Friday, No. 1 rang in at 7:51 and left at 12:01; he rang in at 12:56 and rang out at 5:02; time, 8 hr.

**4.** Copy the following pay roll, enter the time for Friday and Saturday (from the above slips), find the amount of the pay roll; make a change memorandum and a pay-roll memorandum.

PAY ROLL FOR THE WEEK ENDING APRIL 27, 19—

| No. | Name | Number of Hours' Work Each Day |  |  |  |  |  | Total No. of Hours | Wages per Hour | Total Wages | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | M. | T. | W. | T. | F. | S. |  |  |  |  |
| 1 | A. B. Comer | 8 | 8 | 8 | 8 |  |  |  | $55\frac{5}{9}$¢ |  |  |
| 2 | W. D. Ball | 8 | 8 | 8 | 8 |  |  |  | $44\frac{4}{9}$¢ |  |  |
| 3 | A. M. Snow | 8 | 8 | 8 | 8 |  |  |  | $44\frac{4}{9}$¢ |  |  |
| 4 | R. O. Mark | 8 | 8 | 8 | 8 |  |  |  | $33\frac{1}{3}$¢ |  |  |
| 5 | Miss Mary Cane | 8 | $7\frac{1}{2}$ | 8 | 8 |  |  |  | $33\frac{1}{3}$¢ |  |  |
| 6 | Miss Ellen Kyle | 8 | $7\frac{1}{2}$ | 8 | 8 |  |  |  | 35¢ |  |  |
| 7 | D. M. Garson | 8 | $7\frac{1}{2}$ | 8 | 8 |  |  |  | 35¢ |  |  |
| 8 | S. D. Lane | 8 | $7\frac{1}{2}$ | $7\frac{1}{2}$ | 8 |  |  |  | 25¢ |  |  |
| 9 | Miss Cora Knapp | 8 | 8 | $7\frac{1}{2}$ | 8 |  |  |  | $22\frac{2}{9}$¢ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

## EXPRESSAGE AND FREIGHTAGE

These express rates went into effect February 1, 1914, in conformity with the order of the Interstate Commerce Commission:

| | Between New York and Chicago | | Between New York and New Orleans | | Between New York and Denver | |
|---|---|---|---|---|---|---|
| | *1st class* | *2d class* | *1st class* | *2d class* | *1st class* | *2d class* |
| 5 lb. | 31¢ | 31¢ | 41¢ | 41¢ | 47¢ | 47¢ |
| 6 | 33 | 32 | 46 | 46 | 53 | 53 |
| 7 | 35 | 32 | 50 | 48 | 58 | 57 |
| 8 | 38 | 32 | 54 | 48 | 64 | 57 |
| 9 | 40 | 32 | 59 | 48 | 69 | 57 |
| 10 | 42 | 32 | 63 | 48 | 75 | 57 |
| 15 | 53 | 40 | 84 | 63 | 1.02 | 77 |
| 20 | 64 | 48 | 1.06 | 80 | 1.30 | 98 |
| 25 | 75 | 57 | 1.27 | 96 | 1.57 | 1.18 |

| | Between Chicago and Boston | | Between Chicago and Galveston | | Between Chicago and Portland, Ore. | |
|---|---|---|---|---|---|---|
| | *1st class* | *2d class* | *1st class* | *2d class* | *1st class* | *2d class* |
| 5 lb. | 31¢ | 31¢ | 39¢ | 39¢ | 63¢ | 63¢ |
| 6 | 34 | 33 | 43 | 43 | 72 | 72 |
| 7 | 36 | 33 | 47 | 45 | 81 | 80 |
| 8 | 38 | 33 | 51 | 45 | 89 | 80 |
| 9 | 41 | 33 | 55 | 45 | 98 | 80 |
| 10 | 43 | 33 | 59 | 45 | 1.06 | 80 |
| 15 | 54 | 41 | 78 | 59 | 1.50 | 1.13 |
| 20 | 66 | 50 | 98 | 74 | 1.93 | 1.45 |
| 25 | 77 | 58 | 1.17 | 88 | 2.36 | 1.77 |

Under the new system the country is divided into blocks, each block covering a certain designated territory, and fixed rates are made for certain weights and distances, not as under the old system, at so much per pound.

The tables given herewith suggest the manner in which charges are made for given amounts to certain points.

*First-class* matter includes all merchandise; *second-class* matter applies to *food* and *drink*, and these terms are elaborated and explained in the instructions to express agents. On small packages the difference in charges for first-class matter or second-class matter is very slight (sometimes the same), but for large packages and long distances the difference is marked.

### WRITTEN EXERCISE

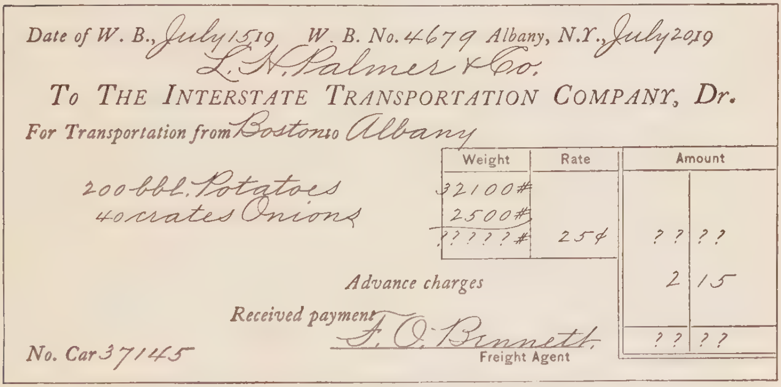
1. Find the total cost of sending the following packages:

    10 lb. first class from New York to Chicago.
    5 lb. second class from Chicago to Boston.
    15 lb. first class from Chicago to Galveston.
    20 lb. second class from New York to Denver.
    10 lb. first class from New York to New Orleans.

2. Find the total cost of sending the following packages:

    15 lb. first class from Chicago to Galveston.
    25 lb. second class from Chicago to Portland, Ore.
    7 lb. second class from Boston to Chicago.
    9 lb. first class from Chicago to New York.
    10 lb. first class from Galveston to Chicago.

3. Find the amount of the following freight bill:



Date of W. B., July 15 19 W. B. No. 4679 Albany, N.Y., July 20 19

L. H. Palmer & Co.

To THE INTERSTATE TRANSPORTATION COMPANY, Dr.

For Transportation from Boston to Albany

| | Weight | Rate | Amount | |
|---|---|---|---|---|
| 200 bbl. Potatoes | 32100# | | | |
| 40 crates Onions | 2500# | | | |
| | ?????# | 25¢ | ?? | ?? |
| Advance charges | | | 2 | 15 |
| | | | ?? | ?? |

Received payment F. O. Bennett, Freight Agent

No. Car 37145

Bulky goods are generally sent by freight. The articles are divided, according to quantity and character, into different classes and are subject to different rates. All railroads follow some official classification. All official classifications divide freight into six different classes.

Such bulky articles as furniture, uncased, is subject to a classification called *double first-class*. There is so much unoccupied space that the first-class rate is doubled.

Such freight as organs and pianos in cases, furniture, statuary, etc., is generally designated as first-class matter. Baled hay, iron, etc., in car loads, is generally designated as fifth-class matter. Building blocks, brick, etc., in car-load lots, is generally designated as sixth-class matter. First-class rates are the highest and sixth-class rates are the lowest charged.

Between most points, shipments weighing less than 100 lb. are charged as 100 lb., irrespective of weight.

## BOSTON & ALBANY RAILROAD

### LOCAL FREIGHT TARIFF BETWEEN

### BOSTON, MASS.

AND

| Miles | Stations | Rate per 100 Lb. | | | | | | Miles | Stations | Rate per 100 Lb. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Classes | | | | | | | | Classes | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | | | 1 | 2 | 3 | 4 | 5 | 6 |
| 21 | So. Framingham | 14¢ | 12¢ | 10¢ | 8¢ | 6¢ | 5¢ | 98 | Springfield . . | 28¢ | 24¢ | 20¢ | 15¢ | 11¢ | 9¢ |
| 32 | Westboro . . | 16¢ | 14¢ | 11¢ | 9¢ | 6¢ | 5¢ | 108 | Westfield . . | 29¢ | 25¢ | 20¢ | 16¢ | 12¢ | 10¢ |
| 44 | Worcester . . | 18¢ | 15¢ | 13¢ | 10¢ | 7¢ | 6¢ | 146 | Athol . . . . | 33¢ | 28¢ | 23¢ | 18¢ | 13¢ | 11¢ |
| 62 | Webster . . . | 22¢ | 19¢ | 15¢ | 12¢ | 9¢ | 7¢ | 150 | Pittsfield . . | 33¢ | 28¢ | 23¢ | 18¢ | 13¢ | 11¢ |
| 83 | Palmer . . . | 26¢ | 22¢ | 18¢ | 14¢ | 10¢ | 9¢ | 202 | Albany . . . | 35¢ | 30¢ | 25¢ | 19¢ | 14¢ | 12¢ |

**4.** Using the table, find the amount of freight to charge on 27,500 lb. sixth-class matter, from Boston to Pittsfield.

**5.** Using the above table, find the amount of freight to charge on 27,290 lb. sixth-class matter and 890 lb. first-class matter from Boston to Albany; to Westfield.

**6.** Using the above table, find the amount of freight to charge on 14,790 lb. fifth-class matter and 2170 lb. second-class matter from Boston to Palmer; to Worcester; to Pittsfield; to Springfield.

**7.** Using the above table, find the amount of freight to charge on 75 lb. first-class matter, 125 lb. second-class matter, 1250 lb. third-class matter, 7290 lb. fourth-class matter, 21,490 lb. fifth-class matter, and 64,640 lb. sixth-class matter from Boston to South Framingham; to Westboro; to Webster; to Springfield; to Athol; to Albany.

### ORAL REVIEW EXERCISE

*Each problem on pages 167 and 168 should be completed in approximately four minutes. Without copying, find the cost of:*

1.

32 yd. at $6\frac{1}{2}$¢
240 yd. at $1.12$\frac{1}{2}$
25 yd. at $2.50
19 yd. at $4\frac{1}{2}$¢
57 yd. at $66\frac{2}{3}$¢
36 yd. at $3\frac{1}{3}$¢
$24\frac{1}{2}$ yd. at 8¢
190 yd. at 25¢
75 yd. at 28¢
75 yd. at $12\frac{1}{2}$¢
42 yd. at $2\frac{1}{2}$¢
246 yd. at 11¢
48 yd. at $10\frac{1}{2}$¢
28 yd. at 6¢
144 yd. at $12\frac{1}{2}$¢

2.

176 yd. at $12\frac{1}{2}$¢
177 yd. at 50¢
210 yd. at $28\frac{4}{7}$¢
291 yd. at $33\frac{1}{3}$¢
104 yd. at $2\frac{1}{2}$¢
98 yd. at 7¢
90 yd. at $6\frac{2}{3}$¢
45 yd. at $1.33$\frac{1}{3}$
45 yd. at 45¢
706 yd. at $33\frac{1}{3}$¢
$22\frac{1}{2}$ yd. at 8¢
146 yd. at 11¢
74 yd. at $12\frac{1}{2}$¢
78 yd. at 25¢
44 yd. at $1.25

3.

149 yd. at 25¢
150 yd. at $1.33$\frac{1}{3}$
120 yd. at $1.25
169 yd. at 11¢
162 yd. at $12\frac{1}{2}$¢
48 yd. at $6\frac{1}{4}$¢
$33\frac{1}{3}$ yd. at 9¢
174 yd. at 30¢
$29\frac{1}{2}$ yd. at 3¢
117 yd. at 9¢
18 yd. at $3\frac{1}{3}$¢
$182\frac{1}{2}$ yd. at 10¢
179 yd. at 20¢
64 yd. at $12\frac{1}{2}$¢
167 yd. at 50¢

4.

33 yd. at $33\frac{1}{3}$¢
96 yd. at $37\frac{1}{2}$¢
129 yd. at 11¢
75 yd. at $1\frac{1}{3}$¢
75 yd. at 75¢
27 yd. at 80¢
73 yd. at $1\frac{1}{2}$¢
65 yd. at 65¢
94 yd. at $33\frac{1}{3}$¢
63 yd. at $11\frac{1}{9}$¢
139 yd. at 50¢
64 yd. at $62\frac{1}{2}$¢
98 yd. at $16\frac{2}{3}$¢
225 yd. at 25¢
67 yd. at $33\frac{1}{3}$¢

5.

48 yd. at $1.50
36 yd. at $1.25
55 yd. at $1\frac{1}{2}$¢
143 yd. at 50¢
36 yd. at 70¢
55 yd. at 55¢
95 yd. at 30¢
112 yd. at $14\frac{2}{7}$¢
47 yd. at $33\frac{1}{3}$¢
53 yd. at $33\frac{1}{3}$¢
17 yd. at $66\frac{2}{3}$¢
104 yd. at $12\frac{1}{2}$¢
80 yd. at $6\frac{2}{3}$¢
77 yd. at 30¢
99 yd. at 40¢

6.

688 yd. at $1.10
$52\frac{1}{2}$ yd. at 10¢
156 yd. at 25¢
85 yd. at 85¢
144 yd. at $16\frac{2}{3}$¢
34 yd. at 90¢
125 yd. at 20¢
53 yd. at 25¢
29 yd. at $66\frac{2}{3}$¢
99 yd. at 25¢
88 yd. at $37\frac{1}{2}$¢
176 yd. at $12\frac{1}{2}$¢
25 yd. at 25¢
88 yd. at $9\frac{1}{11}$¢
57 yd. at 50¢

**7.**

128 yd. at 11¢
73 yd. at 60¢
76 yd. at 60¢
177 yd. at 11¢
72 yd. at $1\frac{1}{4}$¢
270 yd. at $11\frac{1}{9}$¢
36 yd. at $1\frac{1}{2}$¢
185 yd. at 25¢
225 yd. at 20¢
39 yd. at $\$1.33\frac{1}{3}$
49 yd. at $66\frac{2}{3}$¢
42 yd. at $\$2.50$
25 yd. at $\$1.10$.
22 yd. at $6\frac{1}{2}$¢.
125 yd. at 36¢
108 yd. at 9¢
216 yd. at $12\frac{1}{2}$¢

**8.**

219 yd. at 11¢
85 yd. at 30¢
94 yd. at 25¢
89 yd. at $1\frac{1}{2}$¢
28 yd. at $2\frac{1}{2}$¢
112 yd. at $6\frac{1}{4}$¢
191 yd. at 50¢
$78\frac{1}{2}$ yd. at 10¢
306 yd. at $33\frac{1}{3}$¢
122 yd. at $2\frac{1}{2}$¢
32 yd. at $\$1.75$
92 yd. at $12\frac{1}{2}$¢
224 yd. at 7¢
240 yd. at $2\frac{1}{2}$¢
276 yd. at 11¢
68 yd. at 8¢
125 yd. at 12¢

**9.**

65 yd. at 80¢
83 yd. at 40¢
145 yd. at 11¢
63 yd. at $1\frac{1}{3}$¢
151 yd. at $33\frac{1}{3}$¢
49 yd. at 75¢
124 yd. at $\$1.25$
180 yd. at $12\frac{1}{2}$¢
360 yd. at $37\frac{1}{2}$¢
24 yd. at $\$1.12\frac{1}{2}$
165 yd. at $6\frac{2}{3}$¢
175 yd. at 8¢
171 yd. at 11¢
$132\frac{1}{2}$ yd. at 10¢
110 yd. at 25¢
51 yd. at $12\frac{1}{2}$¢
301 yd. at 25¢

**10.**

62 yd. at $4\frac{1}{2}$¢
189 yd. at $11\frac{1}{9}$¢
78 yd. at 40¢
334 yd. at 7¢
255 yd. at 5¢
78 yd. at 11¢
118 yd. at 11¢
187 yd. at 25¢
88 yd. at $33\frac{1}{3}$¢
96 yd. at $37\frac{1}{2}$¢
$405\frac{1}{2}$ yd. at 10¢
232 yd. at 11¢
36 yd. at $\$1.33\frac{1}{3}$
256 yd. at 6¢
143 yd. at 9¢
64 yd. at $\$1.25$
51 yd. at $\$1.33\frac{1}{3}$

**11.**

144 yd. at $87\frac{1}{2}$¢
25 yd. at $\$1.62$
14 yd. at $\$1.14\frac{2}{7}$
512 yd. at 6¢
$117\frac{1}{2}$ yd. at 10¢
45 yd. at $1\frac{1}{2}$¢
155 yd. at 12¢
247 yd. at 50¢
92 yd. at $12\frac{1}{2}$¢
55 yd. at $16\frac{2}{3}$¢
37 yd. at $10\frac{1}{2}$¢
42 yd. at $\$1.25$
24 yd. at $\$1.66\frac{2}{3}$
178 yd. at 7¢
321 yd. at 11¢
24 yd. at $\$1.87\frac{1}{2}$
36 yd. at $\$1.66\frac{2}{3}$

**12.**

97 yd. at 30¢
36 yd. at $\$1.16\frac{2}{3}$
225 yd. at 8¢
$143\frac{1}{2}$ yd. at 4¢
55 yd. at $10\frac{1}{2}$¢
15 yd. at $1\frac{2}{3}$¢
235 yd. at 20¢
56 yd. at $33\frac{1}{3}$¢
48 yd. at $12\frac{1}{2}$¢
84 yd. at $12\frac{1}{2}$¢
165 yd. at 25¢
192 yd. at $12\frac{1}{2}$¢
145 yd. at 5¢
231 yd. at 8¢
148 yd. at $16\frac{2}{3}$¢
121 yd. at 25¢
$110\frac{1}{2}$ yd. at 10¢

# DENOMINATE NUMBERS

## CHAPTER XII

### DENOMINATE QUANTITIES

### REVIEW OF THE COMMON TABLES[1]

#### ORAL EXERCISE

**1.** Which of the following numbers are concrete? which are abstract? which are denominate?

*a.* 16
*b.* 24 yr.
*c.* 64 hr.
*d.* 12 men
*e.* 15 desks
*f.* 150
*g.* 21 yd.
*h.* 65 A.
*i.* 17 books
*j.* 34 houses
*k.* 36 min.
*l.* 5 yd. 2 ft.
*m.* 3 yr. 4 mo.
*n.* 10 T. 75 lb.
*o.* 5 A. 61 sq. rd.

**2.** Define an abstract number; a concrete number; a denominate number; a simple number; a compound number.

**3.** Which of the numbers in question 1 are simple? which are compound?

#### ORAL EXERCISE

**1.** Repeat the table of avoirdupois weight.

**2.** Repeat the table of long measure; of surveyors' long measure; of square measure; of surveyors' square measure.

**3.** Repeat the table of cubic measure; of dry measure; of liquid measure; of time; of angular measure; of United States money; of English money.

**4.** Name a number expressing distance; two numbers expressing area; two expressing value; three expressing capacity.

**5.** How many statute miles in a degree of the earth's surface at the equator? how many geographical miles? How many feet in a statute mile? how many inches?

---

[1] Tables of weights and measures may be found in the Appendix B.

## REDUCTION

### ORAL EXERCISE

1. Change 42 ft. to inches; to yards.
2. Express 15 yd. as feet; as inches.
3. Reduce 80 qt. to gallons; to pints.
4. Change 128 qt. to pecks; to bushels.
5. Express 120 pt. as quarts; as gallons.
6. What part of a yard is 2 ft.? $\frac{1}{2}$ ft.? $\frac{1}{4}$ ft.?
7. Reduce 5 bu. to pecks; to quarts; to pints.

### Reduction Descending

**174. Example.** Reduce 4 T. 75 lb. to ounces.

Solution. Since 1 T. = 2000 lb., 4 T. = 4 times 2000 lb. = 8000 lb.; and with the 75 lb. added this = 8075 lb. Since 1 lb. = 16 oz., 8075 lb. = 8075 times 16 oz. = 129,200 oz., the required result.

8075 times 16 oz. = 16 times 8075 oz.; therefore 8075 times 16 oz. is found as shown in the margin.

```
  2000
     4
  8075
    16
129200, No. of oz.
```

### WRITTEN EXERCISE

*Reduce:*

1. 115′ 6″ to inches.
2. 12 bu. 4 qt. to pecks.
3. £16 15*s.* to shillings.
4. 211 rd. 3 ft. to inches.
5. $3\frac{1}{2}$ rd. to feet.
6. $1\frac{1}{2}$ T. to ounces.
7. 12 A. to square feet.
8. 161 cd. to cubic feet.

### ORAL EXERCISE

1. How many pecks in $\frac{1}{4}$ bu.? in $\frac{1}{2}$ bu.?
2. Change .25 A. to square rods; .375 A.; 75 A.
3. Reduce $\frac{1}{2}$ gal. to pints. Express $\frac{1}{2}$ rd. as inches; as yards.

### WRITTEN EXERCISE

*Reduce:*

1. $\frac{2}{3}$ mi. to feet.
2. .75 cd. to cubic feet.
3. $\frac{119}{160}$ A. to square feet.
4. $\frac{7}{8}$ yd. to inches.
5. .375 mi. to feet.
6. $\frac{1}{12}$ hr. to seconds.

### REDUCTION ASCENDING

**175. Example.** Express 176 qt. dry measure in higher denominations.

SOLUTION. Since 8 qt. = 1 pk., divide by 8 and obtain as a result 22 pk. Since 4 pk. = 1 bu., divide by 4 and obtain as a result 5 bu. 2 pk.

$176 \div 8 = 22$, or 22 pk.
$22 \div 4 = 5$ and a remainder of 2, or 5 bu. 2 pk.

#### WRITTEN EXERCISE

*Reduce to higher denominations:*

**1.** 3840 ft.
**2.** 1054 pt.
**3.** 14,400 sec.
**4.** 2000 sq. in.
**5.** 816 pk.
**6.** 106,590 ft.
**7.** 43,560 sq. in.
**8.** 27,900 lb. avoir.
**9.** 15,120″.
**10.** 51,200 cu. ft.
**11.** 145,152 cu. in.
**12.** 27,900 oz. avoir.

#### ORAL EXERCISE

**1.** Reduce $\frac{1}{2}$ ft. to the fraction of a yard.

**2.** Change .16 cwt. to the decimal of a ton.

**3.** What part of a yard is 1 in.? 2 in.? $\frac{1}{2}$ in.?

**4.** What decimal part of an acre is 16 rd.? 40 rd.?

**5.** What part of 35 bu. is 7 bu.? of $1\frac{1}{2}$ bu. is $\frac{1}{2}$ bu.?

#### WRITTEN EXERCISE

**1.** Reduce $1\frac{7}{8}$ in. to the fraction of a foot; of a yard.

**2.** Reduce 10*s.* 9*d.* to the fraction of a pound sterling.

SOLUTION. There are 12*d.* in a shilling and 20*s.* in a pound sterling, or 240*d.*

To find what fractional part of a pound sterling 10*s.* and 9*d.* are, use the following statement:

10*s.* 9*d.* = 129*d.* £1 = 240*d.* Therefore, 10*s.* 9*d.* = $\frac{129}{240}$ of a £, or £.5375.

10*s.* and 9*d.* = 129*d.*
£1 = 240*d.*
$\frac{129}{240} = .5375$,
or £.5375

**3.** Reduce 4 yd. $1\frac{1}{2}$ ft. to the decimal of a rod.

**4.** Reduce 10*s.* 6*d.* 2 far. to the decimal of a pound sterling.

**5.** Reduce 5 T. 721 lb. to tons and decimal of a ton; 6 T. 1750 lb.; 12 T. 290 lb.; 29,240 lb.; 28,390 lb.

**6.** Find the cost of 1750 lb. of coal at $6.25 per ton; of 2170 lb.; of 690 lb.; of 1360 lb.; of 3240 lb.; of 32590 lb.

## ADDITION AND SUBTRACTION

### ORAL EXERCISE

*State the sum of:*

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 12 ft. 1 in. | 5 lb. 8 oz. | 15 rd. 5 ft. | 10 mi. 8 rd. |
| 6 3 | 6 3 | 17 2 | 8 40 |

*State the difference between:*

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| 90 mi. 300 rd. | 75 rd. 12½ ft. | 30 yd. 2 ft. | 44 bu. 3 pk. |
| 75 120 | 26 4½ | 17 1½ | 29 1 |

### WRITTEN EXERCISE

*Find the sum of:*

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| £140 6*s.* | £139 5*s.* | 84 T. 75 lb. | 279 T. 840 lb. |
| 159 3 | 214 5 | 96 14 | 364 210 |
| 162 4 | 921 3 | 78 79 | 872 220 |
| 139 2 | 141 7 | 37 41 | 146 140 |
| 167 4 | 10 9 | 19 63 | 214 180 |
| 129 3 | 171 8 | 84 79 | 926 230 |
| 136 4 | 215 7 | 97 13 | 210 420 |
| 147 2 | 321 5 | 87 125 | 75 750 |

*Find the difference between:*

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| 11 mo. 17 da. | 11 mo. 1 da. | 8 mo. 14 da. | 9 mo. 17 da. |
| 8 31 | 9 31 | 2 29 | 2 31 |

**9.** An English merchant had on hand Jan. 1 goods valued at £5927 10*s.*; during the following six months he bought goods at a cost of £4920 10*s.* and sold goods to the amount of £7926 4*s.* If the value of the goods on hand July 1 of the same year was £4120 10*s.*, what was the gain or the loss in English money? in United States money?

## MULTIPLICATION AND DIVISION

### ORAL EXERCISE

*Multiply:*

**1.** 3 ft. by 6.
**2.** $1\frac{1}{2}$ mi. by 8.
**3.** 9 lb. 4 oz. by 2.
**4.** 18 lb. 1 oz. by 9.
**5.** 17 yd. 2 in. by 9.
**6.** 19 gal. 1 qt. by 3.

*Divide:*

**7.** 27 yd. by 9.
**8.** 225 ft. by $7\frac{1}{2}$ ft.
**9.** 48 ft. 6 in. by 2.
**10.** 540 yd. by 18 yd.
**11.** 164 lb. 12 oz by 4.
**12.** 640 mi. 160 rd. by 20.

**176. Examples.** **1.** How much hay in 8 stacks each containing 5 T. 760 lb.?

SOLUTION. 8 times 760 lb. = 6080 lb. = 3 T. 80 lb.; write 80 in place of pounds and carry 3. 8 times 5 T. = 40 T.; 40 T. + 3 T. carried = 43 T. The required result is therefore 43 T. 80 lb.

$$\begin{array}{r}5\text{ T. }760\text{ lb.}\\ 8\\ \hline 43\text{ T. }\ 80\text{ lb.}\end{array}$$

**2.** An importer paid £ 87 10*s.* for 50 pc. of bric-a-brac. What was the cost per piece?

SOLUTION. Since 50 pc. cost £ 87 10*s.*, 1 pc. costs $\frac{1}{50}$ of £ 87 10*s.* $\frac{1}{50}$ of £ 87 = £ 1 with an undivided remainder of £ 37; write £ 1 in the quotient and add £ 37 to the next lower denomination; £ 37 10*s.* = 750*s.* $\frac{1}{50}$ of 750*s.* = 15*s.*

$$\begin{array}{r|l}\multicolumn{2}{r}{£\ 1\quad 15s.}\\ 50 & £\ 87\quad 10s.\end{array}$$

**3.** At 10*s.* 6*d.* per yard, how many yards can be bought for £ 15 15*s.*?

SOLUTION. The dividend and divisor are concrete numbers; therefore reduce them to the same denomination before dividing. £ 15 15*s.* = 3780*d.*, 10*s.* 6*d.* = 126*d.* 3780*d.* ÷ 126*d.* = 30; that is 30 yd. can be bought.

£ 15 15*s.* = 3780*d.*
10*s.* 6*d.* = 126*d.*
3780*d.* ÷ 126*d.* = 30, no. of yd.

### ORAL EXERCISE

**1.** At 72 ¢ per gross what will 2 doz. buttons cost? 4 doz.? 7 doz.?

**2.** How many 3-oz. packages can be put up from 4 lb. of pepper?

**3.** Find the cost of 3 T. of bran at 30 ¢ per hundredweight; of 5 T. at 50 ¢ per hundredweight.

**4.** How many 1-lb. packages can be put up from 15 T. of breakfast food?

**5.** When coal is $6 per ton what will 7000 lb. cost? 6400 lb.? 3600 lb.?

**6.** Find the cost of 2400 lb. of flour at $2.25 per hundredweight; of 4400 lb.; of 3200 lb.

**7.** At $12\frac{1}{2}$ ¢ per quire what will 480 sheets of paper cost? 240 sheets? 2880 sheets? 720 sheets?

**8.** I buy 3 qt. of milk per day. If I pay 8 ¢ per quart, what is my bill for July and August?

**9.** I bought 3 gro. pens at 60 ¢ a gross and sold them at the rate of 2 for 1 ¢; what was my gain or loss?

**10.** I bought $3\frac{1}{2}$ bu. of apples at $1.00 per bu. and sold them at 50 ¢ a peck. What was my gain?

**11.** I sold $4\frac{1}{2}$ cd. of wood for $27 and thereby lost $9 on the cost. What was the cost per cord?

### WRITTEN EXERCISE

**1.** Find the cost of 10 pwt. 7 gr. of old gold at $1.25 per pennyweight; of 12 pwt. 4 gr. at $1.10 per pennyweight.

**2.** I bought $3\frac{1}{8}$ A. of city land at $125 an acre and sold it at 50 ¢ per square foot. Did I gain or lose and how much?

**3.** Give the length of a double-track railroad that can be laid with 352,000 rails 30 ft. long.

**4.** I bought a barrel of cranberries containing $2\frac{1}{2}$ bu. at $4 per bushel and retailed them at 15 ¢ a quart. Did I gain or lose and how much?

**5.** From a farm of 375 A. I sold $25\frac{3}{4}$ A. What is the remainder worth at $125 per acre?

# PERCENTAGE AND ITS APPLICATIONS

## CHAPTER XIII

### PERCENTAGE

#### ORAL EXERCISE

**1.** .50 may be read *fifty hundredths*, *one half*, or *fifty per cent.* Read each of the following in three ways: .25, .30, $12\frac{1}{2}\%$.

**2.** Read each of the following in three ways: $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{25}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{4}$, 2 %, $2\frac{1}{2}\%$, 125 %, $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $66\frac{2}{3}\%$, 250 %, 375 %.

**3.** 50 % of a number is .50 or $\frac{1}{2}$ of the number. What is 50 % of \$600? 25 %? $12\frac{1}{2}\%$? 10 %? 40 %? 20 %? 75 %?

**177.** **Per cent** is a common name for hundredths.

**178.** The **symbol** % may be read *hundredths* or *per cent.*

**179.** **Percentage** is the process of computing by *hundredths* or *per cents.*

#### ORAL EXERCISE

*Express as per cents:*

**1.** .28.
**2.** .37.
**3.** $.00\frac{1}{2}$.
**4.** $.14\frac{2}{7}$.
**5.** $.33\frac{1}{3}$.
**6.** $.28\frac{4}{7}$.
**7.** $.62\frac{1}{2}$.
**8.** .0075.
**9.** .5.
**10.** .2.

*Express as decimal fractions:*

**11.** 20 %.
**12.** 45 %.
**13.** 72 %.
**14.** 18 %.
**15.** $\frac{1}{2}\%$.
**16.** $\frac{1}{4}\%$.
**17.** 125 %.
**18.** 250 %.
**19.** $\frac{1}{50}\%$.
**20.** 375 %.

*Express as common fractions:*

**21.** 1 %.
**22.** 2 %.
**23.** $2\frac{1}{2}\%$.
**24.** $3\frac{1}{2}\%$.
**25.** $133\frac{1}{3}\%$.
**26.** $266\frac{2}{3}\%$.
**27.** $87\frac{1}{2}\%$.
**28.** $112\frac{1}{2}\%$.
**29.** $\frac{1}{2}\%$.
**30.** 175 %.

*Express as per cents:*

**31.** $\frac{1}{5}$.
**32.** $\frac{1}{9}$.
**33.** $\frac{1}{11}$.
**34.** $\frac{9}{10}$.
**35.** $1\frac{1}{4}$.
**36.** $2\frac{3}{4}$.
**37.** $\frac{6}{5}$.
**38.** $1\frac{7}{8}$.
**39.** $\frac{5}{6}$.
**40.** $\frac{24}{6}$.

### IMPORTANT PER CENTS AND THEIR FRACTIONAL EQUIVALENTS

| PER CENT | FRACTIONAL VALUE | PER CENT | FRACTIONAL VALUE | PER CENT | FRACTIONAL VALUE | PER CENT | FRACTIONAL VALUE |
|---|---|---|---|---|---|---|---|
| $12\frac{1}{2}\%$ | $\frac{1}{8}$ | 75 % | $\frac{3}{4}$ | $83\frac{1}{3}\%$ | $\frac{5}{6}$ | $6\frac{1}{4}\%$ | $\frac{1}{16}$ |
| 25 % | $\frac{1}{4}$ | 100 % | 1 | 20 % | $\frac{1}{5}$ | $6\frac{2}{3}\%$ | $\frac{1}{15}$ |
| $37\frac{1}{2}\%$ | $\frac{3}{8}$ | $16\frac{2}{3}\%$ | $\frac{1}{6}$ | 40 % | $\frac{2}{5}$ | $8\frac{1}{3}\%$ | $\frac{1}{12}$ |
| 50 % | $\frac{1}{2}$ | $33\frac{1}{3}\%$ | $\frac{1}{3}$ | 60 % | $\frac{3}{5}$ | $11\frac{1}{9}\%$ | $\frac{1}{9}$ |
| $62\frac{1}{2}\%$ | $\frac{5}{8}$ | $66\frac{2}{3}\%$ | $\frac{2}{3}$ | 80 % | $\frac{4}{5}$ | $14\frac{2}{7}\%$ | $\frac{1}{7}$ |

**180.** The **terms** used in percentage are the base, the rate, and the percentage. The **base** is the number of which a per cent is taken; the **rate**, the number of hundredths of the base to be taken; the **percentage,** the result obtained by taking a certain per cent of the base.

In the expression "12 % of \$50 is \$ 6," \$ 50 is the base, 12 %, the rate, and \$ 6, the percentage.

**181.** The base plus the percentage is sometimes called the **amount**; the base minus the percentage, the **difference.**

## FINDING THE PERCENTAGE

**182. Example.** What is 15 % of \$ 660?

SOLUTION. 15 % of a number equals .15 of it. .15 of \$ 660 = \$ 99, the required result.

$$\begin{array}{r} \$\,660 \\ .15 \\ \hline \$\,99.00 \end{array}$$

**183.** Obviously, *the product of the base and rate equals the percentage.*

The *base* may be either *concrete* or *abstract.* The *rate* is always *abstract.* The *percentage* is always of the *same name as the base.*

### ORAL EXERCISE

1. What aliquot part of 1 is $.12\frac{1}{2}$? .25? .50? $.16\frac{2}{3}$? $.33\frac{1}{3}$? .20? $.06\frac{1}{4}$? $.06\frac{2}{3}$? $.08\frac{1}{3}$? $.11\frac{1}{9}$? $.14\frac{2}{7}$? $37\frac{1}{2}\%$? $62\frac{1}{2}\%$? $66\frac{2}{3}\%$?

2. Formulate a short method for finding $12\frac{1}{2}\%$ of a number.

SOLUTION. $12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$; hence, to find $12\frac{1}{2}\%$ of a number, *divide by 8.*

3. State a short method for finding 25 % of a number; 50 %; $16\frac{2}{3}\%$; $33\frac{1}{3}\%$; 20 %; $6\frac{1}{4}\%$; $6\frac{2}{3}\%$; $8\frac{1}{3}\%$; $11\frac{1}{9}\%$.

To guard against absurd answers in exercises of this character estimate the results in advance as explained on pages 44 and 128.

**4.** Find $50\%$ of 960. Also $25\%$; $37\frac{1}{2}\%$; $12\frac{1}{2}\%$; $62\frac{1}{2}\%$; $75\%$; $16\frac{2}{3}\%$; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$; $83\frac{1}{3}\%$; $20\%$; $40\%$; $60\%$; $6\frac{1}{4}\%$.

**5.** By inspection find:

*a.* $50\%$ of \$792.
*b.* $37\frac{1}{2}\%$ of \$320.
*c.* $12\frac{1}{2}\%$ of \$880.
*d.* $16\frac{2}{3}\%$ of \$669.
*e.* $25\%$ of \$1729.
*f.* $6\frac{2}{3}\%$ of \$6600.
*g.* $6\frac{1}{4}\%$ of 3296.
*h.* $83\frac{1}{3}\%$ of 4560.
*i.* $66\frac{2}{3}\%$ of 2460.
*j.* $33\frac{1}{3}\%$ of 2793.
*k.* $8\frac{1}{3}\%$ of 24,960.
*l.* $20\%$ of 12,535.

### ORAL EXERCISE

**1.** Find $10\%$ of 720; of \$15.50; of 120 men; of \$127.50.

**2.** What aliquot part of $10\%$ is $5\%$? $2\frac{1}{2}\%$? $1\frac{1}{4}\%$? $3\frac{1}{3}\%$? $1\frac{2}{3}\%$?

**3.** Formulate a short method for finding $1\frac{1}{4}\%$ of a number.

SOLUTION. $1\frac{1}{4}\%$ of a number is $\frac{1}{8}$ of $10\%$ of the number; hence, to find $1\frac{1}{4}\%$ of a number, *point off one place to the left and divide by 8.*

**4.** State a short method for finding $5\%$ of a number; $2\frac{1}{2}\%$; $3\frac{1}{3}\%$; $1\frac{2}{3}\%$.

**5.** By inspection find:

*a.* $5\%$ of 720.
*b.* $2\frac{1}{2}\%$ of 840.
*c.* $3\frac{1}{3}\%$ of 1560.
*d.* $1\frac{1}{4}\%$ of 1840.
*e.* $1\frac{2}{3}\%$ of \$366.
*f.* $2\frac{1}{2}\%$ of \$720.
*g.* $3\frac{1}{3}\%$ of \$3900.
*h.* $1\frac{2}{3}\%$ of 120 mi.
*i.* $1\frac{1}{4}\%$ of 1632 A.

### ORAL EXERCISE

**1.** Compare $24\%$ of \$25 with $25\%$ of \$24; $24\%$ of \$2500 with $25\%$ of \$2400. What is $32\%$ of \$25?

SOLUTION. $32\%$ of \$25 = $25\%$ of \$32 = $\frac{1}{4}$ of \$32 = \$8, the required result.

**2.** What is $125\%$ of \$880?

SOLUTION. $125\% = 1.25 = \frac{1}{8}$ of 10; $\frac{1}{8}$ of \$8800 (10 times \$880) = \$1100.

**3.** Find $125\%$ of 400; of 640; of 3200; of 160; of 1280.

**4.** Formulate a short method for finding $166\frac{2}{3}\%$ of a number; $333\frac{1}{3}\%$ of a number; $250\%$ of a number.

**5.** Compare $88\%$ of 12,500 bu. with $125\%$ of 8800 bu.

**6.** Find $32\%$ of \$125; of \$1250; of \$12,500; of \$125,000.

**7.** Find $250\%$ of \$720; of \$3200; of \$28,800; of \$64,800.

### ORAL EXERCISE

*By inspection find:*

1. 48 % of 250.
2. 32 % of 125.
3. 128 % of 250.
4. 16 % of 2500.
5. 180 % of 625.
6. 160 % of 875.
7. 240 % of 7500.
8. 125 % of $240.40.

### WRITTEN EXERCISE

**1.** A farmer sold 640 bu. wheat, receiving $1.05 per bushel for 87½ % of it and 85¢ per bushel for the remainder. What was the total amount received?

**2.** A grocer compromised with his creditors, paying 60 % of the amount of his debts. If he owed A $756, B $1250, and C $3750, how much did each receive?

**3.** A merchant sold 360 bbl. apples for $1200. If he received $3.50 per barrel for 66⅔ % of the apples, what was the price received per barrel for the remainder?

**4.** A man bought a house for $12,864.75; he expended for improvements 33⅓ % of the first cost of the property, and then sold it for $20,000. Did he gain or lose, and how much?

**5.** A commission merchant bought 1200 bbl. apples and after holding them for 3 mo. found that his loss from decay was 10 %. If he sold the remainder at $3.75 per barrel, how much did he receive?

**6.** A merchant prepaid the following bills and received the per cents of discount named: 4 % on bill of $875.50; 6 % on bill of $378.45; 2 % on bill of $940.50; 3½ % on bill of $400. What was the net amount paid?

## FINDING THE RATE

### ORAL EXERCISE

1. 8 is what part of 40? what per cent of 40?
2. 90 is what per cent of 270? of 360? of 450?
3. 70 is what per cent of 560? of 630? of 700?
4. The base is 900 and the percentage 450; what is the rate?

**184. Example.** \$35.50 is what per cent of \$284?

SOLUTIONS. *a.* \$35.50 is $\frac{3550}{28400}$ or $\frac{1}{8}$ of \$284. \$284 is 100% of itself; hence, \$35.50, which is $\frac{1}{8}$ of \$284, must be $\frac{1}{8}$ of 100%, or 12½%. Or,

*b.* Since the product of the base and the rate is the percentage, the quotient obtained by dividing the percentage by the base is the rate.

(*a*)

$$\frac{3550}{28400} = \frac{1}{8} = 12\tfrac{1}{2}\,\%$$

(*b*)

$$284\overline{)35.50}\quad .125 = 12\tfrac{1}{2}\,\%$$

**185.** Obviously, *the percentage divided by the base equals the rate.*

### ORAL EXERCISE

*What per cent of:*

1. 95 is 19?
2. 4.8 is 1.2?
3. \$35 is \$17½?
4. 225 A. are 75 A.?
5. 34 bu. are 34 bu.?
6. 34 bu. are 68 bu.?
7. 1.6 is .008?
8. 1 yd. is 1 ft.?
9. 2 da. are 8 hr.?
10. 4 T. are 3000 lb.?
11. 1 yr. are 4 mo.?
12. 2 mi. are 80 rd.?

### WRITTEN EXERCISE

1. A man bought a house for \$7500 and sold it for \$8700. What per cent did he gain?

2. In a certain city, school was in session 190 da. A lost 38 da. What per cent of the school year did he attend?

3. An agent sold a piece of property for \$8462.50 and received \$338.50 for his services. What per cent did he receive?

4. A commission agent sold 28,600 bu. of grain at 50¢ per bushel and received for his services \$357.50. What per cent did he receive on the sales made?

5. Smith and Brown engaged in business, investing \$18,000. Smith invested \$10,440, and Brown the remainder. What per cent of the total capital did each invest?

6. An agent for a wholesale house earned \$165.55 during the month of May. If the goods sold amounted to \$1505, what per cent did he receive on the sales made?

## FINDING THE BASE

### ORAL EXERCISE

**1.** What is 5% of 240 bu.?

**2.** 12 bu. is 5% of how many bushels?

**3.** 160 is 8% of what number? 4%? 2%? 1%? $\frac{1}{2}$%? $\frac{1}{4}$%?

**4.** The multiplicand is 400 and the multiplier 10; what is the product? The product is 2000 and the multiplicand 100; what is the multiplier? The product is 4000 and the multiplier 20; what is the multiplicand?

**5.** In percentage what name is given to the product? to the multiplicand? to the multiplier? When the base and rate are given, how is the percentage found? When the percentage and base are given, how is the rate found? When the percentage and rate are given, how is the base found?

**186. Example.** 37.5 is 25% of what number?

SOLUTION. 25% or $\frac{1}{4}$ of the number $= 37.5$
$\therefore$ the number $= 37.5 \div \frac{1}{4} = 150.$

**187.** Obviously, *the quotient of the percentage divided by the rate equals the base.*

### WRITTEN EXERCISE

**1.** N invested 30% of the capital of a firm, H 35%, and W the remainder, $1400. What was the capital of the firm?

**2.** During the month of May the sales of a clothing merchant amounted to $4864.24, which was 8% of the total sales for the year. What were the total sales for the year?

**3.** B sold his city property and took a mortgage for $4375, which was 17$\frac{1}{2}$% of the value of the property. If the balance was paid in cash, what was the amount of cash received?

**4.** In compromising with his creditors, a man finds that his assets are $270,900, and that this sum is 43% of his entire indebtedness. What will be the aggregate loss to his creditors?

**5.** The aggregate attendance in the schools of a certain city for 1 da. was 43,225 students. If this number was 95% of the number of students belonging, how many students were absent?

## PER CENTS OF INCREASE

### ORAL EXERCISE

**1.** If $2\frac{1}{2}$ times a number is 50, what is the number?

**2.** If 2.5 times a number is 75, what is the number?

**3.** If 250% of a number is \$1250, what is the number?

**4.** If 250% of a number is 150, what is the number? If 250% is 125, what is the number?

**5.** If 300% of a number is \$5400, what is the number?

**188. Examples.** **1.** A man sold a farm for \$3900 and thereby gained 30%. How much did the farm cost?

SOLUTION. $1.30 \text{ of the cost} = \$3900.$
$\therefore \text{ the cost} = \$3900 \div 1.30 = \$3000.$

**2.** What number increased by $33\frac{1}{3}\%$ of itself equals 180?

SOLUTION. $\frac{4}{3} \text{ of the number} = 180$
$\therefore \text{ the number} = 180 \div \frac{4}{3} = 135.$

### ORAL EXERCISE

*What number increased by:*

**1.** 10% of itself is 220?
**2.** 25% of itself is 125?
**3.** 50% of itself is 300?
**4.** 75% of itself is 700?
**5.** $6\frac{1}{4}\%$ of itself is 170?
**6.** $12\frac{1}{2}\%$ of itself is 180?
**7.** $66\frac{2}{3}\%$ of itself is 135?
**8.** $6\frac{2}{3}\%$ of itself is 480?
**9.** 125% of itself is 900?
**10.** $37\frac{1}{2}\%$ of itself is 440?
**11.** $11\frac{1}{9}\%$ of itself is 300?
**12.** $14\frac{2}{7}\%$ of itself is 328?
**13.** 200% of itself is 2700?
**14.** 300% of itself is 2800?

### WRITTEN EXERCISE

**1.** I sold 375 bu. of wheat for \$427.50, thereby gaining 20%. How much did the wheat cost me per bushel?

**2.** A fruit dealer sold a quantity of oranges for \$6.75. If his gain was $12\frac{1}{2}\%$, what did the oranges cost him?

**3.** My savings for March increased $33\frac{1}{3}\%$ over February. If my savings for March were \$84.36, what were my savings for February and March?

## PER CENTS OF DECREASE

### ORAL EXERCISE

**1.** What per cent of a number is left after taking away $33\frac{1}{3}\%$ of it? What fractional part?

**2.** If $\frac{2}{3}$ of a number is 600, what is the number? If $66\frac{2}{3}\%$ of a number is 75, what is the number?

**3.** A man spent $40\%$ of his money and had $60 remaining. How much had he at first? How much did he spend?

**189. Examples.** **1.** A man sold a horse for $332, thereby losing $17\%$. What was the cost?

Solution. 0.83 of the cost = $332.
$\therefore$ the cost = $\$332 \div 0.83 = \$400$.

**2.** What number decreased by $25\%$ of itself equals $375?

Solution. $\frac{3}{4}$ of the number = $375.
$\therefore$ the number = $\$375 \div \frac{3}{4} = \$500$.

### ORAL EXERCISE

*What number diminished by:*

**1.** $6\frac{1}{4}\%$ of itself equals 75?
**2.** $8\frac{1}{3}\%$ of itself equals 440?
**3.** $6\frac{2}{3}\%$ of itself equals 280?
**4.** $\frac{1}{4}$ of itself equals 750?
**5.** $\frac{1}{2}\%$ of itself equals 99.5?
**6.** $1\%$ of itself equals 49.5?

### WRITTEN EXERCISE

**1.** Of what number is 9581.88 $77\%$?

**2.** A merchant sold 1200 bu. of potatoes for $640, which was $16\frac{2}{3}\%$ less than he paid for them. What was the cost per bushel?

**3.** In selling a carriage for $75 a merchant lost $25\%$ on the cost. What was the asking price if the carriage was marked to gain $25\%$?

**4.** A newsboy sold 92 papers on Tuesday. If this number was $23\frac{1}{3}\%$ less than the number sold on Monday, how many papers were sold on the two days?

**5.** A dealer sold a quantity of apples at $6 per barrel, and by so doing lost $16\frac{2}{3}\%$. If he paid $309.60 for the apples, how many barrels did he buy?

### ORAL REVIEW EXERCISE

**1.** By inspection find $12\frac{1}{2}\%$ of the following numbers:

*a.* $872.
*b.* 648 bu.
*c.* 1264 A.
*d.* 960 mi.
*e.* $2464.
*f.* 2696 A.
*g.* 1624 ft.
*h.* 1832 mi.
*i.* $1688.
*j.* 2072 A.
*k.* 11,464 mi.
*l.* 37,128 mi.
*m.* $24.72.
*n.* $168.48.
*o.* $176.24.
*p.* $2184.32.

**2.** By inspection find 10% of each of the above numbers; 25%; 125%; 20%.

**3.** State the missing term in each of the following:

| No. | Base | Rate | Percentage | No. | Base | Rate | Percentage |
|---|---|---|---|---|---|---|---|
| *a.* | $600 | $7\frac{1}{2}\%$ | ? | *f.* | 966 | $16\frac{2}{3}\%$ | ? |
| *b.* | $650 | ? | $39 | *g.* | ? | $8\frac{1}{3}\%$ | 15 bu. |
| *c.* | ? | 4% | $18 | *h.* | 1275 | $6\frac{2}{3}\%$ | ? |
| *d.* | 900 | ? | 720 | *i.* | ? | $6\frac{1}{4}\%$ | 21 mi. |
| *e.* | ? | 4% | 20 | *j.* | 400 | ? | 600 |

**4.** By inspection find 10% of each of the following:

*a.* $264.
*b.* $920.
*c.* $720.
*d.* $840.
*e.* $750.
*f.* $364.
*g.* $232.
*h.* $144.
*i.* $288.
*j.* $2448.
*k.* $1432.
*l.* $3624.

**5.** By inspection find $1\frac{1}{4}\%$ of each of the above numbers; $1\frac{2}{3}\%$; 1000%; 125%; $166\frac{2}{3}\%$.

### WRITTEN REVIEW EXERCISE

**1.** A collector charged 4% on all amounts collected. If he remitted to his customers in one month $3720.48, how much did he receive for his services?

**2.** A father left to his son 60% of his estate and to his daughter the remainder, $9390.88. What was the value of the estate and how much did the son receive?

**3.** A farmer planted 1 bu. 3 pk. of oats on an acre of ground and harvested 56 bu. What per cent of the yield was the planting? What per cent of the planting was the yield?

**4.** A merchant paid the following charges on a bill of goods: cartage $12.45, freight $65.32, insurance $41. If the charges represent 5% of the face of the bill, what was the gross cost of the goods?

**5.** A man had 6 A. of land; to one party he sold a piece 25 rd. by 20 rd., and to another party 140 sq. rd. What per cent of the field remained unsold?

**6.** In a recent year 191,571,750 lb. of fish were landed in Boston and Gloucester, and of this quantity 103,460,410 lb. were landed in Gloucester and 88,111,340 in Boston. What per cent of the total was furnished by each city? (Correct to the nearest .01.)

**7.** A owned property valued at $12,000 from which he received a yearly rental of $960. If he paid taxes amounting to $160, insurance $75.50, and made repairs amounting to $184.50, what per cent net income did he receive?

**8.** B owns a field 80 rd. square. During a certain year this field yielded on an average 25 bu. of wheat to an acre. The wheat when sold at $1 a bushel produced an amount equal to 25% of the value of the field. What was the value of the field?

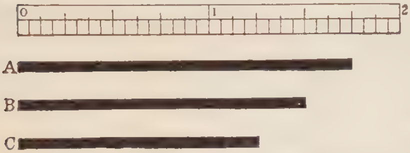
**9.** A landowner rented a field to a tenant and was to receive as rent $16\frac{2}{3}$% of the grain raised. The owner of the field sold his share of the grain for 84¢ per bushel, receiving $298.20. If the tenant sold his share of the grain for the same price per bushel, how much did he receive?

**10.** In a single year the cost of the cotton yarn used in the manufacture of hosiery and knit goods in the state of New York, in round numbers, was $13,825,000; in the state of Illinois, $1,550,000. The cost of the cotton yarn used in Illinois was what per cent less than the cost of the cotton yarn used in New York, in a year? (Correct to the nearest .01.)

**11.** By a recent census report it was shown that the value of all personal property in the state of New York was approximately $500,000,000 and the value of all the real estate approximately $10,000,000,000. Draw parallel lines making a comparison of the personal property and the real estate. The real estate is what per cent greater than the personal property? The personal property is what per cent less than the real estate?

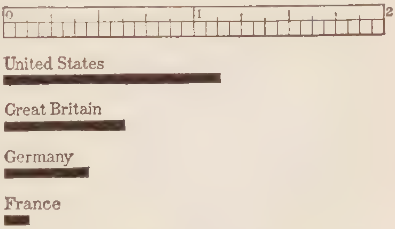
**12.** A young man entered a bank as cashier and at the end of the first year his salary was increased 25 %; at the end of the second year he was given an increase of 20 %; and at the end of the third year he was given an increase of 25 %, which made his salary $ 4500. What salary did he receive at first?

**13.** A government statistician collected facts regarding wages and income from nearly two thousand private manufacturing concerns, and reported the following: the average wages of all employees, men, women, and children, per year was $ 263.06, and the average net profit for each employer was $ 2273. What per cent greater was the income of each employer than of each employee? (Correct to the nearest .01.)



**14.** The population of three cities during a certain year is illustrated by the accompanying lines, which are drawn on a scale of 12,500 inhabitants to each $\frac{1}{8}$ of an inch. What is the population of A, B, and C, respectively? The population of each city is what per cent of the population of the three cities?

**15.** The annual coal production in the United States, Great Britain, Germany, and France for a certain year is illustrated in the accompanying rectangles, drawn on the scale of 50,000,000 short tons to each $\frac{1}{8}$ of an inch. During that year, how many tons did the United States, Great Britain, Germany, and France, respectively, produce? The production of each country is what per cent of the production of the four countries? In the same year the rest of the world produced approximately 200,000,000 short tons. Illustrate graphically the world's coal production for that year. What was the world's approximate production this year?



### A REVIEW EXERCISE

Illustrate the following problems by the use of graphs. Graph forms are given on pages 126, 133. Use the form suggested by the instructor.

1. Illustrate graphically problem 25, page 88. Use the even number of thousands for each month.

2. In a recent year the railway mileage, single-track, of the world was as follows: America, 325,000 mi.; Europe, 200,000 mi.; Asia, 63,000 mi.; Africa, 23,000 mi.; Australia, 19,000 mi. Illustrate graphically, showing the total mileage, and the relation that each country bears to the total.

3. In a recent year there were enrolled in the schools and colleges of the United States 20,000,000 students, grouped according to ages as follows: 5 yr., 400,000; 6 to 9 yr., 6,200,000; 10 to 14 yr., 9,000,000; 15 to 17 yr., 3,000,000; 18 to 20 yr., 1,000,000; 21 to 24 yr., 400,000. Illustrate graphically. Each group is what per cent of the total?

4. The number of cattle, other than milch cows, on farms and ranches in the United States, as reported by the decennial censuses, for the years named were as follows: 1870, 13,500,000; 1880, 22,500,000; 1890, 33,500,000; 1900, 50,000,000; 1910, 41,000,000. Illustrate graphically. What do these figures suggest regarding the cost of living as applied to beef?

5. The following figures represent the latest estimates of the wealth of the nations named. The figures given represent billions of dollars: United States, 130; Great Britain and Ireland, 80; France, 65; Germany, 60; Russia, 40; Austria-Hungary, 25; Italy, 20; Belgium, 9; Spain, $5\frac{1}{2}$; Netherlands, 5; Portugal, $2\frac{1}{2}$. Switzerland, $2\frac{1}{2}$. Illustrate graphically.

6. In a recent year the cities of the United States which had a population of over 100,000 expended $100,000,000 in various school expenses, according to the following geographical divisions: North Atlantic Division, $54,000,000; North Central Division, $30,000,000; South Atlantic Division, $2,700,000; South Central Division, $3,000,000; Western Division, $10,300,000. Illustrate graphically.

### A WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

**1.** A gardener planted 1 qt. of corn and harvested 5 bu. What per cent of the planting was the harvest?

**2.** A bookkeeper made an investment on which he lost 15%. If the sum returned to him was \$1912.50, what was the investment?

**3.** A piece of cloth, unfinished, cost 6¢ per yard. It costs .0075¢ per yard to bleach it, and then it sells for $7\frac{3}{4}$¢ per yard. The selling price is what per cent advance over the total cost?

**4.** A merchant paid the following bills less the discounts named: \$85.50 less 2%; \$141.50 less 3%; \$117.95 less 1%; \$225.40 less $1\frac{1}{2}$%; \$47.50 less $2\frac{1}{2}$%. What was the total sum paid? What was the total discount allowed?

**5.** On Monday a man deposited in the bank \$184.96. On Wednesday he deposited a sum $12\frac{1}{2}$% greater than the deposit of Monday; he then drew a check for 50% of his total deposit. What was the amount of the check?

**6.** A merchant's sales increased the second month of his business 25% over the first month; the third month they increased 20% over the second month; the fourth month they decreased 10% from the sales of the third month. What were the sales for each month if they were \$3240 for the fourth month?

**7.** A farmer used 1200 lb. of potato fertilizer per acre, on a 16-acre field of potatoes. The fertilizer cost \$24.125 per ton, less 5% for cash payment. If the unfertilized land produced 60 bu. of potatoes per acre, and the fertilized land produced 150 bu. per acre, what per cent of increase was realized by using the fertilizer if the potatoes sold for 80¢ per bushel?

**8.** A man bought a piece of land, and at the end of the first year it had increased in value 25%; at the end of the second year it had increased an additional 8% in value; at the end of the third year it had increased an additional 5% in value. What did he pay for the property if at the end of the third year it was worth \$2551.50?

# CHAPTER XIV

## COMMERCIAL DISCOUNTS

### ORAL EXERCISE

**1.** A set of Scott's works is marked $12. If I buy it at this price, less $16\frac{2}{3}\%$, what does it cost me?

**2.** I buy $90 worth of goods on 30 da. time, or 5% off for cash. What cash payment will settle the bill?

**3.** I owe B $600, due in 30 da. He offers to allow me 5% discount if I pay cash to-day. I accept his offer and give him a check for the amount. What was the amount of the check?

**190.** A reduction from the catalogue (list) price of an article, from the amount of a bill of merchandise, or from the amount of a debt, is called a **commercial** or **trade discount.**

Business houses usually announce their terms upon their bill heads. The space allowed for recording the terms is usually limited, and bookkeepers find it necessary to use symbols and abbreviations to indicate them. Thus, if a bill is due in 30 da. without discount, the terms may be written $^{N}/_{30}$, or Net 30 da.; if the bill is due in 30 da. without discount, but an allowance of 2% is made for payment within 10 da., the terms may be written $^{2}/_{10}$, $^{N}/_{30}$, or 2% 10 da., net 30 da.

**191.** Manufacturers, jobbers, and wholesale dealers usually have printed price lists for their goods. To obviate the necessity of issuing a new catalogue every time the market changes, these lists are frequently printed higher than the actual selling price of the goods, and made subject to a trade discount.

**192.** The fluctuations of the market and the differences in the quantities purchased by different customers frequently give rise to two or more discounts called a **discount series.**

Large purchasers sometimes get better prices and terms than small purchasers. Thus, the average customer might be quoted the regular prices less a trade discount of 25%, while an especially large buyer might be quoted the regular prices less trade discounts of 25% and 10%.

**193.** When two or more discounts are quoted, one denotes a discount off the list price, another, a discount off the remainder, and so on.

The order in which the discounts of any series is considered is not material. Thus, a series of 25 %, 20 %, and 10 % is the same as one of 20 %, 10 %, and 25 %, or one of 10 %, 25 %, and 20 %.

**194.** Catalogue prices are generally estimated on the basis of credit sales, and a cash purchaser is given the usual trade discount and a special discount for early payment. This latter discount has the effect of encouraging prompt payments.

The list price is sometimes called the **gross price** and the price after the discount has been deducted the **net price.**

## FINDING THE NET PRICE

**195. Example.** The list price of a dozen pairs of shoes is \$45. If this price is subject to a discount series of 20 % and 10 %, what is the net selling price?

SOLUTION. 20 % or $\frac{1}{5}$ of \$45 = \$9, the first discount.
\$45 − \$9 = \$36, the price after the first discount.
10 % or $\frac{1}{10}$ of \$36 = \$3.60, the second discount.
\$36 − \$3.60 = \$32.40, the net selling price.

### ORAL EXERCISE

*Find the net price:*

| | LIST PRICE | TRADE DISCOUNT | | LIST PRICE | TRADE DISCOUNT | | LIST PRICE | TRADE DISCOUNTS |
|---|---|---|---|---|---|---|---|---|
| **1.** | \$4 | 25 % | **8.** | \$6 | 40 % | **15.** | \$4 | 25 % and $33\frac{1}{3}$ % |
| **2.** | \$15 | 20 % | **9.** | \$4 | $12\frac{1}{2}$ % | **16.** | \$30 | $33\frac{1}{3}$ % and 25 % |
| **3.** | \$90 | $33\frac{1}{3}$ % | **10.** | \$24 | $8\frac{1}{3}$ % | **17.** | \$35 | 20 % and 25 % |
| **4.** | \$20 | 10 % | **11.** | \$42 | $16\frac{2}{3}$ % | **18.** | \$45 | 20 % and $16\frac{2}{3}$ % |
| **5.** | \$50 | 50 % | **12.** | \$35 | 20 % | **19.** | \$50 | 20 % and 25 % |
| **6.** | \$2.50 | 20 % | **13.** | \$100 | 25 % | **20.** | \$100 | 20 % and 10 % |
| **7.** | \$4.50 | $16\frac{2}{3}$ % | **14.** | \$720 | $33\frac{1}{3}$ % | **21.** | \$600 | $16\frac{2}{3}$ % and 20 % |

**22.** A piano listed at \$750 is sold less $33\frac{1}{3}$ %, 20 %, and 10 %. What is the net cost to the purchaser?

**23.** A dealer offers cloth at \$3.50 per yard subject to a discount of 20 %. How many yards can be bought for \$56?

### WRITTEN EXERCISE

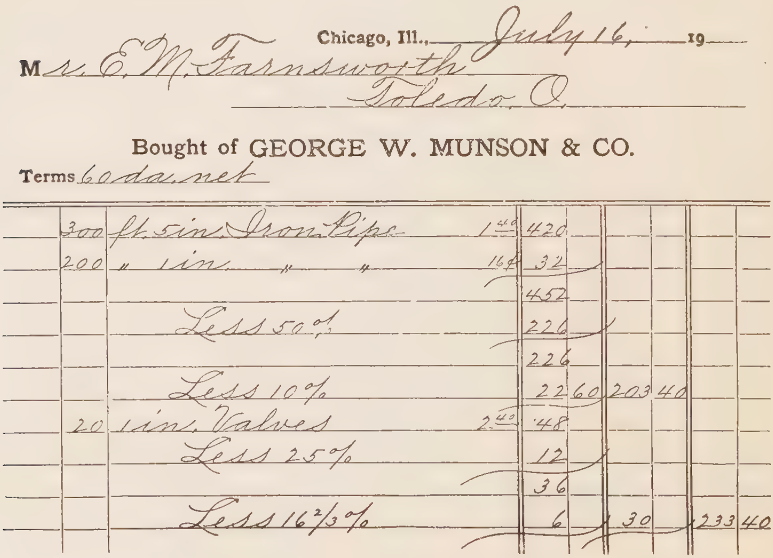
*Find the net price :*

| | Gross Selling Price | Trade Discounts | | Gross Selling Price | Trade Discounts |
|---|---|---|---|---|---|
| **1.** | $3360 | 25 % and 10 % | **4.** | $2500 | 20 %, 10 %, and 5 % |
| **2.** | $3510 | $33\frac{1}{3}$ % and 20 % | **5.** | $5400 | 25 %, 20 %, and 10 % |
| **3.** | $4500 | 20 % and $16\frac{2}{3}$ % | **6.** | $3960 | $33\frac{1}{3}$ %, 20 %, and $16\frac{2}{3}$ % |

**7.** The list price of cloth is $4.80 per yard, but this price is subject to discounts of 25 % and 20 %. How many yards can be bought for $288?

**8.** A hardware dealer sold 25 doz. 5-in. files at $2.50 and 25 doz. 12-in. files at $7.50, less 50 % and 10 %; 150 machine bolts at $21.50 per C, less 20 % and 10 %. What was the net amount of the bill?

**9.** Study the following model. Copy and find the net amount of the bill, using the discounts named in the bill, and the following prices: 5-in. pipe, $1.45; 1-in. pipe, 17 ¢; valves, $2.67.



Chicago, Ill., July 16, 19

Mr. E. M. Farnsworth
Toledo, O.

Bought of GEORGE W. MUNSON & CO.

Terms 60 da. net

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 300 | ft. 5 in. Iron Pipe | $1^{40}$ | 420 | | | | | |
| | 200 | " 1 in. " " | 16¢ | 32 | | | | | |
| | | | | 452 | | | | | |
| | | Less 50 % | | 226 | | | | | |
| | | | | 226 | | | | | |
| | | Less 10 % | | 22 | 60 | 203 | 40 | | |
| | 20 | 1 in. Valves | $2^{40}$ | 48 | | | | | |
| | | Less 25 % | | 12 | | | | | |
| | | | | 36 | | | | | |
| | | Less 16 2/3 % | | 6 | | 30 | | 233 | 40 |

**10.** One firm offers a piano for $400, subject to discounts of 20% and 20%; another offers the same piano for $400 less discounts of 25% and 15%. Which is the better offer? How much better?

**11.** A jobber bought a quantity of goods listed at $3600, subject to discounts of 25% and 20%. He sold the goods at the same list price, subject to discounts of 20% and 10%. Did he gain or lose, and how much?

**12.** Make out bills for the following, using the current date and the name and address of some dealer whom you know. Terms in each case, 60 da. net.

*a.* You bought 12 doz. hand saws, # 27, at $18.50; $7\frac{1}{2}$ doz. mortise locks, # 271, at $4.25; 25 doz. pocket knives, # 27, at $7.50; and $1\frac{3}{4}$ doz. cheese knives at $8.25. Discount: 25%, 10%.

*b.* You bought $41\frac{1}{2}'$ of $2''$ extra strong iron pipe at 70¢; $94\frac{1}{4}'$ of $1\frac{1}{4}''$ extra strong iron pipe at $31\frac{1}{2}$¢; $153\frac{1}{4}'$ of $\frac{1}{4}''$ iron pipe at $6\frac{1}{2}$¢; $88\frac{1}{2}'$ of $\frac{3}{8}''$ iron pipe at $7\frac{3}{4}$¢. Discount: 25%, 10%.

*c.* You bought 25 kitchen tables at $3.25; 25 dining-room tables at $8.75; 15 doz. dining-room chairs at $12.50; 12 antique rockers at $12.25; and 15 oak bedroom sets at $32.50. Discount: $16\frac{2}{3}$%, 5%.

## FINDING A SINGLE RATE OF DISCOUNT EQUIVALENT TO A DISCOUNT SERIES

**196. Example.** What single rate of discount is equivalent to a discount series of 25%, $33\frac{1}{3}$%, and 10%?

Solution. Represent the list price by 100%. Then, 75% equals the price after the first discount, 50% the price after the second discount, and 45% equals the net selling price. 100%, the list price, minus 45%, the net selling price, equals 55%, the single rate of discount equivalent to the given discount series.

A single discount equivalent to a discount series may often be determined mentally (see §§ 197, 198).

$$\begin{array}{l}
1.00 \\
\underline{\phantom{0}.25} \ (25\% \text{ of } 100\%) \\
\phantom{0}.75 \\
\underline{\phantom{0}.25} \ (33\tfrac{1}{3}\% \text{ of } 75\%) \\
\phantom{0}.50 \\
\underline{\phantom{0}.05} \ (10\% \text{ of } 50\%) \\
\phantom{0}.45 \\
100\% - 45\% = 55\%
\end{array}$$

### WRITTEN EXERCISE

**1.** Find a single rate of discount equivalent to a discount series of 50 %, 25 %, 20 %, and 10 %.

**2.** Which is the better for the buyer and how much, a single discount of 65 % or a discount series of 25 %, 20 %, and 20 %?

**3.** The net amount of a bill of goods was $ 450 and the discounts allowed were 25%, 33⅓%, and 10 %. Find the total discount allowed.

**4.** I allowed a customer discounts of 50%, 10 %, and 10 % from a list price. What per cent better would a single discount of 65 % have been?

**5.** Goods were sold subject to trade discounts of 25 %, 20 %, and 10 %. If the total discount allowed was $ 460, what was the net selling price of the goods?

**6.** A quantity of goods was sold subject to trade discounts of 20 % and 20 %. The terms were 60 da. net or 5 % off for payment within 10 da. If a cash payment of $ 1026 was required 3 da. after the date of the bill, what was the list price of the goods sold?

**197.** Since the first of a series of discounts is computed on 100 % of the list price, and the second on 100 % minus the first discount, it follows that *the sum of any two separate discounts exceeds the equivalent single discount by the product of the two rates per cent.*

Thus, in a discount series of 20 % and 20 % the apparent single discount is the sum of the two separate discounts or 40 %; but since the second discount is not computed on 100 %, but on 80 %, 40 % exceeds the true single discount by 20 % of 20 %, or 4 %; and the equivalent single discount is 40 % minus 4 %, or 36 %. Hence,

**198.** To find the single discount equivalent to a series of two discounts:

*From the sum of the separate discounts subtract their product, and the remainder will be the equivalent single discount.*

When two separate discounts cannot be reduced to a single discount *mentally*, proceed as in § 196; when they can, proceed as in § 198.

### ORAL EXERCISE

*State a single rate of discount equivalent to a discount series of:*

**1.** 10% and 10%.
**2.** 20% and 20%.
**3.** 30% and 30%.
**4.** 40% and 40%.
**5.** 50% and 50%.
**6.** 20% and 10%.
**7.** 30% and 10%.
**8.** 40% and 10%.
**9.** 50% and 10%.
**10.** 60% and 10%.
**11.** 30% and 20%.
**12.** 40% and 20%.
**13.** 50% and 20%.
**14.** 60% and 20%.
**15.** 25% and 10%.
**16.** 35% and 40%.
**17.** 50% and 5%.
**18.** 10% and 5%.
**19.** 20% and 5%.
**20.** 40% and 5%.
**21.** 25% and 30%.
**22.** 25% and 40%.
**23.** 20% and 15%.
**24.** 40% and 15%.
**25.** 35% and 20%.
**26.** 45% and 20%.
**27.** 55% and 20%.
**28.** 60% and 25%.
**29.** 40% and 25%.
**30.** 60% and 15%.
**31.** 25% and $33\frac{1}{3}$%.
**32.** 45% and $33\frac{1}{3}$%.
**33.** 25% and 8%.
**34.** $8\frac{1}{3}$% and 24%.
**35.** $8\frac{1}{3}$% and 36%.
**36.** 35% and 10%.
**37.** 20% and $12\frac{1}{2}$%.
**38.** 40% and $12\frac{1}{2}$%.
**39.** 60% and $12\frac{1}{2}$%.
**40.** 12% and $12\frac{1}{2}$%.
**41.** 24% and $16\frac{2}{3}$%.
**42.** $16\frac{2}{3}$% and 20%.
**43.** $14\frac{2}{7}$% and 35%.
**44.** $16\frac{2}{3}$% and 25%.
**45.** $33\frac{1}{3}$% and 15%.
**46.** $66\frac{2}{3}$% and 15%.
**47.** $11\frac{1}{9}$% and 18%.
**48.** 36% and $11\frac{1}{9}$%.

**199.** When a discount series consists of three separate rates, the first two may be combined as in § 198 and then the result and the third may be combined in the same manner.

**200. Example.** Find a single rate of discount equivalent to a discount series of 25%, 20%, and 20%.

SOLUTION. — Combine the first two by thinking 25% + 20% − 5% = 40%, the single discount equivalent to the series 25% and 20%. 20% + 40% − 8% = 52%, or the single rate equivalent to the discount series 25%, 20%, and 20%.

### ORAL EXERCISE

*State a single rate of discount equivalent to a discount series of:*

**1.** 20%, 25%, and 20%.
**2.** 20%, 15%, and 10%.
**3.** 20%, 20%, and 20%.
**4.** 10%, 10%, and 10%.
**5.** 20%, 20%, and 10%.
**6.** 25%, $33\frac{1}{3}$%, and 10%.
**7.** 20%, 10%, and 10%.
**8.** 40%, 10%, and 10%.
**9.** 50%, 10%, and 10%.
**10.** 30%, 10%, and 10%.
**11.** 20%, 25%, and 10%.
**12.** 20%, 20%, and 25%.

**201.** When it is not desirable to show the separate discounts, the net selling price may be found as shown in the following example.

**202. Example.** A mahogany sideboard listed at $175 is sold subject to trade discounts of 20% and 25%. Find the net cost to the purchaser.

Solution. By inspection determine that a discount of 40% is equivalent to a series of 25% and 20%. Represent the gross cost by 100%. Then 100% − 40% = 60%, the net cost to the purchaser; that is, the net cost of the sideboard is 60% of the list price. 60% of $175 = $105, the net cost to the purchaser.

$$100\% - 40\% = 60\%$$
$$60\% \text{ of } \$175 = \$105$$

### ORAL EXERCISE

*By inspection find the net cost of articles listed at:*

1. $400, less 20% and 25%.
2. $300, less 20% and 20%.
3. $600, less 10% and 10%.
4. $200, less 30% and 30%.
5. $1000, less 50% and 50%.
6. $1000, less 30% and 10%.
7. $200, less 60% and 25%.
8. $400, less 20% and 40%.

### WRITTEN EXERCISE

**1.** Find the net selling price of a piano listed at $450, less 20% and 20%.

**2.** Find the net selling price of an oak sideboard listed at $125, less 25%, 33$\frac{1}{3}$%, and 10%.

**3.** I bought 125 cultivators listed at $8.50, each subject to trade discounts of 20% and 25%. If I paid freight $30.50 and drayage $7.90, how much did the cultivators cost me?

**4.** The net cost of an article was increased $30 by freight, making the actual cost of it $630. What was the list price of the article, the rates of discount being 25% and 33$\frac{1}{3}$%?

**5.** You desire to buy 24,000 ft. choice cypress: one firm quotes you $60 per thousand feet, less trade discounts of 20% and 5%; another firm offers you the same lumber at $75 per thousand feet, less 33$\frac{1}{3}$% and 8%. The terms offered by both firms are $^1/_{10}$, $^N/_{30}$. You accept the better offer and pay cash. How much does the lumber cost you?

### WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

**1.** If goods are bought 25% below the list price and sold at the list price, what is the advance per cent over the cost?

**2.** If goods are bought 20% below the list price and sold at the list price, what is the advance per cent over the cost?

**3.** If goods are bought 10% below the list price and sold at 10% above the list price, what is the advance per cent over the cost?

**4.** If goods are bought at 20% and $12\frac{1}{2}$% below the list price, and sold at 10% below the list price, what is the advance per cent over the cost?

**5.** A hardware dealer bought a machine listed at $24, less $16\frac{2}{3}$% and 10%, and sold it at the list price. At what per cent above cost did he mark the selling price?

**6.** A jobber wished to buy at such a discount from the manufacturer's list price that he could make an advance of 25% over cost, and still sell at the manufacturer's list price. What would the jobber pay for $1000 worth of goods?

**7.** A gentleman wished to buy a carriage. One dealer offered him a discount of $33\frac{1}{3}$% and 10% from the list price, and another dealer offered him 20%, 10%, and 10% from the list price. If the list price is $450, what will be the cost of the carriage if it is bought at the better discount?

**8.** Aug. 5, you buy of Gray, Salisbury & Son, New York City, 4000 lb. raisins at 16 ¢, less trade discounts of 25%, 20%, and 10%. Terms: $^2/_{10}$, $^N/_{30}$. You pay cash for freight $3.20. If you pay the bill Aug. 7, what will the raisins cost you?

**9.** You desire to buy 200 lb. nutmeg. You find that S. S. Pierce Co., of your city, offer this article at 75 ¢ per lb., less a discount of 25%, and that Smith, Perkins & Co., New York City, offer it at 70 ¢ per lb., less discounts of 15% and 10%. The freight from New York to your city on a package of this kind is $1.50. The terms offered by both firms are: $^1/_{10}$, $^N/_{30}$. You accept the better offer and pay cash. How much does the nutmeg cost you?

# CHAPTER XV

## GAIN AND LOSS

### ORAL EXERCISE

**1.** What is $33\frac{1}{3}\%$ of $660? How much is gained on goods bought for $900 and sold at a profit of $33\frac{1}{3}\%$?

**2.** What per cent greater is $75 than $60? what per cent less is $60 than $75? Goods bought for $100 are sold for $150. What is the gain per cent?

**3.** What per cent less is $80 than $100? what per cent more is $100 than $80? Goods bought for $100 are sold for $90. What is the loss per cent?

**4.** If $800 is increased by 25% of itself, what is the result? Goods bought for $1400 are sold at a profit of 25%. What is the selling price?

**5.** If $1500 is decreased by $33\frac{1}{3}\%$ of itself, what is the result? Goods bought for $2700 are sold at a loss of $33\frac{1}{3}\%$. What is the selling price?

**6.** State a brief method for finding a gain of $6\frac{1}{4}\%$; a gain of $6\frac{2}{3}\%$; a gain of $8\frac{1}{3}\%$; a gain of 10%; a gain of $1\frac{1}{4}\%$; a gain of $1\frac{2}{3}\%$; a gain of $2\frac{1}{2}\%$; a gain of $3\frac{1}{3}\%$.

**7.** State a brief method for finding a loss of $11\frac{1}{9}\%$; a loss of $12\frac{1}{2}\%$; a loss of $14\frac{2}{7}\%$; a loss of $16\frac{2}{3}\%$; a loss of 20%; a loss of 25%; a loss of $9\frac{1}{11}\%$; a loss of $37\frac{1}{2}\%$.

**8.** State a brief method for finding a gain of $33\frac{1}{3}\%$; a gain of $22\frac{2}{9}\%$; a gain of 50%; a gain of $66\frac{2}{3}\%$; a gain of 75%.

**203.** The **gains** and **losses** resulting from business transactions are frequently estimated at some *rate per cent* of the cost, or of the money or capital invested.

Since no new principles are involved in this subject, illustrative examples are unnecessary.

## FINDING THE GAIN OR LOSS

### ORAL EXERCISE

*By inspection find the gain or loss:*

|  | Cost | Per Cent of Gain |  | Cost | Per Cent of Loss |  | Cost | Per Cent of Gain |
|---|---|---|---|---|---|---|---|---|
| **1.** | $2900 | 50% | **9.** | $1500 | 10% | **17.** | $7500 | 20% |
| **2.** | $1600 | 75% | **10.** | $1600 | $1\frac{1}{4}\%$ | **18.** | $1400 | 25% |
| **3.** | $5600 | $28\frac{4}{7}\%$ | **11.** | $3000 | $1\frac{2}{3}\%$ | **19.** | $2200 | $9\frac{1}{11}\%$ |
| **4.** | $2700 | $33\frac{1}{3}\%$ | **12.** | $4800 | $2\frac{1}{2}\%$ | **20.** | $8100 | $11\frac{1}{9}\%$ |
| **5.** | $2400 | $37\frac{1}{2}\%$ | **13.** | $3600 | $3\frac{1}{3}\%$ | **21.** | $6400 | $12\frac{1}{2}\%$ |
| **6.** | $1400 | $42\frac{6}{7}\%$ | **14.** | $3200 | $6\frac{1}{4}\%$ | **22.** | $2800 | $14\frac{2}{7}\%$ |
| **7.** | $3200 | $62\frac{1}{2}\%$ | **15.** | $4500 | $6\frac{2}{3}\%$ | **23.** | $9600 | $16\frac{2}{3}\%$ |
| **8.** | $2100 | $66\frac{2}{3}\%$ | **16.** | $8400 | $8\frac{1}{3}\%$ | **24.** | $3600 | $22\frac{2}{9}\%$ |

**25–48.** Find the selling price in each of the above problems.

### WRITTEN EXERCISE

**1.** An importation of silks invoiced at £40 10*s.* was sold at a profit of 25%. Find the amount (in United States money) of the gain.

**2.** An importation of German toys invoiced at 43,750 marks was sold at a gain of $33\frac{1}{3}\%$. Find the amount (in United States money) of the gain.

**3.** An article that cost $1 was marked 10% above cost. In order to effect a sale, it was afterward sold for 10% below the marked price. Find the gain or loss on 250 of the articles.

**4.** A man bought a city lot for $1150 and built a house on it costing $2650. He then sold the house and lot at a gain of 5%. How much did he gain and what was his selling price?

**5.** A man bought a quantity of silk for $450, a quantity of fancy plaids for $120, and a quantity of velvet for $90. He sold the silk at a gain of 25%, the plaids at a loss of 5%, and the velvet at a gain of $33\frac{1}{3}\%$. What was his gain, and how much did he realize from the sale of the three kinds of material?

## FINDING THE PER CENT OF GAIN OR LOSS

### ORAL EXERCISE

*By inspection find the per cent of gain or loss:*

| | Cost | Gain | | Cost | Loss | | Cost | Selling Price | | Selling Price | Gain |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | $100 | $10 | **7.** | $60 | $15 | **13.** | $80 | $90 | **19.** | $300 | $60 |
| **2.** | $150 | $50 | **8.** | $40 | $10 | **14.** | $90 | $80 | **20.** | $115 | $23 |
| **3.** | $140 | $70 | **9.** | $90 | $45 | **15.** | $60 | $75 | **21.** | $102 | $17 |
| **4.** | $140 | $140 | **10.** | $70 | $14 | **16.** | $75 | $60 | **22.** | $420 | $60 |
| **5.** | $200 | $400 | **11.** | $80 | $16 | **17.** | $10 | $50 | **23.** | $300 | $200 |
| **6.** | $300 | $750 | **12.** | $15 | $10 | **18.** | $50 | $10 | **24.** | $700 | $100 |

### WRITTEN EXERCISE

**1.** A milliner bought hats at $15 a dozen and retailed them at $3 each. What per cent was gained?

**2.** A stationer bought paper at $2 a ream and retailed the same at a cent a sheet. What was his per cent of gain?

**3.** A dry-goods merchant bought gloves at $7.50 a dozen pair and retailed them at $1.25 a pair. What was his per cent of gain?

**4.** A merchant imported 50 gro. of table knives at a cost of $1125. Two months later he found that the sales of table knives aggregated $920 and that the value of the stock unsold was $435. Did he gain or lose, and what per cent?

**5.** An importer bought a quantity of silk goods for £400 5*s.* After disposing of a part of the goods for $1200 he took an account of the stock remaining unsold and found that at cost prices it was worth $1047.82. Did he gain or lose, and what per cent?

**6.** Jan. 1, F. E. Smith & Co. had merchandise on hand valued at $2500. During the month they purchased goods costing $6000 and sold goods amounting to $7500. If the stock on hand at cost prices Feb. 5 was worth $2500, what was the per cent of gain on the sales?

## FINDING THE COST

### ORAL EXERCISE

*By inspection find the cost:*

| | Loss | Rate of Loss |
|---|---|---|
| **1.** | $150 | 10 % |
| **2.** | $100 | $1\frac{1}{4}$ % |
| **3.** | $200 | $1\frac{2}{3}$ % |
| **4.** | $450 | $2\frac{1}{2}$ % |
| **5.** | $220 | $6\frac{1}{4}$ % |
| **6.** | $115 | $8\frac{1}{3}$ % |

| | Gain | Rate of Gain |
|---|---|---|
| **7.** | $35 | 20 % |
| **8.** | $79 | 25 % |
| **9.** | $12 | $11\frac{1}{9}$ % |
| **10.** | $19 | $16\frac{2}{3}$ % |
| **11.** | $44 | $22\frac{2}{9}$ % |
| **12.** | $15 | $33\frac{1}{3}$ % |

| | Selling Price | Rate of Gain |
|---|---|---|
| **13.** | $1050 | 5 % |
| **14.** | $2040 | 2 % |
| **15.** | $3600 | 20 % |
| **16.** | $1400 | $16\frac{2}{3}$ % |
| **17.** | $1800 | $12\frac{1}{2}$ % |
| **18.** | $2400 | $33\frac{1}{3}$ % |

| | Selling Price | Rate of Loss |
|---|---|---|
| **19.** | $950 | 5 % |
| **20.** | $900 | 50 % |
| **21.** | $150 | $6\frac{1}{4}$ % |
| **22.** | $550 | $16\frac{2}{3}$ % |
| **23.** | $240 | $33\frac{1}{3}$ % |
| **24.** | $490 | $22\frac{2}{9}$ % |

**25.** A man bought a machine for $240.48. For how much must he sell it to gain $12\frac{1}{2}$ %?

**26.** B sold a farm for $2400, thereby losing 25 %. For how much should he have sold it to have gained 10 %?

**27.** By selling a piano at $400 a dealer realizes a gain of $33\frac{1}{3}$ %. What would be the selling price of the piano if sold at a gain of 25 %?

### WRITTEN EXERCISE

**1.** A sleigh was sold for $64.80, which was 10 % below cost. What was the cost?

**2.** An office safe was sold at $102, which was 20 % above cost. What was the cost?

**3.** A merchant marks goods $16\frac{2}{3}$ % above cost. What is the cost of an article that he has marked $21.70?

**4.** An owner of real estate sold 2 city lots for $12,000 each. On one he gained 25% and on the other he lost 25%. What was his net gain or loss from the two transactions?

**5.** A merchant sold a quantity of goods to a customer at a gain of 25%, but owing to the failure of the customer he received in settlement but 88¢ on the dollar. If the merchant gained $645.15, what did the goods cost him?

**6.** A manufacturer sold an article to a jobber at a gain of 25%, the jobber sold it to a wholesaler at a gain of 20%, and the wholesaler sold it to a retailer at a gain of $33\frac{1}{3}$%. If the retailer paid $28 for the article, what was the cost to manufacture it?

**7.** A manufacturer sold an article to a wholesaler at a gain of 20%, the wholesaler sold the same article to a retailer at a gain of $33\frac{1}{3}$%, and the retailer to the consumer at a gain of 25%. If the average gain was $40, what was the cost to manufacture the article?

### WRITTEN REVIEW EXERCISE

**1.** A merchant bought goods at 40% off from the list price and sold the same at 20% and 10% off the list price. What was his gain per cent?

**2.** I bought goods at 50% off from the list price and sold them at 25% and 25% off from the list price. Did I gain or lose, and what per cent?

**3.** Apr. 15 you bought of Baker, Taylor & Co., Rochester, N.Y., 4000 bbl. Roller Process flour listed at $4.50 a barrel, and 2000 bbl. of Searchlight pastry flour listed at $4.75 a barrel. Each list price was subject to trade discounts of 20% and 10%. You paid cash $16,000 and gave your note at 30 da. for the balance. What was the amount of the note?

**4.** May 18 you sold to F. H. Clark & Co., New York City, 2000 bbl. of the Roller Process flour, bought in problem 3, at $33\frac{1}{3}$% above cost. Terms: $^2/_{10}$, $^N/_{30}$. F. H. Clark & Co. paid cash. Find the cash payment.

**5.** May 30 you sold Smith, Perkins & Co., Albany, N.Y., the balance of the flour bought in problem 3, at an advance of $33\frac{1}{3}\%$ on the cost. Terms: $^{2}/_{10}$, $^{N}/_{30}$. The flour was paid for June 8. Find the cash payment.

**6.** What is the net gain on the transactions in problems 3, 4, and 5? the net gain per cent?

**7.** Dec. 15 you bought of E. B. Johnson & Co. 400 bbl. of apples at $2.50 per barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. You paid cash. Find the amount of your payment.

**8.** May 15 you sold F. E. Redmond the apples bought in problem 7, at $4 a barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. At the maturity of the bill Redmond refused payment and you placed the account in the hands of a lawyer who succeeded in collecting 75% of the amount due. If the lawyer's fee for collecting was 4%, what was your net gain or loss?

**9.** A tailor made 25 doz. overcoats with cloth worth $2 a yard. 4 yd. were required for each coat and the cost of making was $48 per dozen. He sold the overcoats so as to gain $33\frac{1}{3}\%$. How much did he receive for each?

**10.** Apr. 12 J. D. Farley & Son, Trenton, N. J., bought of Cobb, Bates & Co., Boston, Mass., a quantity of green Java coffee sufficient to yield 2400 lb. when roasted. If the loss of weight in roasting averages 4%, what will the green coffee cost at 30¢ a pound, less a trade discount of 10%? Arrange the problem in bill form.

**11.** If the coffee in problem 10 is retailed $33\frac{1}{3}\%$ above cost, and there is a loss of 1% from bad debts, what is the gain on the transactions in coffee? the gain per cent?

**12.** The Metropolitan Coal Co., of Boston, Mass., decides to bid on a contract for supplying 2240 T. of coal for the public schools of the city. It can buy the coal at $4.50 per long ton delivered on board track, Boston. It costs on an average 75¢ per short ton to deliver the coal, and there is a waste of $\frac{1}{4}\%$ from handling. Name a bid covering a profit of 20%. Terms: cash.

## FINDING THE PER CENT OF GAIN OR LOSS ON THE SELLING PRICE

### ORAL EXERCISE

1. An article cost $80 and it is sold for $100. What is the sum gained? The gain is what per cent of the cost? of the selling price?

2. An article costs $60 and it is sold for $75. What is the sum gained? The gain is what per cent of the cost? of the selling price?

3. An article is sold for $90. If the gain on the selling price is $33\frac{1}{3}\%$, what was the cost, and what is the gain per cent on the cost price?

**204.** Find by inspection the gain per cent on the selling price:

| | Cost | Selling Price | | Cost | Selling Price |
|---|---|---|---|---|---|
| *a.* | $20 | $30 | *f.* | $120 | $150 |
| *b.* | $30 | $40 | *g.* | $125 | $150 |
| *c.* | $45 | $60 | *h.* | $140 | $160 |
| *d.* | $60 | $75 | *i.* | $150 | $175 |
| *e.* | $50 | $60 | *j.* | $160 | $180 |

This principle may be applied effectively when goods have been marked by a merchant at a certain per cent on the advance of the cost, and then marked down to sell at cost.

**205.** If an article that costs $1 is marked to sell at $1.10, what per cent of reduction will restore the original cost price?

Suggestion. It is evident that a reduction of 10% on the selling price will not restore the original marking of $1.

**206.** Find by inspection the per cent of reduction that must be made to reduce the marked price to the cost price.

| | Cost | Marked Price | | Cost | Marked Price |
|---|---|---|---|---|---|
| *a.* | $1.00 | $1.25 | *d.* | $1.50 | $1.80 |
| *b.* | $1.25 | $1.50 | *e.* | $2.00 | $2.50 |
| *c.* | $1.60 | $2.00 | *f.* | $3.00 | $4.00 |

**207.** Business men are continually dealing with the problem of *overhead charges;* that is, the *cost of doing business.* Overhead charges include such expenses as employees' salaries, rent, insurance, taxes, light and heat, postage, advertising, depreciation, telephone, and many others. To the invoice charges there must be added a certain per cent to cover the cost of doing business.

**208.** The following principle applies to subsequent problems: Divide the invoice cost plus the freight by *100 % minus the overhead charges plus the per cent of profit* $(100\,\% - \overline{\text{charges} + \text{profits}})$; the result will be the selling price. (This statement is based on reckoning the overhead expenses and the gain as a per cent of the selling price.)

### WRITTEN EXERCISE

**1.** An article was invoiced at $33.50; freight charges, $1.50. If the overhead charges amounted to 15 % and the gain to 10 %, what was the selling price?

SOLUTION. 15% + 10% = 25%.
100% − 25% = 75%.
$33.50 + $1.50 = $35, the cost.
$35 ÷ .75 = $46.67, the selling price.
PROOF. 25% of $46.67 = $11.67, overhead charges and gain.
$46.67 − $11.67 = $35, the cost.

**2.** A merchant sold goods amounting to $22,500. If the overhead charges amounted to 18 % and the profits to 8 %, what was the invoice price of the goods if the freight amounted to $350?

**3.** A merchant marked a lot of goods $33\frac{1}{3}$ % above cost, but as he was unable to sell them at the marked price, he decided to reduce the marking to cost. What per cent reduction must be made?

**4.** A machine was invoiced at $53.50; freight charges, $3.50. If the overhead charges of the business amounted to 20 %, and the gain to 10 %, what must be the selling price of the goods?

**5.** An invoice of merchandise amounted to $1204.50; freight charges, $10.50. If the overhead charges amounted to $17\frac{1}{2}$ % and the gain to $7\frac{1}{2}$ %, what must be the selling price?

**6.** A merchant marked a lot of goods at 25 % above cost, but as the goods did not sell at the marked price, he reduced it 25 %, and announced that he was selling at cost. What per cent represents the amount of his error? If the goods thus marked cost $1760.48, what did the merchant lose by his blunder?

## CHAPTER XVI

### MARKING GOODS

**209.** Merchants frequently use some private mark to denote the cost and the selling price of goods. The word, phrase, or series of arbitrary characters employed for private marks is called a **key**.

Many houses use two different keys in marking goods, one to represent the cost and the other the selling price. In this way the cost of an article may not be known to the salesmen, and the selling price may not be known to any except those in some way connected with the business. In large houses, when but one key is used, only the selling price is indicated on the article, it being deemed best to keep the actual cost of the article a secret with the buyers. In small houses, when but one key is used, both the cost and the selling price are frequently written on the article.

**210.** If letters are used to mark goods, any word or phrase containing *ten different letters* may be selected for a key. If arbitrary characters are used, any *ten different characters* may be selected for a key.

Some methods of marking are so complicated that it is necessary to always have a key of the system at hand for reference. Goods are so marked in order that important facts, such as the cost of goods, may be kept strictly private.

**211.** When a figure is repeated one or more times, one or two extra letters called **repeaters** are used to make the key word more secure as a private mark.

**212.** The following illustrates the method of marking goods by letters:

COST KEY

R E P U B L I C A N
1 2 3 4 5 6 7 8 9 0
Repeaters: S and Z

SELLING-PRICE KEY

P E R T H A M B O Y
1 2 3 4 5 6 7 8 9 0
Repeaters: W and D

The cost is generally written above and the selling price below a horizontal line on a tag, or on a paster or box. Gloves No. 271, costing $5 a dozen and selling for $6.25 a dozen, might be marked as shown in the margin. Fractions may be designated by additional letters or characters. Thus, W may be made to represent ½, K ¼, etc. in the above key. In marking goods for the retail trade, *all fractions* of a cent are called another whole cent.

### WRITTEN EXERCISE

**213.** *Using the keys given in § 212, write the cost and the selling price in each of the following problems:*

| | First Cost of Article | Freight | Gain | Loss |
|---|---|---|---|---|
| **1.** | $2.50 | 10% | 20% | |
| **2.** | $1.00 | 10% | 20% | |
| **3.** | .50 | | 33⅓% | |
| **4.** | $4.80 | 20% | | 25% |

| | First Cost of Article | Freight | Gain | Loss |
|---|---|---|---|---|
| **5.** | $16.00 | 2½% | 37½% | |
| **6.** | $40.00 | 5% | 16⅔% | |
| **7.** | $ 3.60 | 2½% | | |
| **8.** | $24.00 | | | 10% |

**214.** *Using the following key, write the cost and the selling price in each of the following problems:*

Cost Key

| ┌ | └ | ┐ | ┘ | ├ | ┤ | ⊏ | ⊐ | ʃ | + |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |

Repeaters: □ ◺

Selling-price Key

| T | ⊥ | ⊔ | ⊓ | E | Ǝ | ᗰ | Ш | I | # |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |

Repeaters: × —

| | First Cost of Article | Charges | Gain | Loss |
|---|---|---|---|---|
| **9.** | $10.00 | 5% | 20% | |
| **10.** | $20.00 | 10% | 50% | |
| **11.** | $30.00 | 6⅔% | | 25% |

| | First Cost of Article | Charges | Gain | Loss |
|---|---|---|---|---|
| **12.** | $15.00 | 6⅔% | 25% | |
| **13.** | $18.00 | 10% | 25% | |
| **14.** | $12.00 | 5% | 33⅓% | |

**215.** Wholesalers and jobbers buy and sell a great many articles by the dozen. Retailers buy a great many articles by the dozen, but generally sell them by the piece. In marking goods, therefore, it is highly important that the student be able to divide by 12 with great rapidity.

To divide by 12 with rapidity, the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, should be *memorized.*

TABLE OF TWELFTHS

| TWELFTHS | SIMPLEST FORM | DECIMAL VALUE | TWELFTHS | SIMPLEST FORM | DECIMAL VALUE |
|---|---|---|---|---|---|
| $\frac{1}{12}$ | | $\$.08\frac{1}{3}$ | $\frac{7}{12}$ | | $\$.58\frac{1}{3}$ |
| $\frac{2}{12}$ | $\frac{1}{6}$ | $.16\frac{2}{3}$ | $\frac{8}{12}$ | $\frac{2}{3}$ | $.66\frac{2}{3}$ |
| $\frac{3}{12}$ | $\frac{1}{4}$ | .25 | $\frac{9}{12}$ | $\frac{3}{4}$ | .75 |
| $\frac{4}{12}$ | $\frac{1}{3}$ | $.33\frac{1}{3}$ | $\frac{10}{12}$ | $\frac{5}{6}$ | $.83\frac{1}{3}$ |
| $\frac{5}{12}$ | | $.41\frac{2}{3}$ | $\frac{11}{12}$ | | $.91\frac{2}{3}$ |
| $\frac{6}{12}$ | $\frac{1}{2}$ | .50 | $\frac{12}{12}$ | 1 | 1.00 |

**216. Example.** What is the cost of one shirt when a dozen shirts cost $19?

SOLUTION. Divide by 12 the same as by any number of one digit and mentally reduce the twelfths in the remainder to their decimal equivalent. Thus, say or think $1\frac{7}{12}$, $\$1.58\frac{1}{3}$, practically $1.58.

### ORAL EXERCISE

*State the cost per article when the cost per dozen articles is:*

1. $25.00.
2. $37.00.
3. $42.00.
4. $64.00.
5. $80.00.
6. $13.00.
7. $7.00.
8. $3.60.
9. $2.40.
10. $5.60.
11. $3.40.
12. $13.20.
13. $23.20.
14. $19.20.
15. $66.60.
16. $38.00.
17. $17.00.
18. $11.00.
19. $9.00.
20. $7.00.
21. $5.00.
22. $7.50.
23. $8.40.
24. $17.50.

### ORAL EXERCISE

1. Hats costing $48 a dozen must be sold for what price each to gain 25%?

2. Rulers bought at $2 a dozen must be retailed at how much each to gain 50%?

3. Note books costing $1.60 per dozen must be retailed at what price each to gain $12\frac{1}{2}\%$?

4. Erasers bought at $3.24 per gross must be retailed at how much each to gain $111\frac{1}{9}\%$?

5. Matches costing $3.60 per gross boxes must be retailed at what price per box to gain 100%?

**6.** Envelopes bought at \$2 per M must be sold at what price per package of 25 to gain 100%?

**7.** Pickles bought at \$1.80 per dozen bottles must be sold at what price per bottle to gain $33\frac{1}{3}$%?

**8.** Mustard costing \$14.40 per gross packages must be retailed at what price per package to gain 20%? to gain 50%?

## LISTING GOODS FOR CATALOGUES

**217.** In listing goods for catalogues dealers generally mark them so that they may allow a discount on the goods and still realize a profit.

**218. Example.** What should be the catalogue price of an article costing \$24 in order to insure a gain of 25% and allow the purchaser a discount of 20%?

SOLUTION. $\frac{1}{4}$ of \$24 = \$6, the gain.
\$30 = the selling price, which is 20% below the catalogue price.
.80 of the catalogue price = \$30.
∴ the catalogue price = \$30 ÷ .80 = \$37.50.

### WRITTEN EXERCISE

**1.** At what price must you mark an article costing \$400 to gain 25% and provide for a 20% loss through bad debts?

**2.** What should be the catalogue price of a library table costing \$25 in order to insure a gain of 20% and allow the purchaser a discount of 25%?

**3.** You list tea costing 30¢ a pound in such a way that you gain $33\frac{1}{3}$% after allowing the purchaser a trade discount of 20%. What is your list price?

**4.** You buy broadcloth at \$3.80 per yard. At what price must you mark it in order that you may allow your cash customers 5% discount and still realize a gain of 20%?

**5.** Having bought a quantity of oranges for \$3.00 per C you mark them so as to gain $33\frac{1}{3}$% and allow for a 20% loss through bad debts. What will be your asking price per dozen?

**6.** At what price must the articles in the following invoice be listed to gain 20 % and allow discounts of 25 % and 20 % ?

*Boston, Mass.,* Nov. 24, *19*

*M*r. Edgar C. Townsend
Rochester, N.Y.

*Bought of WELLS, FOWLER & CO.*

*Terms* Net 30 da.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| #721 | 50 | Oak Bookcases | $8.00 | 400 | | | |
| #924 | 25 | Gentlemen's Chiffoniers | 12.00 | 300 | | | |
| | | | | 700 | | | |
| | | Less 10% | | 70 | | 630 | |

### WRITTEN REVIEW EXERCISE

**1.** Using the word *importance*, with repeaters *s* and *w*, for the buying key, and the words *buy for cash*, with repeaters *t* and *m*, for the selling key, write the cost and selling price of the articles in the following bill. It is desired to gain 25 % on the pens and pencils, 20 % on the cards, and to provide for a loss of 12½ % through bad debts.

**Boston, Mass.,** Oct. 18, **19**

**M**essrs. WHITE & WYCKOFF
Holyoke, Mass.

**Bought of C. E. Stevens & Co.**

**Terms** Net 30 da.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 100 | gro. Pens | $0.80 | 80 | | | |
| | 25 | " Lead Pencils | 3.20 | 80 | | | |
| | 50 | pkg. Record Cards | .40 | 20 | | | |
| | | | | 180 | | | |
| | | Less 12 1/2% | | 22 | 50 | 157 | 50 |

# CHAPTER XVII

## PROPERTY INSURANCE

## FIRE INSURANCE

### ORAL EXERCISE

1. One hundred persons have property valued at $500,000. They pay into a common fund 60¢ per $100 of this sum. What is the amount of the fund?

2. These one hundred persons live in widely separated parts of the country. Is it likely that many of them will suffer losses by fire in the same year?

3. Suppose the losses to this property by fire for a year amount to $2500. What portion of the common fund will remain on hand as a surplus? (No interest.)

4. If this surplus is divided among the hundred persons at the end of the year, how much should A, who paid in $30, receive?

5. What are the companies organized to receive and distribute the fund in problem 1 called?

**219.** **Insurance** is a contract whereby for a stipulated consideration one party agrees to indemnify another for the loss or damage on a specified subject by specified perils, according to certain prescribed terms and conditions.

The best-known forms of property insurance are *fire* insurance and *marine* insurance.

There are also property-insurance companies which insure against loss due to steam-boiler explosions, failure of crops, death of live stock, burglary, injury to business by strikes among employees, and numerous other hazards.

**220.** **Fire insurance** is insurance against loss of property or damage to it by fire.

A contract of fire insurance frequently covers loss by lightning or tornado. It also covers damage resulting from or consequent on a fire, such as the loss resulting from water applied for the purpose of extinguishing flames, also, for the loss when such destruction has been ordered by the proper authorities.

**221.** The **insurer,** also called the **underwriter,** is the one who agrees to indemnify. The **insured** is the one to whom the promise of indemnity is made. The **premium** is the consideration agreed upon to be paid by the insured. The **policy** is the written contract between the insurer and the insured.

**222.** **Fire insurance** is usually conducted under the joint stock or the mutual plan.

In a *joint stock company* capital is subscribed, paid for, and owned by persons called stockholders, who share in the gains and are liable, to the extent of their subscriptions, for all the losses.

A *mutual insurance company* is one in which all the policy holders share the gains and bear the losses in proportion to the amount of the premiums they pay to that particular company, and their fire funds consist of the reserve earnings and the results of investments.

**223.** **Policies of insurance** are of various kinds. The *ordinary* policy is a contract of indemnity, that is, a contract in which the amount paid in case of loss does not exceed a certain specified sum; this sum is determined by evidence after the loss occurs. A *valued* policy is one that states in advance the amount to be paid in case of loss.

Further subdivisions of policies are as follows: *specific*, one that covers a particular kind of property, as a single building; *blanket*, one that covers several items of property, as a group of buildings and the contents; *fixed*, one that covers property at some particular defined location; *floating*, one that covers specified property while in transit or in various defined localities; *open*, one which, while it affixes the extreme limit of the amount and duration of the risk, is yet open to secure endorsements granting insurance in various amounts and places at any time and for any period that may be agreed upon at the time of the endorsement; this policy is used largely to protect such stocks as grain in elevators or as the contents of warehouses, and the records are usually kept in a book known as an *open policy book*.

**224.** The **standard forms of contract** used in fire insurance policies are prescribed by the state.

These forms not only define the maximum amount and the term for which the company is liable but also the consideration paid by the insured, the conditions under which the contract will become void, the methods to be followed in the settlement of a loss, and the procedure to effect the cancellation of the contract.

If a loss either total or partial occurs under such a policy, the company is bound to pay only so much of the sum stated in the policy as will indemnify the insured; *e.g.* if a building insured for $3000 is damaged by fire $400, only the actual loss, $400, can be recovered; but if the same building were damaged by fire $3500, the company could not be held for more than $3000, the sum stated in the policy.

**225. Average and co-insurance clauses.** Where a number of detached properties are insured under one policy, it is customary to attach what is known as an **average clause** which specifies that the amount of insurance covering any one particular piece of property shall bear such proportion to the total amount of insurance on the whole as the value of that special piece of property bears to the value of all of the properties so covered.

**226.** Many fire-insurance policies contain what is known as a **co-insurance,** or a **reduced-rate, clause.** Under this clause the insured party agrees to keep his property insured for a certain percentage of its value; failing to do this, the company or companies insuring him are liable only for that proportion of a loss which the amount they insure bears to the specified percentage of the sound value of the property covered.

Thus, the value of a piece of property is $10,000, and the insured agrees to keep it insured for 80% of its value, or $8000, but fails to do so and carries only $6000 insurance. Should a loss occur, the company will pay only three fourths ($\frac{6000}{8000}$) of the amount of such loss.

**227.** The **rate** in fire insurance is the amount to be paid to secure $100 of indemnity for one year.

The rate is based on the character of the risk; the greater the likelihood of fire the higher the rate.

When policies are written for a period of more than one year, a reduction is usually made in figuring the premium. Illustrations: on city dwellings the premium for five years is charged for four times the annual rate; if written for three years, for two and one-half times the annual rate.

Rates are expressed by the number of cents charged for $100 of insurance. When over $1 per hundred, the rate is often stated in dollars and cents.

*Short* rates are those used for a term of less than one year; they are proportionately higher than the annual rates.

### ORAL EXERCISE

1. What is the cost of \$6500 insurance at 80¢ per \$100?

2. What is the premium on a \$4000 policy at \$1.50 per \$100?

3. What is the cost of \$6000 insurance at 75¢ per \$100?

4. B insures a \$6000 barn for $\frac{2}{3}$ value at 50¢ per \$100. What quarterly premium should he pay?

5. A insures a \$6000 house for $\frac{4}{5}$ value, at 50¢ per \$100. What is the semiannual premium?

6. Goods worth \$3000 are insured for $\frac{2}{3}$ value. If the annual premium is \$30, what is the rate?

7. I insure \$2400 worth of merchandise for $\frac{5}{6}$ of its value at 60¢ per \$100. What premium must I pay?

8. I insure a stock of goods worth \$8000 for \$6000 at 2%. The goods become damaged by fire to the extent of \$3000. Under an ordinary policy how much can I recover? What will be my net loss, premium included?

9. A brick schoolhouse is insured at 50¢ per \$100, the annual premium is \$50, and the face of the policy $\frac{5}{8}$ of the value of the building. What is the value of the building?

### ORAL EXERCISE

*State the premium in each of the following problems:*

| | Face of Policy | Rate | | Face of Policy | Rate |
|---|---|---|---|---|---|
| 1. | \$1600 | $1\frac{1}{2}$% | 3. | \$3500 | \$1.10 per \$100 |
| 2. | \$1000 | $1\frac{1}{4}$% | 4. | \$5000 | \$1.20 per \$100 |

*State the face of the policy in each of the following problems:*

| | Premium | Rate | | Premium | Rate |
|---|---|---|---|---|---|
| 5. | \$9 | 2% | 7. | \$13.50 | \$1.35 per \$100 |
| 6. | \$15 | $1\frac{1}{2}$% | 8. | \$24.00 | \$1.60 per \$100 |

*State the rate of insurance in each of the following problems:*

| | Face of Policy | Premium | | Face of Policy | Premium |
|---|---|---|---|---|---|
| 9. | \$1700 | \$25.50 | 11. | \$3200 | \$130.00 |
| 10. | \$1850 | \$37.00 | 12. | \$6500 | \$40.00 |

**228.** The following is an extract from a tariff, or rate, book for the properties shown on the map which follows this schedule.

### MAIN STREET, SOUTH SIDE

| No. | | Flat Rate | 80% Rate |
|---|---|---|---|
| 189 | John Smith & Co. | | |
| | Frame carriage factory | $1.75 *c* | $1.23 *c* |
| | Contents | 1.75 *c* | 1.23 *c* |
| 193 | John Smith | | |
| | Frame dwelling | $0.25 *a* | —— |
| 197 | Frame stable (private) | 1.00 *a* | —— |
| 199 | William Brown | | |
| | Frame store and dwelling | $0.40 *c* | $0.28 *c* |
| | Contents of grocery store | 0.40 *c* | 0.28 *c* |
| | Contents of dwelling | 0.40 *a* | 0.28 *c* |
| 203–205 | James Robinson | | |
| | Brick mercantile building | $0.70 *a* | $0.50 *a* |
| | Robinson & Co. | | |
| | Department store | $0.70 *c* | $0.50 *c* |
| | Offices second and third floors | 0.70 *c* | 0.50 *c* |

### STATE STREET, NORTH SIDE

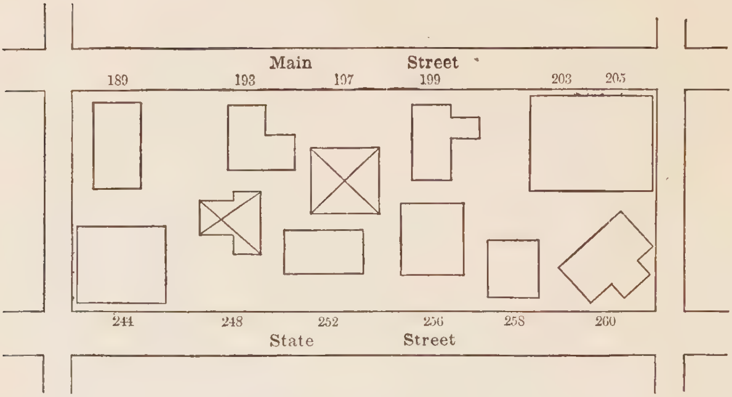
| | | | |
|---|---|---|---|
| 244 | James Green | | |
| | Brick store and dwelling | $0.25 *a* | $0.17½ *a* |
| | National Butter Co. | 0.25 *c* | 0.17½ *c* |
| | Dwelling | 0.25 *c* | —— |
| 248 | Thomas White | | |
| | Frame stable | $1.00 *c* | $0.70 *c* |
| | *White's Livery* | 1.00 *c* | 0.70 *c* |
| 252 | Thomas White | | |
| | Frame dwelling and contents | $0.25 *a* | $0.17½ *a* |
| 256 | Town of Jonesville | | |
| | Brick high school | $0.50 *a* | $0.35 *a* |
| | Contents | 0.50 *a* | 0.35 *a* |
| 258 | Samuel Parker | | |
| | Brick dwelling | $0.17 *a* | $0.12 *a* |
| 260 | State Street Baptist Society | | |
| | Brick church building | $0.50 *a* | $0.35 *a* |
| | Organ and other contents | 0.50 *a* | 0.35 *a* |

The letter *a* after the rate indicates that the insurance on this property can be written for more than one year; that is, at two and one-half times the rate, for a three-year policy, and at four times the rate, for a five-year policy.

The letter *c* after the rate indicates that the insurance on this property can be written for one year, or for a number of years, at yearly rates.

The city block, page 214, contains properties insured under the above schedule of rates.

Diagram of a City Block



### WRITTEN EXERCISE

These problems apply to the properties shown on the above diagram; also to the tariff of rates in the preceding schedule.

The flat rate is used unless the co-insurance clause is mentioned.

1. The frame carriage factory at 189 Main Street is worth \$7000. The contents are worth \$8000; both are insured at $\frac{4}{5}$ of their value. What is the amount of the annual premium?

2. The frame dwelling at 193 Main Street is worth \$3400, and the contents, \$1200. The frame stable owned by the same party at 197 Main Street is worth \$1500, and the contents, \$1100. All of this property is insured for 1 yr. at a $\frac{5}{6}$ valuation. What is the annual premium? What will it cost to insure it for 3 yr.?

3. The store and dwelling at 199 Main Street are worth \$4800. The contents of the store are worth \$2400, and of the dwelling, \$800. What will it cost to insure the property for 1 yr.?

4. The brick mercantile building at 203–205 Main Street is worth \$20,000. The contents of the first floor are worth \$4500, and of the second and third floors, \$7500. All are insured at a 75% valuation for 1 yr. What is the amount of the premium? A fire occurs, and the building and the contents are damaged to the extent of \$4500. If the policies contained an 80% co-insurance clause, how much will the insuring company have to pay?

**5.** Suppose that the building described in problem 4 was insured in Company A for $18,000 at the tariff rate, and the contents in Company B for $10,000 at a rate of 75 ¢; that each company had an 80 % co-insurance clause attached to its policy; that the building was damaged to the extent of $3000, and the contents, $2500. How much would each company have to pay? What would be the net loss to the owner of the building? to the owner of the contents? (Premium included in each case, but no interest.)

**6.** The brick church at 260 State Street is worth $10,000, and the contents, $3500. The property is insured for 1 yr. for $8100. If the policy contains an 80 % co-insurance clause, what is the net loss to the insurance company (premium included) if the property is wholly destroyed by fire?

**7.** If the brick school building at 256 State Street is worth $15,000 and the contents are worth $7500, what will it cost under the term rule to insure it for 5 yr. for 80 % of its value?

**8.** For insuring the frame buildings at 252 and 248 State Street, and the contents of each for $\frac{3}{4}$ of their value, the owner pays an annual premium of $22.50. If the frame stable and the contents are worth $\frac{2}{7}$ of the frame dwelling and the contents, what is the value of each building, including the contents?

**9.** The brick store and the dwelling at 244 State Street are worth $15,000; the property is insured in three companies for $\frac{4}{5}$ of its value. Company A carries $\frac{1}{3}$ of the line at the tariff rate; Company B, $\frac{2}{5}$ of the line at a 50 ¢ rate; Company C, the remainder of the line at a $66\frac{2}{3}$ ¢ rate. What is the total premium paid? The building is damaged by fire to the amount of $6000. What amount will each company pay?

**10.** I insured a block of buildings in the Ætna Insurance Company for $75,000 at an annual rate of 75 ¢. The Ætna afterwards reinsured $15,000 of its liability under my policy in the Continental Insurance Company at 75 ¢, and $20,000 in the German American Insurance Company at the same rate. The building was damaged by fire $20,000. What was the net loss of each of the three companies?

# INTEREST AND BANKING

## CHAPTER XVIII

### INTEREST

#### ORAL EXERCISE

**1.** A borrows $100 of B for 1 yr. At the end of the year what will A probably pay B besides the face of the loan?

**2.** C puts $100 in a savings bank and leaves it for 1 yr. What can he draw out at the end of the year besides the money deposited?

**3.** If you wished to borrow money of a bank in your town, what rate of interest would you have to pay?

**4.** If you loaned a man $500 for 1 yr., what would you require him to give you as evidence of the loan and security for its payment?

**229.** The compensation paid for the use of money is called **interest.** Interest is computed at a certain per cent of the sum borrowed. This per cent of interest is called the **rate**, and the sum upon which it is computed, the **principal.**

The rate of interest allowed by law is called the **legal rate.** Persons may agree to pay less than this rate, but not more, unless a higher rate by special agreement is permitted by statute. When an obligation is interest-bearing and no rate is mentioned, the legal rate will be understood. An agreement for interest greater than that allowed by law is called **usury.**

**230.** In the **commercial world,** 12 mo. of 30 da. each, or 360 da., are reckoned as 1 yr.

This method is not exact, but it is the most common because the most convenient. It has been legalized by statute in some states and is generally used in all the states.

## SIMPLE INTEREST

### The Day Method

#### ORAL EXERCISE

**1.** How many days in a commercial year?

**2.** What part of a year is 60 da.? 6 da.? What is the interest on $1 for 1 yr. at 6%? for 60 da.? for 6 da.?

**3.** How do you find .01 of a number? .001 of a number? What is the interest on $120 for 60 da. at 6%? for 6 da.?

**4.** State a short method for finding the interest on any principal for 60 da. at 6%; for 6 da.

**5.** 1 da. is what part of 6 da.? What is $\frac{1}{6}$ of .001? What is the interest on $1200 for 1 da. at 6%? on $180? on $1500?

**6.** State a short method for finding the interest on any principal for 1 da. at 6%.

**231.** In the foregoing exercise it is clear that *0.001 of any principal is equal to the interest for 6 da. at 6%;* or *0.001 of any principal is equal to 6 times the interest for 1 da. at 6%.*

#### ORAL EXERCISE

**1.** Find the interest on each of the following for 6 da. at 6%.

*a.* $250.
*b.* $870.
*c.* $358.
*d.* $350.
*e.* $560.
*f.* $435.
*g.* $430.
*h.* $470.
*i.* $678.
*j.* $320.
*k.* $100.
*l.* $185.
*m.* $290.
*n.* $150.
*o.* $325.
*p.* $990.
*q.* $890.
*r.* $750.
*s.* $580.
*t.* $625.

**2.** Find the interest on each of the above amounts for 12 da. at 6%; for 18 da.; for 24 da.

**3.** Find the interest on each of the following for 1 da. at 6%.

*a.* $360.
*b.* $450.
*c.* $300.
*d.* $420.
*e.* $660.
*f.* $900.
*g.* $540.
*h.* $240.
*i.* $600.
*j.* $180.
*k.* $720.
*l.* $500.
*m.* $480.
*n.* $780.
*o.* $400.
*p.* $120.
*q.* $840.
*r.* $200.
*s.* $330.
*t.* $960.

**4.** Find the interest on each of the above amounts for 3 da. at 6%; for 2 da.

**232. Example.** Find the interest on $450 for 54 da. at 6 %.

Solution. Pointing off three places to the left gives $0.45, or 6 times the interest for 1 da. Multiplying this result by 54 gives $24.30, or 6 times the interest for 54 da. Dividing this result by 6 gives $4.05, the required interest.

$$54 \times \$0.45 = \$24.30$$
$$\$24.30 \div 6 = \$4.05$$

By arranging the numbers as shown in the margin and canceling the work is greatly shortened.

$$\frac{\overset{9}{\cancel{54}} \times \$0.45}{\cancel{6}} = \$4.05$$

### WRITTEN EXERCISE

*At 6 % find the interest on each of the following problems. Reduce the time expressed in months and days to days.*

| | Principal | Time | | Principal | Time | | Principal | Time |
|---|---|---|---|---|---|---|---|---|
| **1.** | $620 | 54 da. | **7.** | $900.00 | 29 da. | **13.** | $375.80 | 2 mo. 15 da. |
| **2.** | $175 | 84 da. | **8.** | $865.45 | 93 da. | **14.** | $300.00 | 3 mo. 19 da. |
| **3.** | $645 | 42 da. | **9.** | $700.00 | 96 da. | **15.** | $171.15 | 1 mo. 14 da. |
| **4.** | $300 | 84 da. | **10.** | $974.30 | 62 da. | **16.** | $120.00 | 4 mo. 14 da. |
| **5.** | $600 | 72 da. | **11.** | $178.45 | 40 da. | **17.** | $211.16 | 6 mo. 16 da. |
| **6.** | $502 | 66 da. | **12.** | $438.55 | 50 da. | **18.** | $665.65 | 1 mo. 10 da. |

### ORAL EXERCISE

**1.** What is the interest on $800 for 6 da. at 3 % ?

Solution. 80¢ is the interest for 6 da. at 6 %. 3 % is $\frac{1}{2}$ of 6 %; therefore, $\frac{1}{2}$ of 80¢, or 40¢, is the interest for 6 da. at 3 %.

**2.** If the interest at 6 % is $45, what is the interest for the same time at 3 % ? at 12 % ? at 2 % ? at 1 % ? at $1\frac{1}{2}$ % ?

**3.** Formulate a short method for changing 6 % interest to 8 % interest.

Solution. 8 % is $\frac{1}{3}$ more than 6 %; hence, *the interest at 6 % increased by $\frac{1}{3}$ of itself equals the interest at 8 %.*

**4.** State a short method for changing 6 % interest to 7 % interest; to 5 % ; to 9 % ; to $7\frac{1}{2}$ % ; to $4\frac{1}{2}$ %.

**5.** If the interest at 6 % is $120, what is the interest at 7 % ? at 5 % ? at 8 % ? at 4 % ? at $7\frac{1}{2}$ % ? at $4\frac{1}{2}$ % ?

**233.** In the foregoing exercise it is clear that *6 % interest increased by $\frac{1}{2}$ of itself equals 9 % interest; by $\frac{1}{3}$ of itself, 8 % interest; by $\frac{1}{4}$ of itself, $7\frac{1}{2}$ % interest; by $\frac{1}{6}$ of itself, 7 % interest;* also that

*6 % interest decreased by $\frac{1}{3}$ of itself equals 4 % interest; by $\frac{1}{4}$ of itself, $4\frac{1}{2}$ % interest; by $\frac{1}{6}$ of itself, 5 % interest*; also that

*6 % interest divided by 2 equals 3 % interest; by 3, 2 % interest; by 6, 1 % interest; by 4, $1\frac{1}{2}$ % interest.*

6 % interest multiplied by 2 equals 12 % interest.

6 % interest is changed to 10 % interest by dividing by 6 and removing the decimal point one place to the right; to any other rate by dividing by 6 and multiplying by the given rate.

### WRITTEN EXERCISE

*Using the exact number of days, find the interest on:*

**1.** $2500 from Sept. 18, 1915, to Feb. 6, 1916, at 9 %; at $3\frac{1}{2}$ %; at 4 %; at 3 %.

**2.** $1700 from Nov. 20, 1915, to Jan. 16, 1916, at 8 %; at $2\frac{1}{2}$ %; at $5\frac{1}{2}$ %; at $3\frac{1}{2}$ %; at 4 %.

**3.** $2750 from Dec. 16, 1915, to Jan. 17, 1916, at 7 %; at 2 %; at 4 %; at 5 %; at 1 %; at 10 %.

**4.** $6250 from Dec. 18, 1915, to Feb. 6, 1916, at $7\frac{1}{2}$ %; at 10 %; at $1\frac{1}{2}$ %; at $4\frac{1}{2}$ %; at 9 %; at 8 %; at 7 %; at 3 %.

## THE BANKER'S SIXTY-DAY METHOD

### ORAL EXERCISE

**1.** 60 da. (2 mo.) is what part of a commercial year?

**2.** What is the interest on $1 for 2 mo. at 6 %? for 60 da.?

**3.** How can you find 0.01 of a number? What is the interest on $50 for 60 da. at 6 %? on $370? on $590? on $214.55?

**4.** What fractional part of 60 da. is 30 da.? 20 da.? 15 da.? 10 da.? What is the interest on $1680 for 60 da.? for 30 da.? for 20 da.? for 15 da.? for 10 da.?

**5.** State a simple way to find the interest on any principal for 60 da. at 6 %; for 30 da.; for 20 da.; for 15 da.; for 10 da.

**6.** Read aloud the following, supplying the missing words:

*a.* 60 da. minus $\frac{1}{12}$ of itself equals 55 da.; 60 da. minus —— of itself equals 50 da.; 60 da. minus —— of itself equals 40 da.; 60 da. minus —— of itself equals 45 da.

*b.* 60 da. plus $\frac{1}{12}$ of itself equals 65 da.; 60 da. plus —— of itself equals 70 da.; 60 da. plus —— of itself equals 75 da.; 60 da. plus —— of itself equals 80 da.; 60 da. plus —— of itself equals 90 da.

**7.** What is the interest on $600 for 60 da. at 6%? for 55 da.? for 50 da.? for 40 da.? for 45 da.?

**8.** What is the interest on $1200 for 60 da.? for 65 da.? for 70 da.? for 75 da.? for 80 da.? for 90 da.?

**9.** State a short way to find the interest at 6% for 80 da.; for 90 da.; for 50 da.; for 65 da.; for 55 da.; for 75 da.; for 70 da.; for 40 da.; for 45 da.

**234.** In the above exercise it is clear that *removing the decimal point two places to the left in the principal gives the interest for 60 da. at 6%.*

**235. Examples. 1.** Find the interest on $1950 for 20 da. at 6%.

Solution. Removing the decimal point two places to the left gives the interest for 60 da. 20 da. is $\frac{1}{3}$ of 60 da. $\frac{1}{3}$ of $19.50 = $6.50.

$$\begin{array}{r} \$19.50 \\ \hline \$6.50 \end{array}$$

**2.** What is the interest on $8400.68 for 75 days?

Solution. Removing the decimal point two places to the left gives the interest for 60 da. 75 da. is 60 da. increased by $\frac{1}{4}$ of itself; therefore, $84.0068 increased by $\frac{1}{4}$ of itself or $105.01 is the required interest. In the following exercise determine the separate interest mentally whenever it is possible to do so.

$$\begin{array}{r} \$84.0068 \\ 21.0017 \\ \hline \$105.0085, \text{ or } \$105.01 \end{array}$$

WRITTEN EXERCISE

**1.** Find the total amount of interest at 6% on:

$8400 for 60 da.
$8400 for 30 da.
$8400 for 20 da.
$8400 for 15 da.
$8400 for 12 da.
$8400 for 10 da.
$7900 for 60 da.
$7900 for 30 da.
$7900 for 20 da.
$7900 for 15 da.
$7900 for 12 da.
$7900 for 10 da.

**2.** Find the total amount of interest at 6% on:

$1600 for 60 da.
$1600 for 55 da.
$1600 for 50 da.
$1600 for 45 da.
$1600 for 40 da.
$2800 for 60 da.
$2800 for 65 da.
$2800 for 70 da.
$2800 for 75 da.
$2800 for 80 da.
$2800 for 90 da.
$7200 for 55 da.

**3.** Find the total amount of interest at 6% on:

$1500.60 for 30 da.
$1800.72 for 20 da.
$1200.64 for 15 da.
$8400.60 for 10 da.
$832.60 for 90 da.
$720.18 for 10 da.
$440.70 for 40 da.
$479.64 for 50 da.
$8575.65 for 70 da.
$6282.40 for 15 da.
$1460.84 for 65 da.
$1385.62 for 55 da.

**4.** Find the total amount of interest at 6% on:

$1800.40 for 90 da.
$9200.50 for 80 da.
$3240.64 for 70 da.
$4125.18 for 45 da.
$7500.00 for 55 da.
$8200.00 for 75 da.
$6400.00 for 45 da.
$1200.45 for 30 da.
$216.90 for 20 da.
$432.65 for 15 da.
$832.30 for 10 da.
$926.17 for 20 da.

### ORAL EXERCISE

**1.** What is the interest on $215 for 6 da. at 6%? on $345? on $415? on $827.50? on $425.90? on $4520.60? State a simple way to find the interest on any principal for 6 da. at 6%.

**2.** What part of 6 da. is 3 da.? is 2 da.? is 1 da.? What is the interest on $720 for 6 da.? for 3 da.? for 2 da.? for 1 da.? State a brief method of finding the interest on any principal for 3 da. at 6%; for 2 da.; for 1 da.

**3.** Read aloud the following, supplying the missing words:

*a.* 6 da. minus $\frac{1}{6}$ of itself equals 5 da.; 6 da. minus —— of itself equals 4 da.

*b.* 6 da. plus $\frac{1}{6}$ of itself equals 7 da.; 6 da. plus —— of itself equals 8 da.; 6 da. plus —— of itself equals 9 da.

*c.* State a short method of finding the interest at 6% for 4 da.; for 5 da.; for 7 da.; for 8 da.; for 9 da.

**236.** In the above exercise it is clear that *removing the decimal point in the principal three places to the left gives the interest for 6 da. at 6%.*

**237. Example.** What is the interest on $420 for 8 da. at 6%?

Solution. Removing the decimal point three places to the left gives the interest for 6 da., or $0.42. Since 8 da. is 6 da. plus $\frac{1}{3}$ of itself, $0.42 increased by $\frac{1}{3}$ of itself, or $0.56 is the required interest. In the following exercises determine the separate interests mentally whenever it is possible to do so.

$.420
.140
$.56

### WRITTEN EXERCISE

**1.** Find the total amount of interest at 6% on:

$800 for 6 da.
$800 for 3 da.
$800 for 2 da.
$800 for 1 da.
$720 for 6 da.
$720 for 7 da.
$720 for 8 da.
$720 for 9 da.
$1500 for 6 da.
$1500 for 5 da.
$1500 for 4 da.
$1500 for 9 da.

**2.** Find the total amount of interest at 6% on:

$1168 for 6 da.
$1168 for 3 da.
$1168 for 2 da.
$1168 for 1 da.
$1600 for 6 da.
$1600 for 7 da.
$1600 for 8 da.
$1600 for 9 da.
$2400 for 6 da.
$2400 for 5 da.
$2400 for 4 da.
$2400 for 8 da.

**3.** Find the total amount of interest at 6% on:

$640.50 for 8 da.
$920.10 for 20 da.
$280.40 for 15 da.
$390.60 for 50 da.
$800.10 for 7 da.
$240.80 for 90 da.
$960.70 for 70 da.
$845.60 for 90 da.
$213.80 for 50 da.
$310.40 for 40 da.
$135.90 for 10 da.
$736.18 for 10 da.

### ORAL EXERCISE

**1.** 600 da. is how many times 60 da.? If the interest on $1 for 60 da. at 6% is $0.01, what is the interest for 600 da.?

**2.** Give a rapid method for finding 0.1 of a number. What is the interest on $500 for 600 da. at 6%? on $350? on $214.60? on $359.80? on $4500? on $9243.80? on $750? on $2150?

**3.** What part of 600 da. is 300 da.? 200 da.? 150 da.? 75 da.? 120 da.? 100 da.? 50 da.?

**4.** What is the interest on $1400 for 600 da.? for 300 da.? for 200 da.? for 150 da.? for 75 da.? for 120 da.? for 100 da.? for 50 da.?

**5.** State a brief method of finding the interest for 600 da. at 6 % ; for 300 da. ; for 200 da. ; for 75 da. ; for 50 da. ; for 150 da. ; for 100 da.

**6.** If the interest on $1 for 600 da. is $0.10, what is the interest for 6000 da.? In how many days will any principal double itself at 6 % interest?

**7.** What is the interest on $1 for 6000 da. at 6 %? on $55? on $75.60? on $18.90? on $350? on $725? on $9125.70.

**8.** What is the interest on each of the amounts in problem 7 for 3000 da.? for 2000 da.? for 1000 da? for 1500 da.?

**9.** What is the interest on $2500 for 6000 da.? on $2150? on $7500? on $790? on $155.60?

**10.** What is the interest on each of the amounts in problem 9 for 6 da.? for 60 da.? for 600 da?

**238.** In the above exercise it is clear that *removing the decimal point in the principal one place to the left gives the interest for 600 da. at 6 %; also that any sum of money will double itself in 6000 da. at 6 %.*

### WRITTEN EXERCISE

*Find the interest at 6 % on:*

**1.** $240 for 3000 da.
**2.** $318 for 6000 da.
**3.** $912 for 2000 da.
**4.** $316 for 1500 da.
**5.** $7420.50 for 600 da.
**6.** $67218.90 for 30 da.
**7.** $8400.50 for 400 da.
**8.** $7500.79 for 1500 da.
**9.** $1640 for 150 da.
**10.** $1260.60 for 1 da.
**11.** $17890 for 10 da.
**12.** $1696 for 100 da.

### ORAL EXERCISE

**1.** How many times is 6 da. contained in 18 da.? in 24 da.? in 36 da.? in 42 da.? in 54 da.? in 48 da.?

**2.** What is the interest on $150 for 6 da.? for 18 da.? for 48 da.? for 54 da.? for 36 da.? for 42 da.? for 12 da.?

**3.** What is the interest on $350 for 60 da.? for 180 da.? for 240 da.? for 360 da.? for 420 da.? for 480 da.?

**239. Example.** Find the interest on $375 for 48 da. at 6 %.

Solution. 37½¢ equals the interest for 6 da. 48 da. is 8 times 6 da. Therefore, the interest for 48 da. is 8 times 37½¢, or $3.

$$\begin{array}{r} \$0.375 \\ \hline \$3.000 \end{array}$$

### WRITTEN EXERCISE

**1.** Find the total amount of interest at 6 % on:

$750 for 6 da.
$750 for 12 da.
$750 for 18 da.
$750 for 36 da.
$750 for 42 da.
$750 for 48 da.
$750 for 60 da.
$750 for 180 da.
$750 for 240 da.

**2.** Find the total amount of interest at 6 % on:

$725 for 18 da.
$824 for 36 da.
$729 for 42 da.
$850 for 54 da.
$690 for 6 da.
$129 for 60 da.
$475 for 600 da.
$8600 for 54 da.
$450 for 540 da.
$727 for 180 da.
$286 for 240 da.
$429 for 420 da.

**3.** Find the total amount of interest at 6 % on:

$317.40 for 240 da.
$218.60 for 180 da.
$419.80 for 420 da.
$425.60 for 120 da.
$217.18 for 18 da.
$420.50 for 24 da.
$240.70 for 540 da.
$290.60 for 180 da.
$360.40 for 24 da.
$860.50 for 48 da.
$900.60 for 66 da.
$400.80 for 84 da.

**240.** In some cases it is advisable to find the interest on the principal for 1 da. and then multiply by the number of days.

### ORAL EXERCISE

**1.** What is the interest on $600 for 17 da. at 6 %?

SOLUTION. The interest for one day is .000$\frac{1}{6}$ of the principal, or 10¢. The interest for 17 da. is 17 times 10¢, or $1.70.

**2.** What is the interest on $6000 for 49 da. at 6 %? on $300? on $240? on $3000? on $1800? on $840? on $600?

**3.** State the interest at 6 % on:

*a.* $600 for 19 da.
*b.* $300 for 37 da.
*c.* $240 for 43 da.
*d.* $180 for 27 da.
*e.* $6000 for 37 da.
*f.* $3000 for 43 da.
*g.* $2400 for 67 da.
*h.* $1800 for 89 da.
*i.* $ 900 for 17 da.
*j.* $1500 for 40 da.
*k.* $ 600 for 139 da.
*l.* $ 300 for 179 da.

**241.** Frequently it is well to mentally divide the days into convenient parts of 6 or 60.

Thus, 97 da. = 60 da. + 30 da. + 6 da. + 1 da.; 71 da. = 60 da. + 10 da. + 1 da.; 49 da. = 8 times 6 da. + 1 da.

### ORAL EXERCISE

*Separate the days in the following exercise into 6 da. or 60 da., or into convenient parts of 6 da. or 60 da.*

**1.** 8 da.
**2.** 67 da.
**3.** 27 da.
**4.** 13 da.
**5.** 72 da.
**6.** 43 da.
**7.** 7 da.
**8.** 22 da.
**9.** 11 da.
**10.** 63 da.
**11.** 37 da.
**12.** 23 da.
**13.** 86 da.
**14.** 55 da.
**15.** 84 da.
**16.** 14 da.
**17.** 97 da.
**18.** 99 da.
**19.** 17 da.
**20.** 25 da.
**21.** 85 da.
**22.** 89 da.
**23.** 19 da.
**24.** 29 da.

**242. Examples.** **1.** Find the interest on $840 for 31 da. at 6%.

Solution. 31 da. = 30 da. + 1 da. The interest for 60 da. is $8.40 and for 30 da. $\frac{1}{2}$ of this sum or $4.20. The interest for 6 da. is $0.84 and for 1 da. $\frac{1}{6}$ of this sum or $0.14. Adding $4.20 and $0.14 the result is the required interest, or $4.34.

$$\begin{array}{r} \$8.40 \\ \hline \$4.20 \\ .14 \\ \hline \$4.34 \end{array}$$

**2.** What is the interest on $2500 for 121 da. at 6%?

Solution. 121 da. = $\overline{2 \times 60 \text{ da.}}$ + 1 da. The interest for 60 da. is $25 and for 120 da. twice this sum, or $50. The interest for 6 da. is $2.50 and for 1 da. $\frac{1}{6}$ of this sum, or $0.42. Adding $50 and $0.42 the result is $50.42, the required interest.

$$\begin{array}{r} \$25.00 \\ \hline \$50.00 \\ .42 \\ \hline \$50.42 \end{array}$$

### WRITTEN EXERCISE

*Find the interest:*

| | Principal | Time | Rate | | Principal | Time | Rate |
|---|---|---|---|---|---|---|---|
| **1.** | $420 | 3 mo. | 6% | **11.** | $450 | 4 mo. | $4\frac{1}{2}$% |
| **2.** | $650 | 4 mo. | 5% | **12.** | $600 | 2 mo. | 5% |
| **3.** | $360 | 92 da. | 4% | **13.** | $720 | 8 mo. | 3% |
| **4.** | $250 | 30 da. | 3% | **14.** | $840 | 2 mo. | $1\frac{1}{2}$% |
| **5.** | $380 | 24 da. | 7% | **15.** | $120 | 7 mo. | 6% |
| **6.** | $900 | 55 da. | 6% | **16.** | $280 | 9 mo. | $3\frac{1}{2}$% |
| **7.** | $550 | 47 da. | 3% | **17.** | $885.90 | 20 da. | 3% |
| **8.** | $800 | 29 da. | 5% | **18.** | $240.00 | 21 da. | 6% |
| **9.** | $400 | 90 da. | 4% | **19.** | $420.18 | 25 da. | $2\frac{1}{2}$% |
| **10.** | $270 | 11 da. | 1% | **20.** | $560.17 | 27 da. | 6% |

**243.** It has been observed that 6 times \$800 = 800 times \$6; that 0.01 of \$715 = 715 times \$0.01; etc. Hence,

**244.** *The principal in dollars and the time in days may be interchanged without affecting the amount of interest.*

**245. Example.** Find the interest on \$600 for 179 da. at 6 %.

Solution. \$600 for 179 da. = \$179 for 600 da.; $\frac{1}{10}$ of the principal equals the interest for 600 da.; $\frac{1}{10}$ of \$179 = \$17.90, the required interest.

### ORAL EXERCISE

*State the interest at 6 % on:*

1. \$60 for 27 da.
2. \$30 for 13 da.
3. \$20 for 171 da.
4. \$10 for 186 da.
5. \$15 for 145 da.
6. \$12 for 179 da.
7. \$10 for 131 da.
8. \$100 for 120 da.
9. \$200 for 189 da.
10. \$150 for 192 da.
11. \$360 for 91 da.
12. \$420 for 87 da.
13. \$540 for 21 da.
14. \$660 for 37 da.
15. \$750 for 56 da.
16. \$3600 for 218 da.
17. \$2000 for 183 da.
18. \$1200 for 155 da.
19. \$1800 for 181 da.
20. \$2400 for 218 da.

**246.** \$1500 on interest for 24 da. at 8 % = \$2000 (\$1500 + $\frac{1}{3}$ of itself) on interest for 24 da. at 6 %, or \$1500 on interest for 32 da. (24 da. + $\frac{1}{3}$ of itself) at 6 %. Hence,

**247.** *If either the principal or the time is increased or decreased by any fraction of itself, the interest is increased or decreased by the same fraction.*

**248. Examples.** 1. Find the interest on \$480 for 279 da. at $7\frac{1}{2}$ %.

Solution. $7\frac{1}{2}$ % is $\frac{1}{4}$ more than 6 %. Increase the principal by $\frac{1}{4}$ of itself, and the result is \$600. Interchanging dollars and days, the problem is "Find the interest on \$279 for 600 da." Pointing off one place in the new principal, the result is \$27.90, the required interest.

2. Find the interest on \$2795.84 for 80 da. at $4\frac{1}{2}$%.

Solution. $4\frac{1}{2}$% is $\frac{1}{4}$ less than 6% interest. 80 da. decreased by $\frac{1}{4}$ of itself equals 60 da. The interest on \$2795.84 for 60 da. = \$27.96, the required result.

### ORAL EXERCISE

*State the interest on:*

**1.** \$279.86 for 45 da. at 4 %.
**2.** \$478.65 for 45 da. at 4 %.
**3.** \$769.64 for 48 da. at $7\frac{1}{2}$ %.
**4.** \$217.49 for 80 da. at $4\frac{1}{2}$ %.
**5.** \$767.53 for 80 da. at $4\frac{1}{2}$ %.
**6.** \$2400 for 39 da. at 5 %.
**7.** \$2700 for 37 da. at 4 %.
**8.** \$2400 for 87 da. at $4\frac{1}{2}$ %.
**9.** \$1600 for 95 da. at $4\frac{1}{2}$ %.
**10.** \$3200 for 59 da. at $4\frac{1}{2}$ %.

## THE SIX PER CENT METHOD

**249.** This method is best adapted to finding the interest when the time is *one year*, or *more than one year.*

### ORAL EXERCISE

**1.** If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 2 yr.? for 3 yr.? for 4 yr.? for 6 yr.? for 8 yr.? for 10 yr.?

**2.** If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 1 mo.? for 2 mo.? for 3 mo.? for 6 mo.? for 10 mo.? for 7 mo.? for 8 mo.?

**3.** What is the interest on \$1 for 1 yr. 6 mo. at 6 %? for 2 yr. 6 mo.? for 3 yr. 4 mo.? for 3 yr. 6 mo.? for 4 yr. 8 mo.? for 1 yr. 10 mo.? for 5 yr. 6 mo.? for 2 yr. 9 mo.?

**4.** What is the interest on \$50 for 1 yr. at 6 %? for 1 yr. 6 mo.? for 2 yr.? for 3 yr. 6 mo.? for 2 yr. 8 mo.? for 1 yr. 10 mo.? for 2 yr. 6 mo.? for 4 yr. 6 mo.? for 1 yr. 9 mo.?

**5.** If the interest on \$1 for 1 mo. at 6 % is \$0.005 (5 mills), what is the interest for 1 da.? for 2 da.? for 3 da.? for 4 da.? for 6 da.? for 12 da.? for 18 da.? for 28 da.? for 24 da.?

**6.** What is the interest on \$1 for 1 yr. 1 mo. 1 da. at 6 %? for 2 yr. 3 mo. 3 da.? for 1 yr. 10 mo. 6 da.? for 4 yr. 4 mo. 24 da.? for 1 yr. 5 mo. 12 da.? for 2 yr. 1 mo. 1 da.?

**250.** In the above exercise it is clear that:

\$0.06 = *interest on* \$1 *for* 1 *yr. at* 6 %.
\$0.005 = *interest on* \$1 *for* 1 *mo. at* 6 %.
\$$0.000\frac{1}{6}$ = *interest on* \$1 *for* 1 *da. at* 6 %.

### ORAL EXERCISE

*Find the interest on $1 at 6 % for:*

1. 1 yr. 4 mo. 12 da.
2. 1 yr. 8 mo. 18 da.
3. 1 yr. 7 mo. 24 da.
4. 1 yr. 9 mo. 27 da.
5. 2 yr. 6 mo. 6 da.
6. 3 yr. 4 mo. 9 da.
7. 5 yr. 3 mo. 3 da.
8. 4 yr. 8 mo. 4 da.

*Find the interest at 6 % on:*

9. $250 for 2 yr.
10. $400 for 5 yr.
11. $700 for 4 yr.
12. $300 for 3 yr. 4 mo.
13. $500 for 4 yr. 2 mo.
14. $350 for 3 yr.
15. $450 for 2 yr. 3 mo.
16. $150 for 1 yr. 6 mo.
17. $50 for 1 yr. 2 mo. 6 da.
18. $10 for 2 yr. 6 mo. 6 da.

**251. Example.** What is the interest on $600 for 2 yr. 8 mo. 15 da. at 6 % ?

$0.12 = int. on $1 for 2 yr.
.04 = int. on $1 for 8 mo.
.0025 = int. on $1 for 15 da.
$0.1625 = int. on $1 for the given time.
600 × $0.1625 = $97.50, int. on $600 for 2 yr. 8 mo. 15 da. at 6 %.

SOLUTION. Find the interest on $1 for 2 yr.; on $1 for 8 mo.; on $1 for 15 da. The sum of these interest items equals $0.1625, the interest on $1 for the given time at 6 %. Multiplying this interest by the given number of dollars, 600, the product is the required interest, $97.50.

Sometimes it is shorter to find the interest on $1 for the given time at any given rate, and multiply by the number of dollars in the principal. Thus to find the interest on $400 for 2 yr. 6 mo. at 8 %, take 400 times 20¢ (2½ × 8¢); on $500 for 5 yr. 3 mo. at 4 %, take 500 times 21¢ (5¼ × 8¢); on $600 for 1 yr. 9 mo. at 4 % take 600 times 7¢; etc.

### ORAL EXERCISE

*Find the interest:*

| | PRINCIPAL | TIME | RATE | | PRINCIPAL | TIME | RATE |
|---|---|---|---|---|---|---|---|
| 1. | $400 | 1 yr. 2 mo. | 6 % | 7. | $840 | 1 yr. 6 mo. | 6 % |
| 2. | $500 | 2 yr. 4 mo. | 6 % | 8. | $100 | 3 yr. 6 mo. | 5 % |
| 3. | $300 | 4 yr. 6 mo. | 6 % | 9. | $960 | 4 yr. 2 mo. | 6 % |
| 4. | $250 | 1 yr. 8 mo. | 6 % | 10. | $300 | 3 yr. 4 mo. | 3 % |
| 5. | $200 | 2 yr. 10 mo. | 3 % | 11. | $240 | 2 yr. 6 mo. | 4 % |
| 6. | $300 | 1 yr. 11 mo. | 6 % | 12. | $180 | 1 yr. 8 mo. | 6 % |

## WRITTEN EXERCISE

*Find the interest:*

1. Principal, \$74.90; time, 6 mo. 15 da.; rate, 4 %.
2. Principal, \$986.00; time, 11 mo. 28 da.; rate, $4\frac{1}{2}$ %.
3. Principal, \$1900.00; time, 10 mo. 28 da.; rate, 6 %.
4. Principal, \$87.55; time, 4 yr. 5 mo. 6 da.; rate, 6 %.
5. Principal, \$735.00; time, 4 yr. 4 mo. 4 da.; rate, 6 %.
6. Principal, \$609.50; time, 2 yr. 7 mo. 6 da.; rate, 6 %.
7. Principal, \$875.40; time, 1 yr. 2 mo. 21 da.; rate, $4\frac{1}{2}$ %.
8. Principal, \$1124.75; time, 1 yr. 5 mo. 14 da.; rate, 6 %.
9. Principal, \$1245.00; time, 3 yr. 6 mo. 23 da.; rate, 6 %.
10. Principal, \$1570.00; time, 1 yr. 9 mo. 25 da.; rate, 5 %.

# CHAPTER XIX

## BANK DISCOUNT

### ORAL EXERCISE

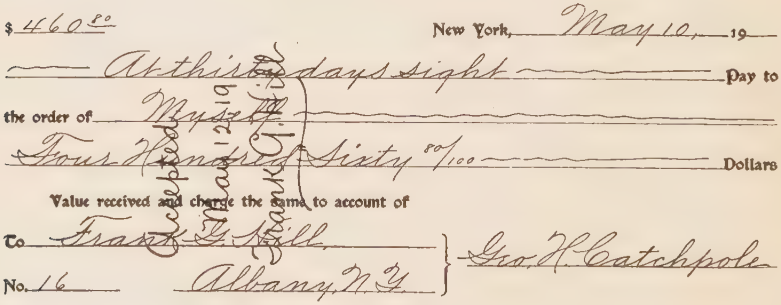
**1.** What is meant by a promissory note? by the face of a note? by the time? by the maker? by the payee?

**2.** How would you word a promissory note for $600, dated at your place to-day, payable in 60 da. at a bank in your place, with interest at 5%, to C. B. Powell, signed by yourself?

**3.** What is meant by negotiable? by indorsing a note? Illustrate a blank indorsement; an indorsement in full; a qualified indorsement.

**252.** A **commercial bank** is an institution chartered by law to receive and loan money, to facilitate the transmission of money and the collection of negotiable paper, and, in some cases, to furnish a circulating medium.

**253.** If the **holder** (owner) of a promissory note wishes to use the money promised before it becomes due, a commercial bank will usually buy the note, provided the holder can show that it will be paid at **maturity**, that is, when it becomes due. This is called **discounting** the note.



$460 80/100 New York, May 10, 19

At thirty days sight Pay to

the order of Myself

Four Hundred Sixty 80/100 Dollars

Value received and charge the same to account of

To Frank G. Hill } Geo. H. Catchpole

No. 16 Albany, N.Y. }

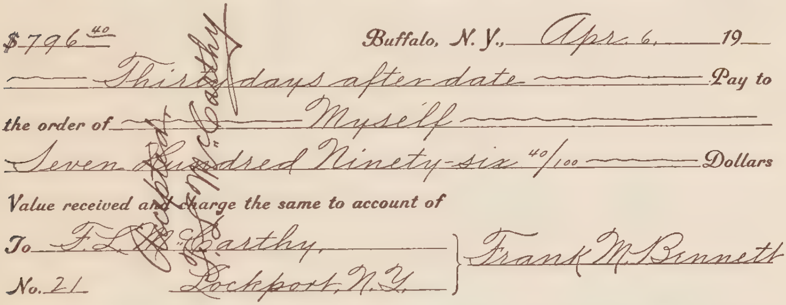
Accepted May 12, 19 Frank G. Hill

**254.** A **commercial draft** is now frequently used instead of the promissory note as security for the payment of goods sold on credit. Such a draft may be defined as a written order in which one person directs another to pay a specified sum of money to the order of himself or to a third person.

The circumstances under which the foregoing draft was drawn are as follows: Geo. H. Catchpole sold Frank G. Hill goods amounting to $460.80. Terms: 30-da. draft. The draft and an invoice were made out and sent to Frank G. Hill by mail. Frank G. Hill accepted the draft, that is, signified his intention to pay it by writing the word *accepted*, the date, and his name across the face. The draft was then returned to Geo. H. Catchpole, who may discount it the same as he would an ordinary promissory note.

The parties to a draft are the **drawer**, the **drawee**, and the **payee**. In the foregoing draft Geo. H. Catchpole is both the drawer and the payee, and Frank G. Hill is the drawee.

A draft payable after sight begins to mature from the date on which it is accepted. An acceptance *must*, therefore, be dated in a draft payable after sight, but it may or may not be dated in a draft payable after date.



$796 40/100 Buffalo, N.Y., Apr. 6, 19

Thirty days after date Pay to

the order of Myself

Seven Hundred Ninety-six 40/100 Dollars

Value received and charge the same to account of

To F. L. McCarthy,

No. 21 Lockport, N.Y. } Frank M. Bennett

Accepted F. L. McCarthy

Some states allow three **days of grace** for the payment of notes and other negotiable paper. Days of grace are obsolete in so many of the states that they are not considered in the exercises in this book. Some states provide that when paper matures on Sunday or a legal holiday it must be paid the day preceding such Sunday or legal holiday; others provide that it must be paid on the day following. To hold all interested parties, the laws of any given state should always be observed. When the time of negotiable paper is expressed in months, **calendar months** are used to determine the date of maturity; but when the time is expressed in days, the **exact number of days** is used. Thus, a note payable 2 mo. after July 15 is due Sept. 15; but a note payable 60 da. after July 15 is due Sept. 13. Paper payable 1 mo. from May 31, Aug. 31, etc., is due June 30, Sept. 30, etc.

**255.** The time from the date of discount to the maturity of paper is called the **term of discount**; the whole sum specified to be paid at maturity, the **value**, or **amount**, of the paper.

The term of discount is usually the *exact number of days* from the date of discount to the date of maturity. Some banks, however, find the term of discount by compound subtraction, and then reduce the time to days; *e.g.* the term of discount on a note due May 6 and discounted Mar. 1 is counted as 2 mo. 5 da., or 65 da. *In this text the term of discount is the exact number of days from the date of discount to the maturity of the paper.*

**256.** The reduction made by a bank for advancing money on negotiable paper not due is called **bank discount.** The value of negotiable paper at maturity, minus the bank discount, is called the **proceeds.**

Bank discount is always the simple interest for the term of discount on the whole sum specified to be paid at maturity.

MATURITY TABLE

| | | |
|---|---|---|
| 1 | Jan. – 1 | 13 |
| 2 | Feb. + 2 | 14 |
| 3 | Mar. – 1 | 15 |
| 4 | Apr. | 16 |
| 5 | May – 1 | 17 |
| 6 | June | 18 |
| 7 | July – 1 | 19 |
| 8 | Aug. – 1 | 20 |
| 9 | Sept. | 21 |
| 10 | Oct. – 1 | 22 |
| 11 | Nov. | 23 |
| 12 | Dec. – 1 | 24 |

**257.** The accompanying **maturity table** is sometimes used by bankers in finding the maturity of notes and drafts. The following examples illustrate its use.

**258. Examples. 1.** Find the maturity of a note payable (*a*) 6 mo. from Apr. 27, 1915; (*b*) 6 mo. from Sept. 25, 1915.

SOLUTIONS. (*a*) Referring to the table, observe that April is the 4th month; adding 4 and 6, the result is 10, and the 10th month (see number on left) is October. The note is therefore due Oct. 27, 1915.

(*b*) September is the 9th month. $9 + 6 = 15$, and the 15th month (see number on right) is March of the next year. The note is therefore due Mar. 25, 1916.

**2.** Find the maturity of a note payable 90 da. from Jan. 18, 1916.

SOLUTION. $1 + 3 = 4$, and the 4th month is April. If the note were payable in 3 mo., it would be due Apr. 18. Referring to the table, note that 2 da. (1 da. + 1 da.) must be subtracted for January and March, and 2 da. added for February. The note is therefore due Apr. 18.

After the student has become familiar with the principles of the table it will not be found necessary to consult it.

### ORAL EXERCISE

*Find the maturity of each of the following notes:*

| | Date | Time | | Date | Time |
|---|---|---|---|---|---|
| **1.** | Apr. 6, 1915 | 30 da. | **6.** | Jan. 30, 1916 | 30 da. |
| **2.** | Oct. 6, 1916 | 3 mo. | **7.** | Jan. 31, 1915 | 30 da. |
| **3.** | Nov. 9, 1915 | 60 da. | **8.** | May 10, 1916 | 90 da. |
| **4.** | Jan. 31, 1916 | 1 mo. | **9.** | June 19, 1916 | 60 da. |
| **5.** | Sept. 18, 1915 | 90 da. | **10.** | Nov. 15, 1916 | 30 da. |

*Find the maturity of each of the following acceptances:*

| | Date | Time after Date | | Date | Time after Date |
|---|---|---|---|---|---|
| **11.** | Apr. 3 | 30 da. | **14.** | Dec. 31 | 2 mo. |
| **12.** | May 5 | 60 da. | **15.** | Jan. 12 | 1 mo. |
| **13.** | Jan. 29 | 1 mo. | **16.** | Feb. 18 | 3 mo. |

*Find the maturity of each of the following acceptances:*

| | Date Accepted | Time after Sight | | Date Accepted | Time after Sight |
|---|---|---|---|---|---|
| **17.** | Aug. 12 | 3 mo. | **20.** | Apr. 25 | 60 da. |
| **18.** | Sept. 18 | 2 mo. | **21.** | May 17 | 3 mo. |
| **19.** | Oct. 30 | 4 mo. | **22.** | June 18 | 30 da. |

### WRITTEN EXERCISE

*Find the maturity and the term of discount:*

| | Date | Time | Discounted |
|---|---|---|---|
| **1.** | Jan. 16, 1916 | 3 mo. | Mar. 1 |
| **2.** | Jan. 31, 1916 | 1 mo. | Feb. 3 |
| **3.** | Feb. 12, 1916 | 90 da. | Mar. 2 |
| **4.** | Feb. 24, 1916 | 60 da. | Apr. 1 |
| **5.** | Mar. 31, 1916 | 90 da. | May 13 |

| | Date of Draft | Time after Date | Date Accepted | Date Discounted |
|---|---|---|---|---|
| **6.** | Feb. 7 | 60 da. | Feb. 8 | Feb. 9 |
| **7.** | Mar. 12 | 30 da. | Mar. 12 | Mar. 15 |

| | Date of Draft | Time after Sight | Date Accepted | Date Discounted |
|---|---|---|---|---|
| **8.** | May 31 | 60 da. | May 31 | June 3 |
| **9.** | Mar. 17 | 90 da. | Mar. 20 | Mar. 21 |

**259.** The following **time table** is frequently used by bankers in finding the exact number of days between any two dates:

TABLE OF TIME

| FROM ANY DAY OF | TO THE SAME DAY OF THE NEXT | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
| JANUARY | 365 | 31 | 59 | 90 | 120 | 151 | 181 | 212 | 243 | 273 | 304 | 334 |
| FEBRUARY | 334 | 365 | 28 | 59 | 89 | 120 | 150 | 181 | 212 | 242 | 273 | 303 |
| MARCH | 306 | 337 | 365 | 31 | 61 | 92 | 122 | 153 | 184 | 214 | 245 | 275 |
| APRIL | 275 | 306 | 334 | 365 | 30 | 61 | 91 | 122 | 153 | 183 | 214 | 244 |
| MAY | 245 | 276 | 304 | 335 | 365 | 31 | 61 | 92 | 123 | 153 | 184 | 214 |
| JUNE | 214 | 245 | 273 | 304 | 334 | 365 | 30 | 61 | 92 | 122 | 153 | 183 |
| JULY | 184 | 215 | 243 | 274 | 304 | 335 | 365 | 31 | 62 | 92 | 123 | 153 |
| AUGUST | 153 | 184 | 212 | 243 | 273 | 304 | 334 | 365 | 31 | 61 | 92 | 122 |
| SEPTEMBER | 122 | 153 | 181 | 212 | 242 | 273 | 303 | 334 | 365 | 30 | 61 | 91 |
| OCTOBER | 92 | 123 | 151 | 182 | 212 | 243 | 273 | 304 | 335 | 365 | 31 | 61 |
| NOVEMBER | 61 | 92 | 120 | 151 | 181 | 212 | 242 | 273 | 304 | 334 | 365 | 30 |
| DECEMBER | 31 | 62 | 90 | 121 | 151 | 182 | 212 | 243 | 274 | 304 | 335 | 365 |

The exact number of days from any day of any month to the corresponding day of any other month, within a year, is found in the column of the last month directly opposite the line of the first month. Thus, from June 6 to Sept. 6 is 92 da.; from Apr. 1 to Oct. 1 is 183 da.; from Aug. 26 to Dec. 26 is 122 da. The exact number of days between *any two dates*, leap years excepted, is found as in the following illustrations:

**260. Examples.** **1.** How many days from Mar. 1 to May 11?

SOLUTION. From Mar. 1 to May 1 is 61 da. From May 1 to May 11 is 10 da. 61 da. + 10 da. = 71 da., the required result.

**2.** How many days from July 26 to Oct. 6?

SOLUTION. From July 26 to Oct. 26 is 92 da. From Oct. 26 back to Oct. 6 is 20 da. 92 da. − 20 da. = 72 da., the required result.

### ORAL EXERCISE

*By the table find the exact number of days from:*

**1.** July 8 to Sept. 8.
**2.** Jan. 6 to Mar. 6.
**3.** Jan. 23 to June 23.
**4.** Feb. 13 to July 13.
**5.** Mar. 11 to Sept. 11.
**6.** Mar. 21 to Aug. 21.
**7.** May 31 to Aug. 1.
**8.** Feb. 23 to Sept. 23.
**9.** Mar. 24 to July 12.
**10.** May 11 to Aug. 31.
**11.** Aug. 15 to Dec. 10.
**12.** Nov. 25 to Mar. 25.

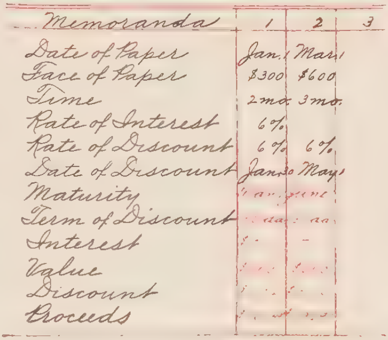
**261. Examples.** 1. Find the proceeds of a note for $3000, payable in 78 da., discounted at 6 %.

Solution. 78 da. = the term of discount.
$39 = the bank discount.
$3000 – $39 = $2961, the proceeds.

2. A note for $6000, payable in 60 da. from May 10, 1915, with interest at 6 %, is discounted May 25, at 6 %. Find the maturity, the term of discount, the bank discount, and the proceeds.

Solution. July 9, 1915 = the maturity.
45 da. = the term of discount.
$60 = the interest on the note for 60 da.
$6060 = the value of the note at maturity.
$45.45 = the bank discount.
$6014.55 = the proceeds.

**262.** The accompanying diagram illustrates a convenient outline for learning the proper method of computing bank discount. It will be observed that the first problem is an interest-bearing note, and the second problem a non-interest-bearing note. The items in black ink are taken from the problem, and the items in red ink are found as previously explained.



| Memoranda | 1 | 2 | 3 |
|---|---|---|---|
| Date of Paper | Jan. 1 | Mar. 1 | |
| Face of Paper | $300 | $600 | |
| Time | 2 mo. | 3 mo. | |
| Rate of Interest | 6% | | |
| Rate of Discount | 6% | 6% | |
| Date of Discount | Jan. 30 | May 1 | |
| Maturity | [illegible] | [illegible] | |
| Term of Discount | [illegible] da. | [illegible] da. | |
| Interest | [illegible] | – | |
| Value | [illegible] | [illegible] | |
| Discount | [illegible] | [illegible] | |
| Proceeds | [illegible] | [illegible] | |

### WRITTEN EXERCISE

1. Assuming that the model draft, page 252, was discounted July 20, at 6%, find the bank discount and the proceeds.

2. Assuming that the model draft, page 230, was discounted May 30, at 6 %, find the bank discount and the proceeds.

3. Assuming that the model draft, page 231, was discounted April 26, at 6 %, find the bank discount and the proceeds.

4. Assuming that the model draft, page 230, was discounted May 15, at 6 %, find the bank discount and the proceeds.

**5.** Assuming that the model draft, page 231, was discounted April 12, at 6%, find the bank discount and the proceeds.

**6.** Find the proceeds of the following **joint note**:

$895.40 BALTIMORE, MD., May 25, 1915.

Six months after date, for value received, we promise to pay to the order of Ralph D. Gibson Eight Hundred Ninety-five $\frac{40}{100}$ Dollars, at Exchange National Bank.

SETH M. BULLARD.

Discounted July 2, 1915, at 5%. ISAAC C. WATKINS.

**7.** Find the proceeds of the following **joint and several note:**

$1000.00 COLUMBUS, O., May 1, 1915.

Three months after date we jointly and severally promise to pay to the order of Wilson N. Burton One Thousand Dollars, at Second National Bank, Columbus, O., with interest at 6%.

Value received. JOHN M. SELLERS.

Discounted June 2, 1915, at 6%. DANIEL W. SHELDON.

**8.** Find the proceeds of the following **firm note**:

$1250.00 ST. LOUIS, MO., Aug. 20, 1915.

Ninety days after date we promise to pay to the order of C. M. Courtwright Twelve Hundred Fifty Dollars, at the National Bank of Redemption, with interest at 5%.

Value received. J. M. COX & SON.

Discounted Sept. 1, 1915, at 6%.

**9.** Sept. 26 you sold R. M. Stein, Portland, Me., a bill of hardware amounting to $2480, less 20%, 25%, and 10%. Terms: $\frac{1}{2}$ by 60-da. note with interest at 6%; $\frac{1}{2}$ on account 60 da. What was the amount of the note which was this day received?

**10.** Oct. 12 you discounted at Union Bank, at 6%, R. M. Stein's note received Sept. 26, the bank giving you credit for the proceeds. If the bank charges $\frac{1}{10}$% for collecting out-of-town paper, what was the amount of the proceeds credited?

A small fee called *collection and exchange* is sometimes charged on discounted paper payable out of town. The charge is by no means uniform, being controlled largely by the size of the depositor's account and the general custom of the banks in any given locality.

**11.** The following is a part of a page from a bank's discount register. Copy it, supplying all missing terms. The notes were all discounted June 17.

| No. | Date of Paper | Time | When Due | Term of Discount | Rate of Discount | Value of Paper | | Disc. | | Coll. & Exch. | | Proceeds Credited | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | Apr. 25 | 3 mo. | | | 6 % | 2000 | 00 | | | | | | |
| 21 | May 1 | 3 mo. | | | 6 % | 3500 | 00 | | | 3 | 50 | | |
| 22 | Apr. 1 | 90 da. | | | 6 % | 1500 | 00 | | | | | | |
| 23 | Apr. 15 | 90 da. | | | 6 % | 900 | 60 | | | | | | |
| 24 | June 15 | 30 da. | | | 6 % | 378 | 90 | | | | 38 | | |

**12.** Sept. 15 your balance in the First National Bank was $1725.90. You immediately offered for discount, at 6%, the following notes, the proceeds of which were to be placed to your credit: E. M. Robinson's 30-day note dated Sept. 1, for $300; C. E. Reardon's note payable 3 mo. from July 25, with interest at 6%, for $427.65; C. W. Allen's 60-day note dated Aug. 1, for $321.17; F. H. Clark's 60-day note dated July 30, for $1500. What was your credit at the bank after discounting the notes?

**13.** April 6, 1915, Peter W. Berger has on deposit in the First National Bank $523.87. He draws a check for $1176.45, and then discounts the following notes at the bank, at 6%, receiving credit for the proceeds. What was the balance of his account after the notes were discounted and credited?

*a.*

$346.50 Hartford, Conn., Mar. 1, 1915.

Ninety days after date I promise to pay Peter W. Berger, or order, Three Hundred Forty-six $\frac{50}{100}$ Dollars, at First National Bank, Hartford, Conn.

Value received. Henry S. Lane.

*b.*

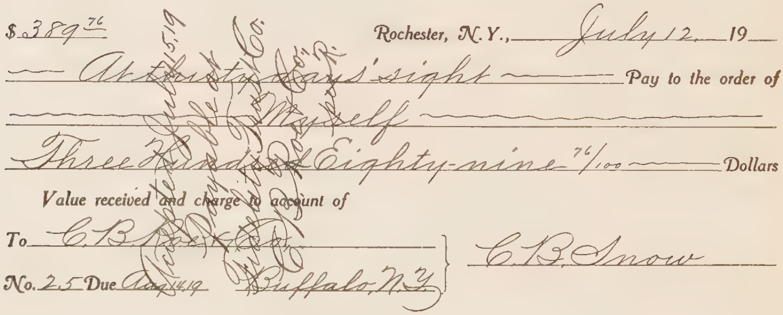
$575.00 Hartford, Conn., Feb. 1, 1915.

Aug. 1, 1915, I promise to pay Peter W. Berger, or order, Five Hundred Seventy-five Dollars, at Second National Bank, Hartford, Conn.

Value received. Samuel D. Skiff.

**14.** July 18, C. B. Snow's bank balance is \$312.90. He discounts at 6% the following drafts, and then issues a check in payment for 5 sewing machines at \$75, less 20% and 25%. What is the amount of his balance after issuing the check?

*a.*



\$389 76/100 Rochester, N.Y., July 12, 19

At thirty days' sight Pay to the order of

Myself

Three Hundred Eighty-nine 76/100 Dollars

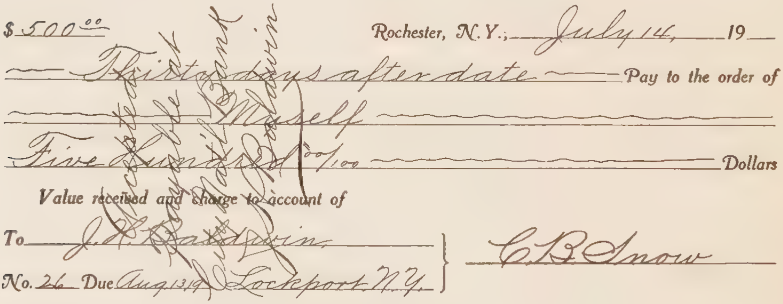
Value received and charge to account of

To C. B. Roe & Co.
No. 25 Due Aug 14 19 Buffalo, N.Y.

C. B. Snow

Accepted July 15, 19
Payable at
First Nat'l Bank
C. B. Roe & Co.

*b.*



\$500 00/100 Rochester, N.Y., July 14, 19

Thirty days after date Pay to the order of

Myself

Five Hundred 00/100 Dollars

Value received and charge to account of

To J. H. Baldwin
No. 26 Due Aug 13 19 Lockport, N.Y.

C. B. Snow

Accepted
Payable at
First Nat'l Bank
J. H. Baldwin

## BANK LOANS

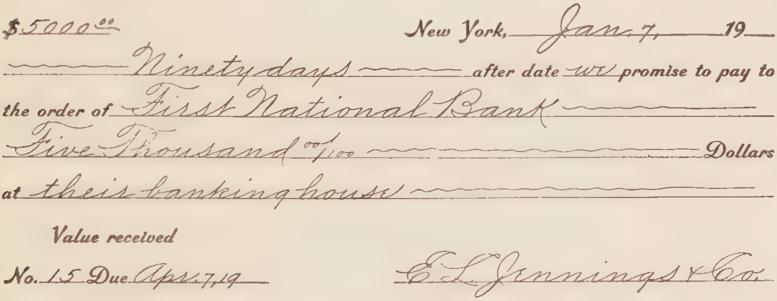
**263.** The foregoing exercises have references to paper bought or discounted by a bank. Money is frequently loaned upon the notes of the borrower, indorsed by some one of known financial ability, or secured by the deposit of stocks, bonds, warehouse receipts, or other collaterals. These notes, if drawn on time, are not interest-bearing, but the bank discounts them by deducting from their face the interest for the full time.

**264.** Loans are sometimes made on **call** or **demand notes**; that is, on notes that can be called or demanded at any time after they are made. These notes are interest-bearing and are drawn for the exact sum loaned.

*Call* or *demand loans* generally bear a lower rate of interest than loans on time. They are made principally to brokers and investors, who use them to pay for stocks; but they are also made to merchants and others to some extent. Business men, however, generally prefer to borrow on time, for they do not wish to be embarrassed by having the loans called in at an unexpected time. *Time loans* are usually drawn for thirty, sixty, or ninety days. If the borrower requires money for a longer period, the bank will usually allow him to renew the note when it falls due.

### WRITTEN EXERCISE

**1.** Jan. 7, 1915, E. L. Jennings & Co. desire to extend their business, and for this purpose borrow money at 6% of the First National Bank of New York, on the following note. How much will the bank place to the credit of E. L. Jennings & Co.?



$5000.00 *New York,* Jan. 7, *19*

Ninety days *after date* we *promise to pay to*

*the order of* First National Bank

Five Thousand 00/100 *Dollars*

*at* their banking house

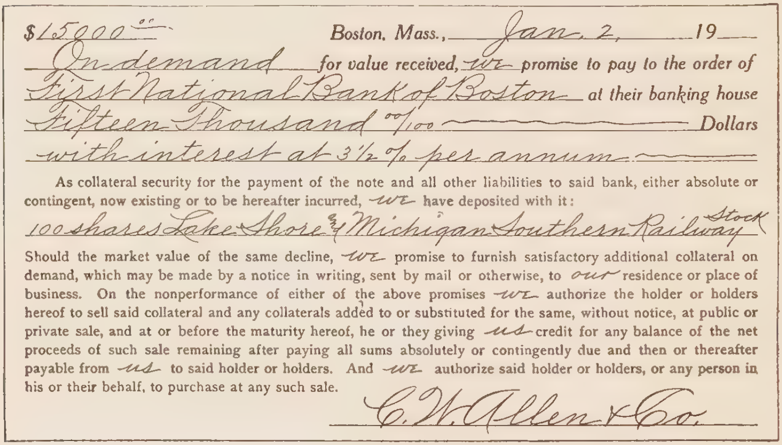
*Value received*

*No.* 15 *Due* Apr. 7, 19 E. L. Jennings & Co.

**2.** You gave the Union National Bank, of your city, your note for $1200, at 60 da., indorsed by Williams & Rogers. How much cash will the bank advance you if discount is deducted at the rate of 6%?

**3.** Howe & Rogers, Buffalo, N. Y., borrowed $12,000 of Merchants National Bank on their demand note secured by 300 shares of Missouri Pacific Railway stock at $50. If the rate of interest was $2\frac{1}{2}$%, how much was required for settlement 39 da. after the loan was made?

**4.** Jan. 2, 1915, C. W. Allen & Co., brokers, borrowed of First National Bank, Boston, Mass., $15,000 on the following **collateral note.** How much was required for full settlement of the loan 57 da. after it was made?



$15000.00 Boston, Mass., Jan. 2, 19

On demand for value received, we promise to pay to the order of First National Bank of Boston at their banking house Fifteen Thousand 00/100 Dollars with interest at 3½% per annum.

As collateral security for the payment of the note and all other liabilities to said bank, either absolute or contingent, now existing or to be hereafter incurred, we have deposited with it: 100 shares Lake Shore & Michigan Southern Railway Stock

Should the market value of the same decline, we promise to furnish satisfactory additional collateral on demand, which may be made by a notice in writing, sent by mail or otherwise, to our residence or place of business. On the nonperformance of either of the above promises we authorize the holder or holders hereof to sell said collateral and any collaterals added to or substituted for the same, without notice, at public or private sale, and at or before the maturity hereof, he or they giving us credit for any balance of the net proceeds of such sale remaining after paying all sums absolutely or contingently due and then or thereafter payable from us to said holder or holders. And we authorize said holder or holders, or any person in his or their behalf, to purchase at any such sale.

C. W. Allen & Co.

## FINDING THE FACE

**265. Example.** I wish to borrow $1980 of a bank. For what sum must I issue a 60-da. note to obtain the amount, discount being at the rate of 6%?

SOLUTION.
Let the face of the note = $1
Then the bank discount = $0.01
And the proceeds = $0.99
But the proceeds = $1980
$1980 ÷ $0.99 = 2000
∴ the face of the note is 2000 × $1, or $2000.

### WRITTEN EXERCISE

**1.** What must be the face of a 30-da. note in order that when discounted at 6% the proceeds will be $1990? of a 60-da. note, same conditions?

**2.** You wish to borrow $3940 cash. What must be the face of a 90-da. note in order that when discounted at 6% the proceeds will be the required sum?

### A WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

*Copy each of the following problems; complete the work, and check the result.*

When a certain number of days is written in the *Discount* column, compute the discount at 6% for the given time. When $\frac{1}{10}$% is written in the *Collection and Exchange* column, compute the collection on the face of the paper.

When copying the problem do not copy the number of days in the discount column, nor the $\frac{1}{10}$% in the collection and exchange column, but compute the discount or the collection and enter the *result* in the required column.

| FACE OF PAPER | DISCOUNT | COLL. & EXCH. | PROCEEDS |
|---|---|---|---|
| **1.** $356.40 | $3.56 | $0.36 | ? |
| 442.50 | 2.21 | 0.44 | ? |
| 365.30 | 60 da. | 0.37 | ? |
| 297.45 | 2.97 | $\frac{1}{10}$% | ? |
| 175.40 | 1.75 | 0.18 | ? |
| 217.50 | 2.18 | $\frac{1}{10}$% | ? |
| 246.30 | 30 da. | —— | ? |
| ? | ? | ? | ? |

| FACE OF PAPER | DISCOUNT | COLL. & EXCH. | PROCEEDS |
|---|---|---|---|
| **2.** $325.65 | $1.63 | $0.33 | ? |
| 150.60 | 90 da. | 0.15 | ? |
| 327.85 | 3.28 | 0.33 | ? |
| 180.96 | 60 da. | $\frac{1}{10}$% | ? |
| 313.46 | 3.13 | 0.31 | ? |
| 286.32 | 1.43 | 0.29 | ? |
| ? | ? | ? | ? |

| FACE OF PAPER | DISCOUNT | COLL. & EXCH. | PROCEEDS |
|---|---|---|---|
| **3.** $422.50 | 30 da. | $\frac{1}{10}$% | ? |
| 384.20 | $3.84 | $0.38 | ? |
| 519.40 | 5.19 | 0.52 | ? |
| 280.50 | 90 da. | 0.28 | ? |
| 375.90 | 1.25 | —— | ? |
| 245.32 | 2.45 | $\frac{1}{10}$% | ? |
| ? | ? | ? | ? |

# CHAPTER XX

## EXCHANGE

## DOMESTIC EXCHANGE

### ORAL EXERCISE

**1.** Mention some objections to sending actual money by express.

**2.** If $50 sent by mail in a registered letter is lost, to what extent are the postal authorities liable?

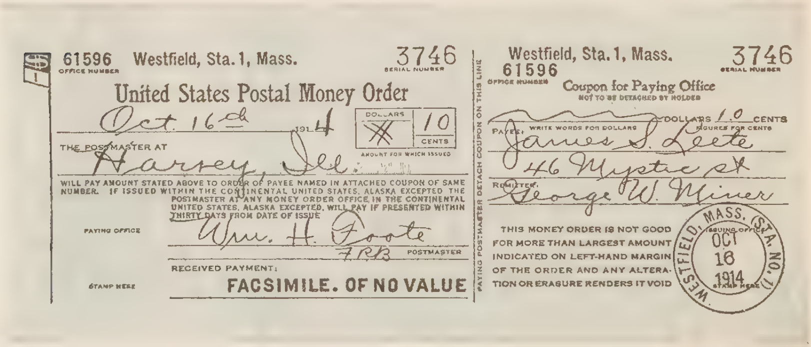
**3.** In what ways may you pay a debt at any distant point without actually sending the money?

**266.** The process of settling accounts at distant points without actually sending the money is called **exchange.**

### Money Orders

**267.** **Money orders,** as issued by post offices, express companies, and banks are frequently used in making payments at a distance.

**268.** A **postal money order** is a government order for the payment of money, issued at one office and payable at another.



61596 Westfield, Sta. 1, Mass. 3746
OFFICE NUMBER SERIAL NUMBER
United States Postal Money Order
Oct. 16th 1914
DOLLARS X CENTS 10
AMOUNT FOR WHICH ISSUED
THE POSTMASTER AT
Harvey, Ill.
WILL PAY AMOUNT STATED ABOVE TO ORDER OF PAYEE NAMED IN ATTACHED COUPON OF SAME NUMBER. IF ISSUED WITHIN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED THE POSTMASTER AT ANY MONEY ORDER OFFICE IN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, WILL PAY IF PRESENTED WITHIN THIRTY DAYS FROM DATE OF ISSUE
PAYING OFFICE
Wm. H. Foote
FRB POSTMASTER
RECEIVED PAYMENT:
STAMP HERE
FACSIMILE. OF NO VALUE

PAYING POSTMASTER DETACH COUPON ON THIS LINE

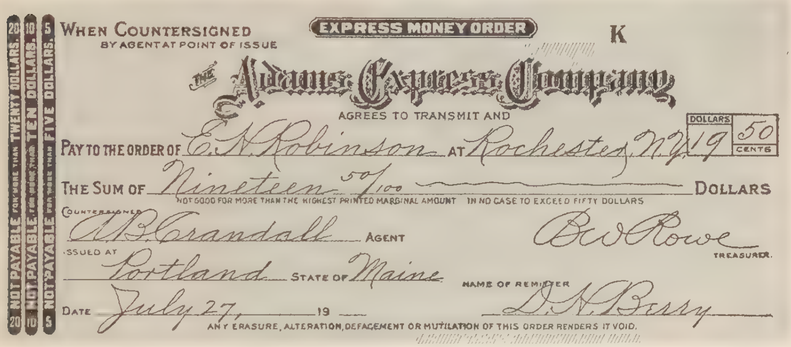
Westfield, Sta. 1, Mass. 3746
61596
OFFICE NUMBER SERIAL NUMBER
Coupon for Paying Office
NOT TO BE DETACHED BY HOLDER
DOLLARS 10 CENTS
WRITE WORDS FOR DOLLARS FIGURES FOR CENTS
PAYEE: James S. Leete
46 Mystic st
REMITTER: George W. Miner
THIS MONEY ORDER IS NOT GOOD FOR MORE THAN LARGEST AMOUNT INDICATED ON LEFT-HAND MARGIN OF THE ORDER AND ANY ALTERATION OR ERASURE RENDERS IT VOID
WESTFIELD, MASS. (STA. NO. 1) ISSUING OFFICE OCT 16 1914 STAMP HERE

The fees **(rate of exchange)** charged for postal money orders are:
For orders for sums

| | | | |
|---|---|---|---|
| Not exceeding $2.50 | 3¢ | Over $30.00 to $40.00 | 15¢ |
| Over $2.50 to 5.00 | 5¢ | Over 40.00 to 50.00 | 18¢ |
| Over 5.00 to 10.00 | 8¢ | Over 50.00 to 60.00 | 20¢ |
| Over 10.00 to 20.00 | 10¢ | Over 60.00 to 75.00 | 25¢ |
| Over 20.00 to 30.00 | 12¢ | Over 75.00 to 100.00 | 30¢ |

The maximum amount for which a single postal money order may be issued is $100. When a larger sum is to be sent, additional orders must be obtained. When an order is issued, the money is not sent from one post office to another. The transfer is merely a matter of bookkeeping, the money being received by the government at one office and paid out at another. If a postal money order is lost, a duplicate may be obtained from the Post Office Department at Washington.

**269.** An **express money order** is an order for the payment of money issued by an express company and payable at any of its agencies.



20 NOT PAYABLE FOR MORE THAN TWENTY DOLLARS. 20
10 NOT PAYABLE FOR MORE THAN TEN DOLLARS. 10
5 NOT PAYABLE FOR MORE THAN FIVE DOLLARS. 5

WHEN COUNTERSIGNED
BY AGENT AT POINT OF ISSUE

EXPRESS MONEY ORDER

K

The Adams Express Company

AGREES TO TRANSMIT AND

PAY TO THE ORDER OF E. N. Robinson AT Rochester, N.Y. DOLLARS 19 CENTS 50

THE SUM OF Nineteen 50/100 DOLLARS
NOT GOOD FOR MORE THAN THE HIGHEST PRINTED MARGINAL AMOUNT IN NO CASE TO EXCEED FIFTY DOLLARS

COUNTERSIGNED A. B. Crandall AGENT

B. W. Rowe TREASURER.

ISSUED AT Portland STATE OF Maine

NAME OF REMITTER D. H. Berry

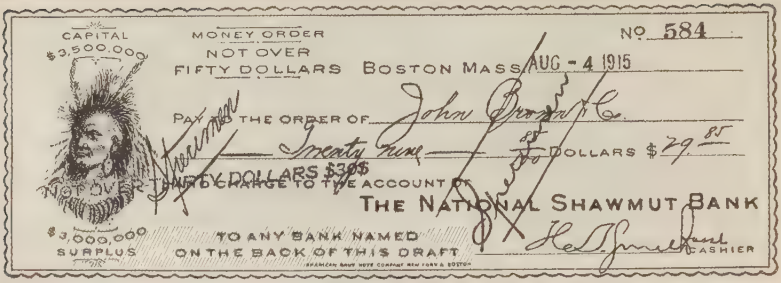
DATE July 27, 19

ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID.

The fees charged for express money orders are the same as those for postal money orders. The maximum amount for which a single express money order may be issued is $50. A postal money order must not bear more than one indorsement, but an express money order may bear any number of indorsements.

**270.** A **bank money order** (see form, page 244) is an order for the payment of money issued by a bank and payable at certain other banks in different parts of the country.

The charge for a bank money order is usually the same as that for a postal money order.



CAPITAL $3,500,000

MONEY ORDER
NOT OVER
FIFTY DOLLARS

No. 584

BOSTON MASS. AUG - 4 1915

PAY TO THE ORDER OF John Brown & Co.

Twenty nine $\frac{85}{100}$ DOLLARS $29.85

NOT OVER THIRTY DOLLARS $30$

AND CHARGE TO THE ACCOUNT OF
THE NATIONAL SHAWMUT BANK

$3,000,000 SURPLUS

TO ANY BANK NAMED
ON THE BACK OF THIS DRAFT

AMERICAN BANK NOTE COMPANY NEW YORK & BOSTON

CASHIER

**271.** A **telegraphic money order** is a telegram of an express or telegraph company, at any given place, ordering the payment of money at another designated place.

## THE UNION TELEGRAPH CO.

INCORPORATED

23,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

ROBERT C. CLOWRY, President and General Manager

**SEND** the following message subject to the terms on back hereof, which are hereby agreed to. Boston, Mass., July 27, 19

To The Union Telegraph Co.

Rochester, N.Y.

Findable Charles Osgood ten East Avenue Fichant Findelkind

The Union Telegraph Co.

These telegrams are usually in cipher; that is, in a language not understood by those who are unfamiliar with the system of abbreviations (**code**) used. The sender and the receiver must each have a code. The following code will illustrate the principle of telegraphing in cipher:

| CODE WORD | MEANING |
|---|---|
| *Fichant* | One hundred dollars |
| *Ficheron* | One thousand dollars |
| *Findable* | Please pay —— of —— your city $——. |
| *Findelkind* | On production by him of positive evidence of his personal identity. |

The principle of a telegraphic money order is the same as that of a postal money order; no money is transferred from one place to another. The charge for an order is usually 1% of the amount to be transmitted plus twice the rate for a single ten-word message.

The following are the rates for a ten-word message from Boston to the places named:

| | | | | | |
|---|---|---|---|---|---|
| New York | $0.30 | Chicago | $0.50 | Galveston | $0.75 |
| Philadelphia | $0.35 | San Francisco | $1.00 | Rochester | $0.40 |

### ORAL EXERCISE

**1.** What was the total cost to the sender of the postal money order, page 242? the express money order, page 243? the telegraphic money order, page 244? the bank money order, page 244?

**2.** What will be the total cost of a postal money order for 27¢? $2.19? $5.28? $10.40? $18.90? $45.10? $35.89? $125 ($100 + $25)? $75.29? $49.82? $127.16?

**3.** What will be the total cost of an express money order for $6.20? $28.80? $19.50? $27.95? $48.90? $65 ($50 + $15)? $111? $37.59? $41.72? $65.59? $114?

**4.** What will be the total cost of a telegraphic money order from Boston to New York for $50? $75? $100? $125? $150? $200? $300? $400? $450? $500? from Boston to Philadelphia? from Boston to San Francisco? from Boston to Chicago?

**5.** Translate the following telegraphic money order: *Findable F. J. Reed, 20 Park St. ficheron findelkind.* How much will it cost for such an order from Boston to Galveston? from Boston to Chicago? from Rochester to Boston?

### WRITTEN EXERCISE

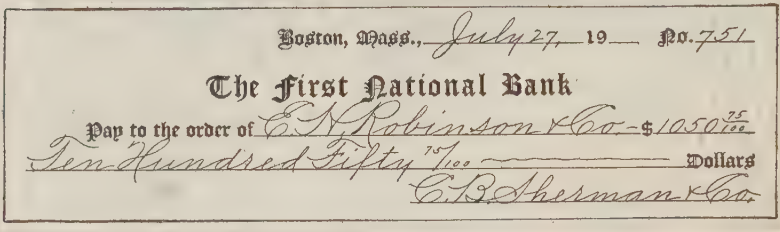
**1.** Find the total cost of 5 postal money orders for the following amounts: $3.10; $8.19; $25.06; $18.50; $20.

**2.** Find the total cost of six express money orders for the following amounts: $1.25; $10; $6.80; $16.25; $80; $19.50.

**3.** Find the total cost of the following telegraphic money orders: one from Boston to New York for $50; one from Boston to Philadelphia for $500; one from Boston to San Francisco for $175; one from Boston to Galveston for $300; one from Boston to Rochester for $250.

## CHECKS AND BANK DRAFTS

**272.** Business men usually keep their money on deposit with a commercial bank or trust company and make most payments, at home and at a distance, by **check**; that is, an order on a bank from one of its depositors for the payment of money.



Boston, Mass., July 27, 19__ No. 751

The First National Bank

Pay to the order of E. H. Robinson & Co. $1050 75/100

Ten Hundred Fifty 75/100 Dollars

C. B. Sherman & Co.

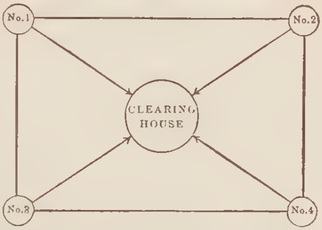
A check may be drawn for any amount so long as it does not exceed the balance on deposit to the credit of the drawer. It may be drawn payable to: (1) the order of a designated payee, in which case the payee must indorse it before the money will be paid over; (2) the payee, or bearer, in which case any one can collect it; (3) "Cash," in which case any one can collect it.

INTERIOR VIEW OF A CLEARING HOUSE

C. B. Sherman & Co. and E. H. Robinson & Co. in the foregoing check both reside in Boston. On receiving the check E. H. Robinson & Co. indorse it and deposit it for credit with their bank, say the National Shawmut Bank. The First National Bank or the National Shawmut Bank, as well as each of the other banks in the city, has many depositors who draw checks upon it which are deposited by the payees in various other city banks; it also receives daily for credit from its own depositors checks drawn upon various other city banks.

Each bank therefore has a daily balance to settle or to be settled with each of the other banks. To some it has payments to make and from others it has payments to receive. If these balances were adjusted in money, each bank would have to send a messenger to each of the debtor banks to present accounts and receive balances. This would be a risky and laborious task. To facilitate the daily exchanges of items and settlements of balances resulting from such exchanges there has been established in every large city a central agency, called a **clearing house**. This agency is an association of banks which pay the expense of conducting it in proportion to the average amount of their clearings.

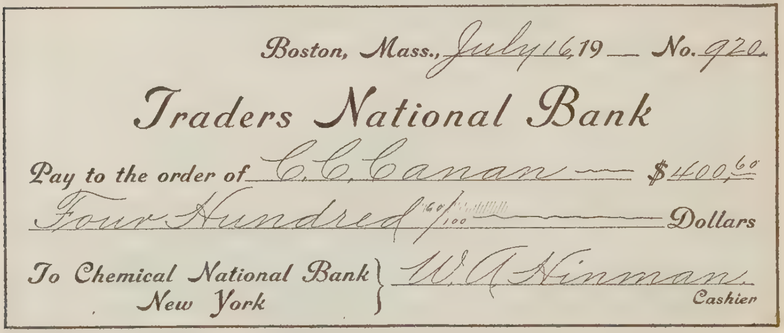
Suppose, for example, that the banks constituting a clearing house are Nos. 1, 2, 3, and 4. No. 1 presents at the clearing house items against Nos. 2, 3, and 4, and Nos. 2, 3, and 4 present items against No. 1. So, likewise, with No. 2 and each of the other banks. In the clearing house there are usually two longitudinal columns containing as many desks as there are banks in the association. At a given time a settling clerk from each bank takes his place at his desk inside of one of the columns and a delivery clerk from each bank takes his place outside the column. Each delivery clerk advances, one desk at a time, and hands over to each settling clerk his exchange items against that bank. After the circuit of the desks has been completed each delivery clerk is at the point from which he started, and each settling clerk has on his desk the claims of all of the other banks against his bank. Each settling clerk then compares his claims against other banks with those of other banks against him and strikes a balance. This balance may be in favor of or against the clearing house. If No. 1 brought claims against Nos. 2, 3, and 4 aggregating $211,000 and Nos. 2, 3, and 4 brought claims against No. 1 aggregating $200,000, there is $11,000 due No. 1 from the clearing house. But if No. 1 brought to the clearing house exchange items aggregating $200,000 and took away exchange items aggregating $211,000, there is $11,000 due the clearing house from No. 1. So, likewise, with No. 2 and each of the other banks. When all of the exchanges have been completed, the clearing house will have paid out the same amount that it has received.



But all checks received by banks are not payable in the city. Suppose that A. W. Palmer, of Chicago, Ill., owes C. B. Andrews, of Westfield, Mass., $500 and that the amount is paid by a check on the City National Bank of Chicago. C. B. Andrews receives the check and offers it for credit at the Farmers and Traders Bank of Westfield, Mass. The Westfield Bank has no account with any Chicago bank, but it has with the First National Bank of Boston, and the check is sent to that bank for credit. The First National Bank wishes to increase its New York balance and the check is forwarded to Chemical National Bank of New York for credit. Chemical National Bank next mails the check to Commercial National Bank of Chicago, the bank with which it has regular dealings in that city. Commercial National Bank sends the check to the clearing house and it is carried to the City National Bank by a messenger from that bank. Thus, all of a depositor's checks will in time be presented to the bank on which they are drawn. When presented, they will be charged to the depositor, canceled, and later returned to him to be filed as receipts.

Banks frequently charge their depositors a small fee (**rate of exchange**) for collecting out-of-town checks. This fee is rarely over $\frac{1}{10}\%$, but there is no uniformity in the matter. Sometimes when a customer keeps a large bank account, no charge whatever is made for the collection.

**273.** It often happens that a person will find it necessary to make payment to one who does not care to take the risk of a private check or to one who should not be called upon to pay the cost of cashing a check. In such cases some other form of instrument of transfer must be used. A very common and convenient method of making a remittance is by means of a **bank draft,** a check of one banking institution upon another.



Boston, Mass., July 16, 19__ No. 920

Traders National Bank

Pay to the order of C. C. Canan $400.60

Four Hundred 60/100 Dollars

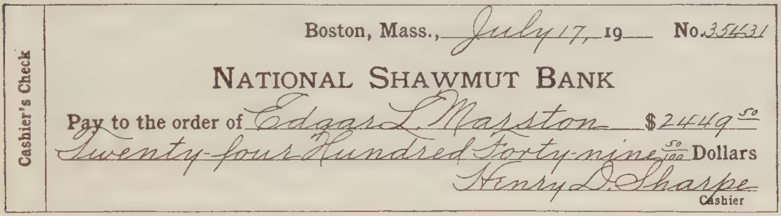
To Chemical National Bank
New York

W. A. Hinman
Cashier

Banks in the different cities frequently keep running accounts with each other and make periodical settlements. At the time of drawing the above draft Traders National Bank of Boston very likely has checks and drafts drawn upon New York banks which it has received from its depositors. These it sends to Chemical National Bank to cover the amount of the draft. Corresponding transactions may also take place in New York. Chemical National Bank may sell its draft on Traders National Bank and, to cover the amount, remit checks and drafts on Boston banks which it has received from its depositors. What is occurring between these two places is also occurring between many other places; but drafts upon New York banks and other financial centers are the most used in making remittances.

A bank draft is sometimes drawn payable to the one to whom it is to be sent. It is better, however, to have it drawn payable to the purchaser, who may indorse it over to the person to whom it is to be sent. In this way the name of the sender appears on the draft, and when canceled, the draft will serve the purpose of a receipt. Banks usually sell drafts at a slight premium on the face. This premium is called **exchange.** It varies somewhat (see page 254), but is seldom more than $\frac{1}{10}\%$.

**274.** There are still other methods of transmitting funds through the instrumentality of a bank. A depositor may exchange his own check for that of a **cashier's check.** The latter, being a check of the cashier on his own bank, would pass among strangers better than a depositor's check.



**Cashier's Check**

Boston, Mass., July 17, 19__ No. 35431

NATIONAL SHAWMUT BANK

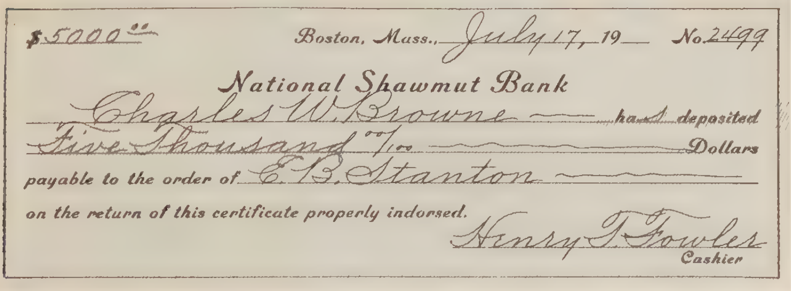
Pay to the order of Edgar L. Marston $2449 $\frac{50}{}$

Twenty-four Hundred Forty-nine $\frac{50}{100}$ Dollars

Henry D. Sharpe
Cashier

In New York City, cashier's checks are occasionally used instead of the New York draft. As New York exchange is in demand in all parts of the country, the expediency of the course is apparent.

**275.** By depositing a sum of money in a bank a person may receive a certificate, called a **certificate of deposit.** This will direct the payment of the sum deposited to any person whom the depositor may name.



$5000.00 Boston, Mass., July 17, 19__ No. 2499

*National Shawmut Bank*

Charles W. Browne has deposited

Five Thousand 00/100 Dollars

payable to the order of E. B. Stanton

on the return of this certificate properly indorsed.

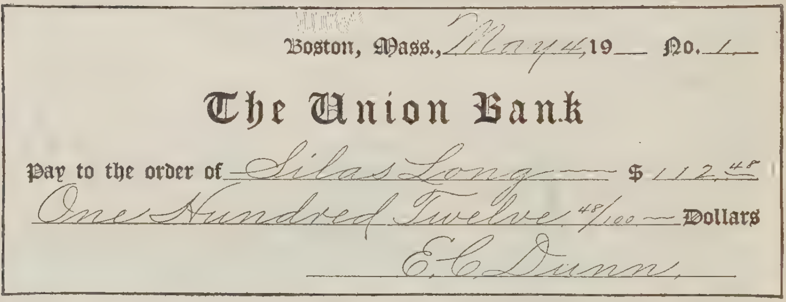
Henry T. Fowler
Cashier

The payee in a certificate of deposit will have no difficulty in getting the certificate cashed or the amount credited to him by his bank.

### ORAL EXERCISE

**1.** Assuming that the bank which cashed the check on page 5 charged $\frac{1}{8}\%$ collection, what was the amount credited to the depositor?

**2.** Silas Long of New York deposited the following check. The bank deducted $\frac{1}{10}\%$ for collection. How much was placed to Silas Long's credit?



Boston, Mass., May 4, 19__ No. 1

The Union Bank

Pay to the order of Silas Long $112.45

One Hundred Twelve 48/100 Dollars

E. C. Dunn.

**3.** B deposited three out-of-town checks in his bank, as follows: \$300; \$700; \$750. If the bank charged $\frac{1}{10}\%$ collection, what amount was placed to B's credit?

**4.** Bring to the class a number of canceled checks, and take several of them and trace them from the time they were issued until they were filed as receipts by the drawer. Show why a canceled check is the best kind of receipt for the payment of money?

**5.** How much did the bank draft on page 248 cost the purchaser if the exchange was at $\frac{1}{10}\%$ premium?

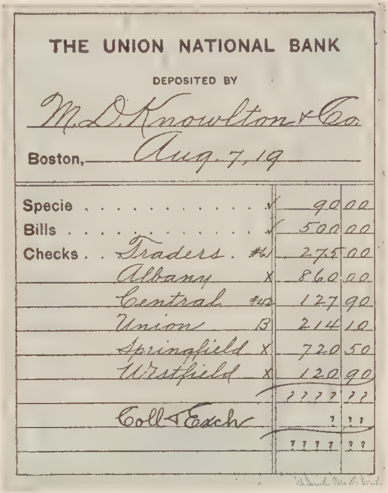
### WRITTEN EXERCISE

**1.** Find the cost of a bank draft for \$3958.75 at $\frac{1}{10}\%$ premium: of a bank draft for \$679.80 at $\frac{1}{20}\%$ premium; of a bank draft for \$768.54 at 50¢ per \$1000 premium.

**2.** To cover the cost of a bank draft bought at $\frac{1}{10}\%$ premium, I gave my bank a check for \$250.25. What was the face of the draft? What was the rate of premium per \$1000?

**3.** How large a bank draft can be bought for $850.85, exchange being at $\frac{1}{10}\%$ premium?

**4.** Find the proceeds of the accompanying deposit, $\frac{1}{10}\%$ collection and exchange being charged on the out-of-town checks.



**THE UNION NATIONAL BANK**

DEPOSITED BY

M. D. Knowlton & Co.

Boston, Aug. 7, 19

| | | | | |
|---|---|---|---|---|
| Specie | | ✓ | 90 | 00 |
| Bills | | ✓ | 500 | 00 |
| Checks | Traders | #61 | 275 | 00 |
| | Albany | X | 860 | 00 |
| | Central | #40 | 127 | 90 |
| | Union | B | 214 | 10 |
| | Springfield | X | 720 | 50 |
| | Westfield | X | 120 | 90 |
| | | | ???? | ?? |
| | Coll & Exch | | ? | ?? |
| | | | ???? | ?? |

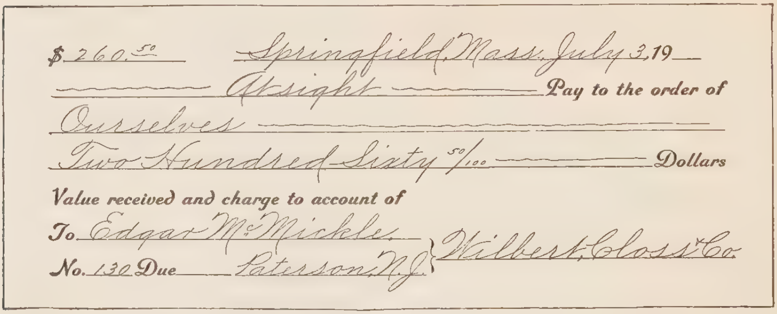
When the receiving teller takes a deposit from a customer, he classifies the items on the deposit ticket, as shown in the accompanying illustration. If the coin and bills passed in count right, these items are checked (✓) on the deposit slip; if a check on a clearing house bank is received, it is marked with the number of that bank in the clearing house; if a check on the teller's bank is received, it is marked "B"; if a check on an out-of-town bank is received, it is marked "X."

**5.** Write a bank draft, using the following data: your address and the current date; drawer, Central National Bank; drawee, Chemical National Bank, New York; amount, $711.94; payee, C. E. Denison; cashier, your name. How large a check will pay for the draft at $\frac{1}{10}\%$ premium? Write the check.

**6.** Suppose that the members of the class whose surnames begin with the letters from A to G inclusive have a deposit with Traders National Bank; that the members whose surnames begin with the letters from H to N inclusive have a deposit with City National Bank; that the members whose surnames begin with O to S inclusive have a deposit with First National Bank; and that the members whose surnames begin with T to Z inclusive have a deposit with Central Bank. Let each student write a check on his bank in favor of one of his classmates, and let this classmate indorse the check and deposit it with his bank. Then form a clearing house, strike a balance between the different banks, and have these balances adjusted by the payment of school money.

## Commercial Drafts

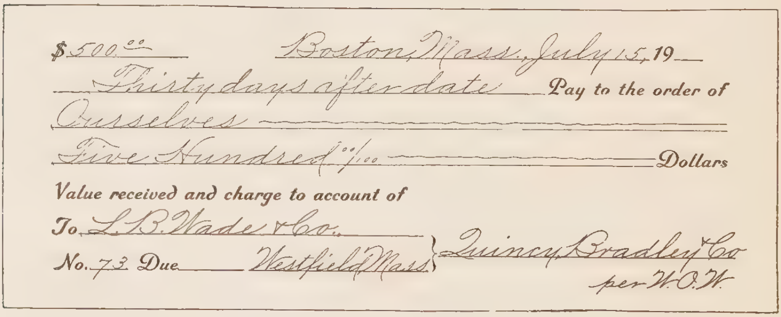
**276.** Business men frequently employ the **commercial draft** as an aid in the collection of accounts that are past due.



$260.50 Springfield, Mass. July 3, 19__
At sight Pay to the order of
Ourselves
Two Hundred Sixty 50/100 Dollars
Value received and charge to account of
To Edgar McMickle
No. 130 Due Paterson, N.J. } Wilbert, Closs & Co.

The above is a common form of draft used for collection purposes. Edgar McMickle owes Wilbert, Closs & Co. $260.50. The amount is due, and Wilbert, Closs & Co. draw a draft on Edgar McMickle and leave it with their Springfield bank for collection. The Springfield bank forwards it to its correspondent in Paterson, and this bank sends it by messenger to Edgar McMickle. When he pays the draft the Paterson bank notifies the Springfield bank, and that bank deducts a small fee (**collection and exchange**) for collecting the draft, and credits Wilbert, Closs & Co. for the proceeds.

**277.** It has been seen (page 231) that the **time draft** is frequently used in connection with sales of merchandise.



$500.00 Boston, Mass. July 15, 19__
Thirty days after date Pay to the order of
Ourselves
Five Hundred 00/100 Dollars
Value received and charge to account of
To L. B. Wade & Co.
No. 73 Due Westfield, Mass. } Quincy, Bradley & Co
per W. O. W.

Suppose Quincy, Bradley & Co. sell L. B. Wade & Co. a bill of merchandise amounting to $500. Terms: 30-da. draft for the amount of the bill. The draft, as above, and the bill in regular form would be drawn up and sent to L. B. Wade & Co. for acceptance. The object of drawing a time draft in connection with sales of merchandise is twofold: (1) when accepted, the draft serves as a written contract; (2) since an acceptance is negotiable, it may be discounted and cash realized upon it before maturity. Such a draft is frequently left with a bank for collection instead of being remitted with the bill. The bank will then first present the draft for acceptance and later for payment.

### ORAL EXERCISE

**1.** If you exchange your check for a cashier's check, is there any charge for the accommodation?

**2.** If the sight draft on page 252 was collected by a bank which charged $\frac{1}{8}\%$ collection, how much was placed to the credit of Wilbert, Closs & Co.?

**3.** You deposited in Shawmut National Bank $5000, received the certificate of deposit shown on page 249, and remitted it to E. B. Stanton on account. Would there be any exchange?

### WRITTEN EXERCISE

**1.** The draft on page 252 was accepted July 17, and discounted July 25. If the bank charged $\frac{1}{10}\%$ collection and 6 % interest, how much was placed to the credit of the drawers?

**2.** Mar. 27 Wilson Bros., Chicago, Ill., drew a 30-da. draft on E. W. King, Toledo, O., in favor of themselves, payable 30 da. after date, for $3500, and mailed it for acceptance. Apr. 1 the draft was received accepted; Apr. 2 it was discounted at City Bank. If the charges were $\frac{1}{20}\%$ collection and 6 % interest, what amount was credited to Wilson Bros.?

**3.** Apr. 17 O. H. Brooks, Buffalo, N.Y., drew a sight draft on Slocum & Co., Hartford, Conn., in favor of himself, for $391, and left it with his bank (First National) for collection. First National Bank sent the draft to its Hartford correspondent (Commercial National), and 5 da. later informed O. H. Brooks that the draft had been collected, and the amount, less $\frac{1}{8}\%$ collection, placed to his credit. If O. H. Brooks's bank balance was $758.62 before the draft was drawn, what was it after the draft was credited? Write the draft and show the indorsements.

**4.** Aug. 9 you sold C. D. Mead & Co., San Francisco, Cal., 39 mahogany sideboards at $162.50, delivered the goods to the Interstate Transportation Co., and received a through **bill of lading** (receipt for the goods and an agreement to transport and deliver them to the consignee or to his order). You then drew a sight draft on C. D. Mead & Co. in favor of your bank, attached the draft to the bill of lading, and left it with your bank for collection. Your bank indorsed the draft and the bill of lading and sent them to First National Bank of San Francisco for collection and credit. Aug. 23 you received advice that the draft had been collected, and the amount, less $\frac{1}{4}$ %, placed to your credit. What was the amount credited?

When First National Bank of San Francisco received the draft, it notified C. D. Mead & Co. They paid the draft, and the bank gave them the bill of lading. When goods are shipped in this manner, the transportation company will not deliver the goods until the consignee presents the bill of lading.

## FLUCTUATION OF RATES OF EXCHANGE

**278.** It has been seen that money orders always sell for more than their face value, and that bank drafts frequently cost a little more than their face value. When exchange costs its face value, it is said to be **at par**; when it costs more than its face value, it is said to be **at a premium**; when it costs less than its face value, it is said to be **at a discount.**

On bank drafts for small sums, say $500 or less, exchange is usually at a uniform premium. This premium is to pay the banks for their trouble and the expense of shipping money to the centers on which the drafts are drawn, when balances at these points become low. But exchange on the trade centers of the country may be at par at one time, at a premium at another, and at a discount at still another. For example, during the late fall months, when the grain crops begin to be sent East, New York is sending a great many checks and drafts to the section of which Chicago is the trade center. Exchange on New York is then very plentiful in Chicago, and if a man in Chicago wished to buy a draft on New York for a large amount, say $10,000 or more, the Chicago banks will sell it to him at a discount. But if a man in New York at that time wished to buy a draft on Chicago for $10,000, he would have to pay a premium, because the New York banks would be anxious not to decrease their Chicago balances.

Early in the spring, when New York importers and jobbers are sending foreign and domestic manufactured goods for distribution in the West, a great many checks and drafts are being sent from the West to New York, and exchange is at a discount in New York and at a premium in Chicago. This principle applies at all trade centers between which exchange operations go on. Smaller places make their settlements in or through larger places, and the main exchange transactions go on between the few leading cities, with converging lines on New York.

The rate of exchange between two cities will never exceed the cost of shipping actual money from one of the cities to the other, except in time of panic or a financial unrest. Thus when the cost of sending money by express from New York to Chicago is \$5 per \$10,000, the discount in New York or the premium in Chicago will not greatly exceed $\frac{1}{20}\%$ (\$5 per \$10,000). To prevent the rates from going any higher the banks will arrange for the shipment of actual money from New York to Chicago.

As a rule no charge is made for cashing bank drafts on the trade centers of the country, like New York, Chicago, and Philadelphia.

**279.** It has been seen that banks frequently charge a small fee for collecting paper payable out of town.

In some cases the rates of collection are more or less arbitrary; in others they are governed by trade movements, the same as rates of exchange. In still others the clearing house association fixes the rate.

### ORAL EXERCISE

*Find the cost of the following bank drafts:*

**1.** \$18,500 at $\frac{1}{20}\%$ discount; at 40¢ per \$1000 premium.

**2.** \$516.90 at $\frac{1}{10}\%$ premium; at 50¢ per \$1000 discount.

**3.** \$1600.80 at 75¢ per \$1000 premium; at $\frac{1}{10}\%$ discount.

**4.** A draft for \$4000 was bought for \$3998. Was exchange at a premium or at a discount, and what rate?

**5.** J. E. Smith & Co. drew at sight on E. M. Barrows for \$250 and made collection through their bank. If the bank charged $\frac{1}{10}\%$ for collection, for what amount did J. E. Smith & Co. receive credit?

**6.** During the late fall many checks and drafts are being sent to the southern cities in payment for shipments of cotton. At such times is exchange on New York likely to be at a discount or at a premium in New Orleans? in New York?

# DIVIDENDS AND INVESTMENTS

## CHAPTER XXI

## STOCKS AND BONDS

### STOCKS

**280.** A **corporation,** or **stock company,** is an association of individuals organized under the laws of a particular state into a body which is by law given the rights and powers and civil liabilities of a person.

Being a mere creature of law a corporation possesses only those properties which its **charter** (the instrument which defines its rights and duties) confers upon it. These are such as are best calculated to effect the object for which it was created. Among the most important is legal continuous existence, irrespective of that of the individuals composing it.

**281.** The **capital stock** of a corporation represents the interest of the individuals who compose the corporate body in the property and profits of the corporation.

The capital stock is divided into a definite number of shares. It is not necessary to give these shares a par value, but it is customary to do so. The usual par value of a share is $100. However, any amount may be the par value.

**282.** A **stock certificate** is a legal instrument which evidences that the holder owns a certain number of shares of stock in the corporation or association issuing the certificates. A **stockholder** is a person who owns one or more shares of stock.

The certificate bears the seal of the corporation and is signed by officers (usually the president and the treasurer or their representatives) authorized by the by-laws to sign stock certificates.

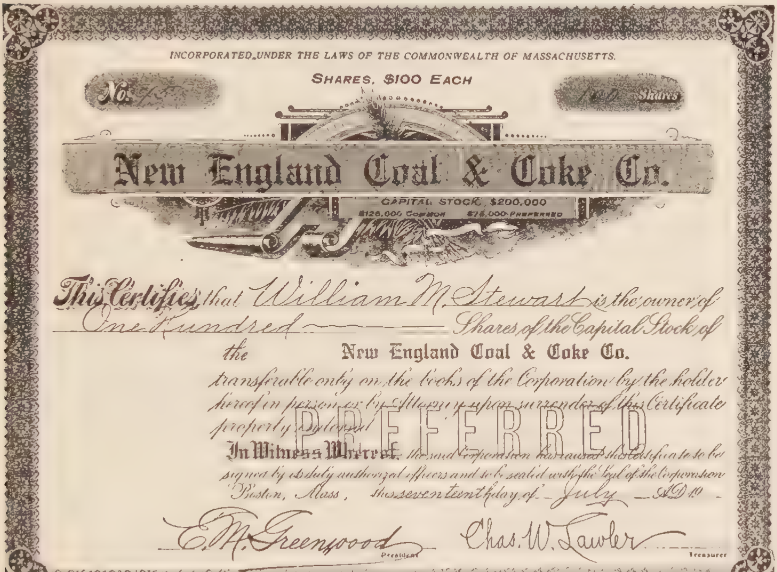
Stockholders elect a few of their number to have general control of the company. The stockholders thus elected constitute the board of directors. This board may delegate its powers to several of its members called an executive committee. The control of a corporation is vested in its board of directors.

**283.** A **dividend** is a sum of money paid by a corporation to its stockholders, each share participating equally. An **assessment** is an amount per share which stockholders are called upon to pay to make up losses or deficiencies.

The board of directors decide upon the rate of dividend, which usually means the rate *per annum on the face value of the stock*, but it may also be expressed as a certain amount in dollars and cents per share.

Shares of stock may be non-assessable.

**284.** A company may divide its stock into two or more classes; the usual division is **preferred** and **common.**



INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

SHARES, $100 EACH

No.

100 Shares

New England Coal & Coke Co.

CAPITAL STOCK, $200,000

$125,000 COMMON $75,000 PREFERRED

**This Certifies** that William M. Stewart is the owner of One Hundred Shares of the Capital Stock of the **New England Coal & Coke Co.** transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly Endorsed.

PREFERRED

**In Witness Whereof,** the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation.

Boston, Mass., this seventeenth day of July A.D. 19

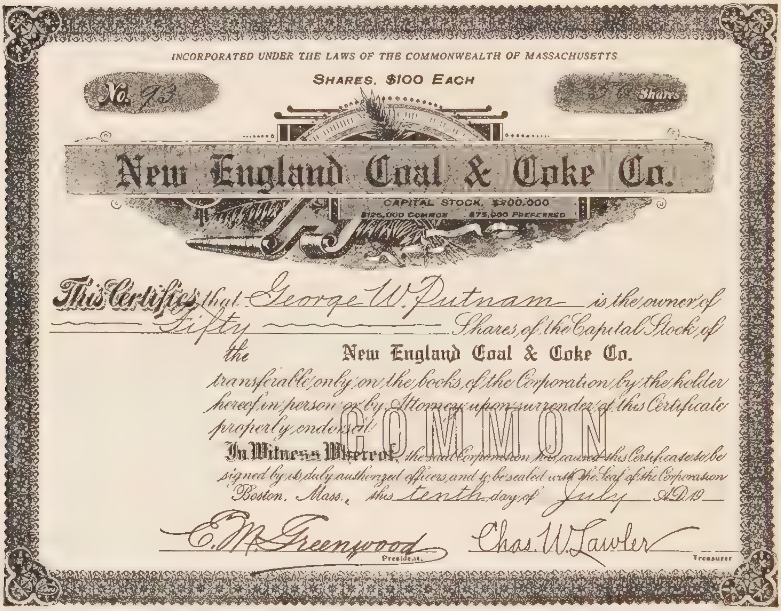
E. M. Greenwood
President

Chas. W. Lawler
Treasurer

**285.** **Preferred stock** is stock which is entitled, out of profits, to a preferential dividend at a fixed rate. In case a company is dissolved, its preferred stock usually has a preferential claim against the assets of the company up to the par value of the preferred stock outstanding.

The prior rights of preferred stocks over common usually make them safer investments, but stocks are not necessarily safe because preferred.

**286. Common stock** is the stock which has the right to the equity in the earnings and the income of the liquidation in the assets of a corporation after the claims of all prior security issues (bonds, notes, preferred stock, etc.) have been satisfied.



INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS

SHARES, $100 EACH

No. 73

50 Shares

New England Coal & Coke Co.

CAPITAL STOCK, $200,000
$125,000 COMMON $75,000 PREFERRED

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the New England Coal & Coke Co. transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation Boston, Mass., this tenth day of July A.D. 19

E. M. Greenwood
President

Chas. W. Lawler
Treasurer

**287.** The **par value** is the face value of stocks; the **market value** is the sum for which the stocks can be sold in the market.

**288.** If a company is prosperous and pays a higher rate of dividend than the money could earn with no greater risk in the general money market, a share may sell for more than its face value; it is then said to be *above par*, or *at a premium*. If the company pays a lower rate of dividend than could be obtained at no greater risk in the general money market, a share may sell for less than its face value; it is then *below par*, or *at a discount*.

**289.** A **stock broker** is a person who buys and sells stocks for other persons on commission.

Stocks are usually bought and sold through stock brokers. The broker's commission is usually $\frac{1}{8}$% of the par value of the stock, or $12\frac{1}{2}$¢ per share. A charge is made for either buying or selling.

**290.** When the price of stock is quoted at 97, 118$\frac{3}{4}$, 160$\frac{1}{2}$, it means that a share whose par value is $100 can be bought for $97, $118.75, $160.50. If a person buys stock through a broker at 160$\frac{1}{2}$, it will cost him $160.50 + $0.12$\frac{1}{2}$ brokerage, or $160.62$\frac{1}{2}$; if he sells stock through a broker for 160$\frac{1}{2}$, he will receive as proceeds $160.50 – $0.12$\frac{1}{2}$, or $160.37$\frac{1}{2}$, per share.

On the New York Stock Exchange quotations are expressed in dollars and eighths of a dollar.

In some other cities quotations are expressed in dollars, in eighths of a dollar, and in sixteenths of a dollar. When stocks are quoted at less than $1 per share, fluctuations are sometimes expressed in cents.

The bulk of transactions in the stock exchanges is in round lots. In New York *round lots* are 100 shares or some multiple thereof; in Boston, 50 shares or some multiple thereof. In these cities any other number of shares than the ones named for each would be called *odd lots*. In Chicago no distinction is made between round and odd lots. Fractional shares of stock which are occasionally in the market are called *scrip*.

### ORAL EXERCISE

**1.** Examine the certificate of stock, page 257. What is the name of the company? From whom did the company get its right to carry on business as a corporation?

**2.** What is the entire capital stock of the company? Into how many shares is this divided? What per cent of the entire stock of the company does the holder of the certificate own?

**3.** What kind of stock is represented by the certificate? What is the difference between common and preferred stock?

**4.** What is the par value of each share? If the market value of each share is $160, what is the certificate worth?

**5.** What sum must be laid aside to provide for the dividends on the preferred stock of the company, the rate being 6 %? How much of this sum will the holder of the certificate receive?

**6.** Examine the stock certificate, page 258. What part of the stock of the company is common stock?

**7.** A 5 % dividend on the common stock would require how much money from the treasury of the company? Of this sum how much would George W. Putnam receive?

## DIVIDENDS AND ASSESSMENTS

### WRITTEN EXERCISE

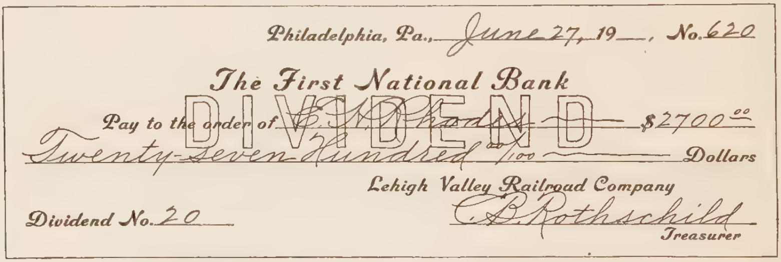
Unless otherwise specified, the par value of a share will be understood to be $100.

**1.** A company with $3,500,000 capital declares an 8% dividend. What does the holder of 250 shares receive?

**2.** B holds 450 shares of Lehigh Valley Railroad stock. When the company declares a dividend of $7\frac{1}{2}\%$, how much will he receive?

**3.** What annual income is derived from investing $48,000 in Union Pacific Railroad stock at 120 if $2\frac{3}{4}\%$ semiannual dividends are declared?

**4.** E. H. Rhodes holds 600 shares of Lehigh Valley Railroad stock. If he received the following check as his annual dividend, what was the rate?



Philadelphia, Pa., June 27, 19__, No. 620

The First National Bank

DIVIDEND

Pay to the order of E. H. Rhodes $2700.00

Twenty-seven Hundred 00/100 Dollars

Lehigh Valley Railroad Company

Dividend No. 20

C. B. Rothschild

Treasurer

**5.** A company with $1,000,000 capital declares quarterly dividends of $1\frac{1}{4}\%$. What are the annual dividends? What is the amount received annually by D, who owns 475 shares?

**6.** A corporation with a capital of $125,000 loses $2500. What per cent of his stock must each stockholder be assessed to meet this loss? How much will it cost A, who owns 150 shares?

**7.** A company with a capital of $750,000 declares a semiannual dividend of $3\frac{1}{4}\%$. How much money does it distribute annually among its stockholders? What is the annual income of a man who owns 200 shares?

**8.** If the Pennsylvania Railroad declares a semiannual dividend of $2\frac{1}{2}\%$ on a capital stock of \$500,000,000, what amount is annually distributed among the stockholders? What is the annual income to J. P. Morgan from this stock if he owns 7,500,000 shares having a par value of \$50 each?

**9.** During a certain year a manufacturing concern with a capital of \$750,000 earns \$75,500 above all expenses. It decides to save \$15,500 of this for emergencies and to divide the remainder in dividends. What is the rate? What would be the amount of A's dividend check if he owns 125 shares?

**10.** The capital stock of the Gramercy Finance Company is \$1,500,000. The gross earnings of the company for a year are \$375,000 and the expenses \$215,000. What even per cent of dividend may be declared and what would be the amount of undivided profits if 10% of the net earnings are first set aside as a surplus fund? (An even per cent is a per cent without a fraction.)

**11.** A railway company has a capital of \$3,500,000 and declares dividends semiannually. During the period from Jan. 1 to July 1 of a certain year the net earnings of the company were \$191,000. Of this amount 10% is carried to surplus fund. What even rate per cent of dividend may be declared on the balance and how much will be carried to undivided profits?

**12.** A company with a capital stock of \$500,000 gains during a certain year \$38,750. It decides to carry \$5000 of the profits to surplus fund and to declare an even per cent of dividends on the remainder. What sum was divided among the stockholders, and what sum was carried to undivided profits account? What was the annual income to F from this stock if he owned 500 shares?

**13.** During a certain year the gross earnings of a railroad having a capital stock of \$100,000,000 were \$65,150,000, and the operating expenses \$45,150,000. If the company declared a semiannual dividend of $3\frac{1}{4}\%$ and carried the balance of the net earnings to undivided profits account, how much was divided among the stockholders? How much was the working capital of the company increased?

**14.** The capital stock of the First National Bank is $3,000,000, and dividends are declared semiannually. The profits of the bank for a certain six months are $185,750. Of this sum 10% is carried to a surplus fund. The directors then vote to declare a semiannual dividend of 3½% and carry the balance of the profits to undivided profits account. What amount was carried to surplus fund account? to dividend account? to undivided profits account?

## BUYING AND SELLING STOCK

**291.** The following is an abbreviated form of the stock quotations for a certain day on the New York Stock Exchange:

TABLE OF SALES AND RANGE OF PRICES

| SALES | STOCKS | OPEN. | HIGH. | LOW. | CLOS. | NET CHANGE |
|---|---|---|---|---|---|---|
| 2,600 | Am. Sugar . . . . | 100½ | 100½ | 99¼ | 99¾ | – ¾ |
| 200 | Am. Sugar (pfd.) . | 110 | 110¼ | 109⅛ | 109⅛ | – ⅞ |
| 10,200 | Atchison . . . . | 95½ | 95½ | 91¾ | 92 | – 3½ |
| 300 | Atchison (pfd.) . . | 100 | 100 | 100 | 100 | |
| 900 | At. Coast Line . . | 121⅛ | 120 | 116 | 116 | – 4⅛ |
| 13,600 | Baltimore & O. . . | 88⅛ | 88⅝ | 87¼ | 88 | – ⅛ |
| 600 | Baltimore & O. (pfd.) | 80½ | 81 | 80¼ | 80¼ | – ¼ |
| 147,100 | Canadian Pacific. . | 193⅝ | 200⅜ | 188½ | 189⅞ | – 3¾ |
| 20,200 | Chic. M. & St. P. . | 98⅜ | 98½ | 94⅛ | 95 | – 3⅜ |
| 300 | Chic. M. & St. P. (pfd.) | 137¾ | 135 | 134⅞ | 134⅞ | – 2 |
| 200 | General Chem. (pfd.) | 108 | 109 | 108½ | 109 | + 1 |
| 2,500 | General Electric . . | 144 | 144 | 141 | 141 | – 3 |
| 15,600 | Gt. Northern (pfd.) . | 122 | 121⅝ | 119 | 119¼ | – 2½ |
| 1,600 | Illinois Central . . | 110 | 110 | 107½ | 107⅝ | – 2⅜ |
| 59,800 | Lehigh Valley . . | 136½ | 136½ | 132¼ | 134¾ | – 1¾ |
| 650 | Louisville & Nash. . | 135¾ | 135¼ | 131¼ | 131¼ | – 4½ |
| 2,100 | Nat'l Biscuit . . . | 131 | 130¼ | 125 | 125 | – 5 |
| 100 | Nat'l Biscuit (pfd.) . | 123 | 123⅞ | 123⅞ | 123⅞ | + ⅞ |
| 69,200 | South. Pacific. . . | 92¼ | 91½ | 86¼ | 87⅛ | – 5⅛ |
| 300 | South. Pacific (pfd.). | 100¼ | 97½ | 97½ | 97½ | – 2¾ |
| 210,100 | Union Pacific . . . | 154⅛ | 154¼ | 148⅞ | 149⅞ | – 4 |
| 400 | Union Pacific (pfd.) . | 82½ | 82½ | 82 | 82 | – ½ |
| 390.100 | U.S. Steel . . . . | 58⅛ | 58⅜ | 56 | 56⅜ | – 2¾ |
| 4,200 | U.S. Steel (pfd.) . . | 109 | 109¼ | 107¼ | 107¼ | – 1½ |

In the first column is shown the number of shares of stock sold; in the second, the name of the stock; in the third, fourth, fifth, and sixth, respectively, the opening, the highest, the lowest, and the closing prices of the day; in the last, the net changes between the closing price of yesterday and to-day. Thus, 2600 shares of American Sugar stock were sold. The opening price was $100.50 per share; the highest price $100.50; the lowest, $99.25; the closing, $99.75, which shows a decline of 75¢ from the preceding day.

### ORAL EXERCISE

**1.** Find in the table (page 262) three cases where a quotation both for common stock and for preferred (*pfd.* stands for *preferred*) stock of the same company is given. Which is worth the more in each case? The par value of all shares is $100.

If the profits of a concern are so great that a large per cent may be paid on the common stock, after paying the fixed rate on the preferred stock, then the common stock may sell for a higher figure than the preferred.

**2.** What would 100 shares of American Sugar (common) cost if bought through a broker at the lowest price for the day, brokerage being $\frac{1}{8}$%.

**3.** What would the seller of the stock realize on the sale?

SUGGESTION. The seller would receive the price for which it was sold minus the brokerage, $\frac{1}{8}$%.

**4.** State the cost, at the opening price in the table, of 100 shares each of the following stocks, assuming that the transactions take place through a broker who charges $\frac{1}{8}$% commission: Baltimore & Ohio; Canadian Pacific; General Electric; Lehigh Valley. (Base the calculations on the common stock.)

**5.** At the highest price in the table, state the amount that would be received from the sale of 100 shares of each of the following stocks, assuming that they were sold through a broker who charged $\frac{1}{8}$% commission: Southern Pacific; U.S. Steel (preferred); Great Northern (preferred); National Biscuit; American Sugar (preferred); Atchison; General Chemical (preferred); Illinois Central; Union Pacific. (If *preferred* is not named, *common* stock is referred to.)

### WRITTEN EXERCISE

*Find the cost, at the closing price in the table, of 2500 shares of the following stocks, including brokerage:*

**1.** Canadian Pacific.
**2.** American Sugar (pfd.).
**3.** National Biscuit (pfd.).
**4.** Baltimore & Ohio (pfd.).
**5.** Atlantic Coast Line.
**6.** United States Steel (pfd.).

*At the closing price for the day find the amount received from the sale of 3500 shares of the following stocks sold through a broker:*

**7.** Illinois Central.
**8.** Louisville & Nashville.
**9.** Southern Pacific.
**10.** Lehigh Valley.
**11.** Atchison (pfd.).
**12.** General Electric.
**13.** Southern Pacific (pfd.).
**14.** Great Northern (pfd.).

**292. Example.** I bought 1000 shares Chicago, Milwaukee, & St. Paul preferred stock, at the lowest price in the table, and sold the same at $140\frac{1}{2}$. Allowing for brokerage both for buying and selling, did I gain or lose, and how much?

Solution. Since I bought through a broker, each share cost me $\$134.87\frac{1}{2} + \$0.12\frac{1}{2}$, or $135; and since I sold through a broker, the proceeds of each share sold was $\$140.50 - \$0.12\frac{1}{2}$, or $\$140.37\frac{1}{2}$. $\$140.37\frac{1}{2} - \$135.00 = \$5.37\frac{1}{2}$, gain on each share. Since $\$5.37\frac{1}{2}$ is gained on each share, 1000 times $\$5.37\frac{1}{2}$, or $5375, is gained on 1000 shares.

$$\begin{array}{r} \$140.37\frac{1}{2} \\ 135.00 \\ \hline \$5.37\frac{1}{2} \\ 1000 \\ \hline \$5375. \end{array}$$

In the following exercise it is understood that all sales and purchases are made through a broker, who charges a commission of $\frac{1}{8}\%$ both for buying and for selling.

### WRITTEN EXERCISE

*Find the gain or loss on 500 shares of each of the following stocks bought at the opening price and sold at the price here given:*

**1.** Illinois Central, $108\frac{1}{4}$.
**2.** General Electric, $147\frac{7}{8}$.
**3.** Southern Pacific (pfd.), 89.
**4.** General Chemical (pfd.), 110.
**5.** American Sugar (pfd.), 103.
**6.** National Biscuit, $134\frac{1}{2}$.
**7.** Baltimore & Ohio, $90\frac{3}{8}$.
**8.** Canadian Pacific, $200\frac{1}{2}$.
**9.** United States Steel (pfd.), $112\frac{1}{2}$.
**10.** Atlantic Coast Line, $115\frac{3}{4}$.
**11.** Great Northern (pfd.), 125.
**12.** National Biscuit (pfd.), $126\frac{1}{4}$.

**13–24.** Find the gain or the loss on 1000 shares of each of the above stocks bought at the lowest price and sold at the highest price, in the table.

**25.** John R. West bought 400 shares of United States Steel, (common) at the opening price in the table and sold it so as to gain $300. What was the quoted price when he sold it?

**26.** I bought some United States Steel (preferred) at the opening price in the table and sold it for $112\frac{1}{2}$. If I gained $650 by the transaction, how many shares did I buy?

**27.** I bought 2500 shares of General Electric at the lowest price in the table, held it for a year, received 5% in dividends, and then sold it at $139\frac{1}{8}$. If money was worth $4\frac{1}{2}$%, did I gain or lose, and how much?

The interest is to be computed on the cost of the stock, the dividend on the par value.

**28.** I gave my broker orders to buy 1500 shares of Atchison (preferred) and to sell 2000 shares of Canadian Pacific. If he bought at the lowest price in the table and sold at the highest price, what balance will he put to my credit?

## BONDS

**293.** A **bond** is an instrument by which a government, a municipality, or a corporation contracts and agrees to pay a specified sum of money on a given date, at a specified rate of interest. — ROLLINS.

Bonds are generally issued at a face value of $1000; less frequently, of $500; occasionally, of $100. All bonds of the same issue usually have the same rights and security.

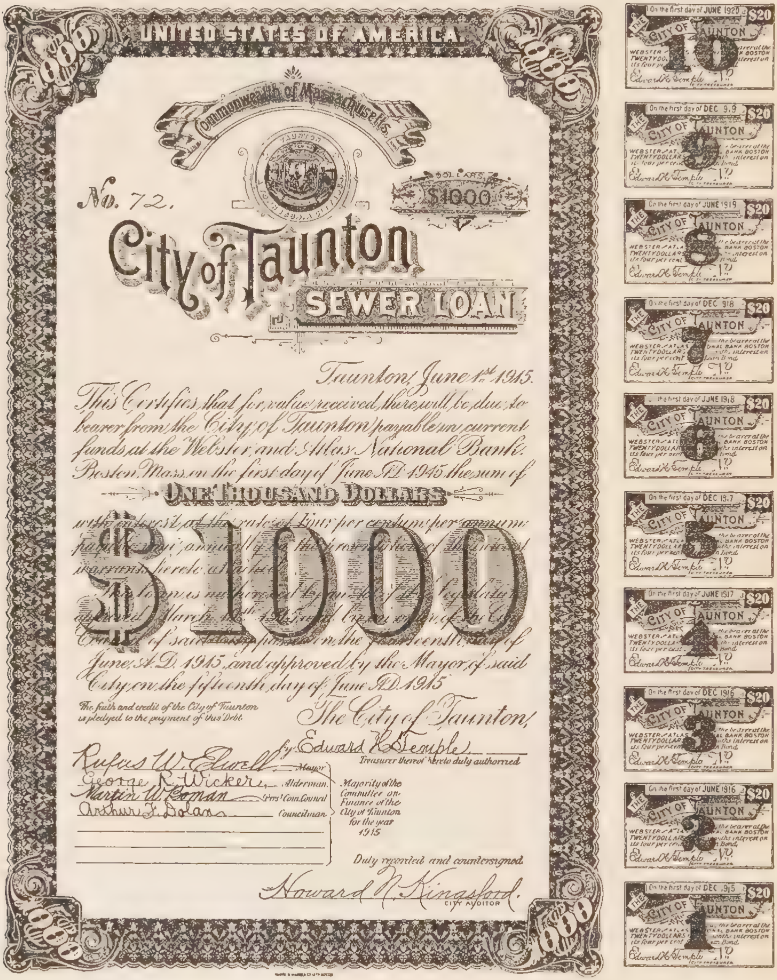
Bonds, the payment of which depends only on the unsecured credit of the issuing company, are called *debenture bonds;* those that have their payment secured by a mortgage on the property of the issuing corporation are called *mortgage bonds;* those that are secured by a deposit with a trustee of collateral are called *collateral trust bonds;* those that provide that the interest on them shall be paid only if earned are called *income bonds.*

Bonds of a national government are called *government bonds;* of a state, a city, a town, or other municipal organization, *municipal bonds.*

The names of the different government bonds are usually derived from the interest they bear and the time when they mature. Thus, "U. S. 2s, 1930," are United States bonds bearing interest at 2% and maturing in 1930.

From the gross earnings of a company the operating expenses are first deducted; from the net earnings are deducted all fixed charges, such as interest on bonds; then the dividends on preferred stock are paid; and finally out of the remainder dividends on the common stock are paid.

**294.** With reference to the **form of contract** for the payment of principal and interest there are two kinds of bonds: *coupon* and *registered.*



UNITED STATES OF AMERICA

Commonwealth of Massachusetts.

No. 72.

DOLLARS $1000

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received, there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1945 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum payable semi-annually on the [illegible] warrants hereto attached. [illegible] March [illegible] Council of said City passed on the fourteenth day of June, A.D. 1915, and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

$1000

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton,

by Edward H. Temple

Treasurer thereof hereto duly authorized

Rufus W. Elwell Mayor

George R. Wicker Alderman.

Martin W. Coman Pres't Com. Council

Arthur J. Dolan Councilman

Majority of the Committee on Finance of the City of Taunton for the year 1915

Duly recorded and countersigned.

Howard N. Kingsford.

CITY AUDITOR

On the first day of JUNE 1920 $20 THE CITY OF TAUNTON 10

On the first day of DEC 1919 $20 THE CITY OF TAUNTON 9

On the first day of JUNE 1919 $20 THE CITY OF TAUNTON 8

On the first day of DEC 1918 $20 THE CITY OF TAUNTON 7

On the first day of JUNE 1918 $20 THE CITY OF TAUNTON 6

On the first day of DEC 1917 $20 THE CITY OF TAUNTON 5

On the first day of JUNE 1917 $20 THE CITY OF TAUNTON 4

On the first day of DEC 1916 $20 THE CITY OF TAUNTON 3

On the first day of JUNE 1916 $20 THE CITY OF TAUNTON 2

On the first day of DEC 1915 $20 THE CITY OF TAUNTON 1

**295.** A **coupon bond** is a bond to which are attached interest notes, or coupons, representing the interest due on the bond at stated periods of payment.

The interest notes may be cut off from the bonds at maturity and the amount of interest which they represent collected through a bank. If these notes are not paid when due, they bear interest at the legal rate.

**296. A registered bond** is a bond which has no separate contract for the payment of the interest. Such a bond must be recorded on the books of the corporation in the name of the holder to whom the interest is sent by check.

Coupon bonds may be made payable either to bearer or registered as to principal only (the first custom prevails generally), and may be transferred by delivery or indorsement accordingly. Registered bonds are always drawn payable to some designated person and can be transferred only by assignment and registry on the books of the corporation.

### ORAL EXERCISE

1. Examine the bond on page 266. With reference to the form of contract, what kind of a bond is it?

2. How many interest notes (coupons) would be attached to the full bond?

3. When was the bond issued? What date (of maturity) should be written on each interest note?

4. What is the face of the bond? What rate of interest does it bear? What sum should be written on each interest note?

5. How may coupon bonds be transferred? registered bonds?

All bonds are bought and sold "and interest"; that is, interest should be reckoned on the par value from the date of the last interest payment to the date of the purchase or sale, at the rate which the bond pays.

6. If the bond on page 266 was quoted at $105\frac{7}{8}$ when it was purchased, how much did it cost, including $\frac{1}{8}\%$ brokerage? How much did the seller realize on it, if sold Aug. 1. 1915?

7. Has the city or town in which you live any bonded indebtedness (indebtedness secured by bonds)? If so, what are these bonds called, and what rate of interest do they pay?

8. What is the difference in the meaning of *government* bond and *municipal* bond? Upon what authority does the government issue bonds? Upon what authority does a town or a city issue bonds? Must the bond issue be approved by the state in which the town or the city is located?

## THE USE OF BOND TABLES

**297.** The use of tables for finding the interest on notes, bonds, etc., is common among bankers and brokers. No interest tables are illustrated in this connection because they are too extended and complex for a textbook.

Referring to the bond table, page 269, the per cents at the top of the table represent the income on the face of a bond at one of the given rates, and the per cents given in the column at the left represent the income that will be realized when a bond is bought at a certain market price. This table is for a bond maturing 20 yr. from date, with interest payable semiannually.

**298. Example.** What will be the net income on a 5 % bond bought at 97.53?

SOLUTION. In the column headed *5 %* find the price named, 97.53, then follow this line to the left and note that in the *Per Cent per Annum* column 5.20 is given; the net income on the price paid for the bond, 97.53, will be 5.2 %.

### ORAL EXERCISE

*Refer to the table and find the cost of:*

**1.** A 6 % bond that will net 6¼ %.

**2.** A 3 % bond that will net 5 %.

**3.** A 4½ % bond that will net 4.8 %.

**4.** A 4 % bond that will net 5½ %.

**5.** A man purchased 4 bonds, as follows: a 3 % bond that would net 4.6 %; a 4½ % bond that would net 4 %; a 5 % bond that would net 4½ %. What did he pay for each bond?

*Refer to the table and find the net income of:*

**6.** A 5 % bond that will cost $91.15.

**7.** A 7 % bond that will cost $125.10.

**8.** A 3 % bond that will cost $79.95.

**9.** A 6 % bond that will cost $106.02.

**10.** A man purchased 5 bonds each of which netted him 5 % income. If the bonds which he bought yielded, on the face value, the following rate of income, what did he pay for each one: 4 %, 3½ %, 5 %, 6 %, and 7 %?

## A BOND TABLE

### 20-YEAR. INTEREST PAYABLE SEMIANNUALLY

| PER CENT PER ANNUM | 3% | 3½% | 4% | 4½% | 5% | 6% | 7% |
|---|---|---|---|---|---|---|---|
| 3.70 | 90.17 | 97.19 | 104.21 | 111.24 | 118.26 | 132.30 | 146.35 |
| 3¾ | 89.51 | 96.50 | 103.50 | 110.49 | 117.48 | 131.46 | 145.44 |
| 3.80 | 88.86 | 95.82 | 102.78 | 109.74 | 116.70 | 130.63 | 144.55 |
| 3⅞ | 87.90 | 94.81 | 101.73 | 108.64 | 115.56 | 129.39 | 143.22 |
| 3.90 | 87.58 | 94.48 | 101.38 | 108.28 | 115.18 | 128.98 | 142.78 |
| 4. | 86.32 | 93.16 | 100.00 | 106.84 | 113.68 | 127.36 | 141.03 |
| 4.10 | 85.09 | 91.86 | 98.64 | 105.42 | 112.20 | 125.76 | 139.32 |
| 4⅛ | 84.78 | 91.54 | 98.31 | 105.07 | 111.84 | 125.37 | 138.90 |
| 4.20 | 83.87 | 90.59 | 97.31 | 104.03 | 110.75 | 124.19 | 137.63 |
| 4¼ | 83.27 | 89.96 | 96.65 | 103.35 | 110.04 | 123.42 | 136.80 |
| 4.30 | 82.68 | 89.34 | 96.00 | 102.66 | 109.33 | 122.65 | 135.98 |
| 4⅜ | 81.80 | 88.42 | 95.04 | 101.65 | 108.27 | 121.51 | 134.75 |
| 4.40 | 81.51 | 88.11 | 94.72 | 101.32 | 107.93 | 121.14 | 134.35 |
| 4½ | 80.35 | 86.90 | 93.45 | 100.00 | 106.55 | 119.65 | 132.74 |
| 4.60 | 79.22 | 85.72 | 92.21 | 98.70 | 105.19 | 118.18 | 131.16 |
| 4⅝ | 78.94 | 85.42 | 91.90 | 98.38 | 104.86 | 117.82 | 130.77 |
| 4.70 | 78.11 | 84.55 | 90.99 | 97.43 | 103.86 | 116.74 | 129.61 |
| 4¾ | 77.57 | 83.98 | 90.39 | 96.80 | 103.20 | 116.02 | 128.84 |
| 4.80 | 77.02 | 83.40 | 89.79 | 96.17 | 102.55 | 115.32 | 128.08 |
| 4⅞ | 76.22 | 82.56 | 88.90 | 95.24 | 101.59 | 114.27 | 126.95 |
| 4.90 | 75.95 | 82.28 | 88.61 | 94.94 | 101.27 | 113.92 | 126.58 |
| 5. | 74.90 | 81.17 | 87.45 | 93.72 | 100.00 | 112.55 | 125.10 |
| 5.10 | 73.86 | 80.09 | 86.31 | 92.53 | 98.76 | 111.20 | 123.65 |
| 5⅛ | 73.61 | 79.82 | 86.03 | 92.24 | 98.45 | 110.87 | 123.29 |
| 5.20 | 72.85 | 79.02 | 85.19 | 91.36 | 97.53 | 109.87 | 122.22 |
| 5¼ | 72.34 | 78.49 | 84.64 | 90.78 | 96.93 | 109.22 | 121.51 |
| 5.30 | 71.85 | 77.97 | 84.09 | 90.21 | 96.33 | 108.57 | 120.81 |
| 5⅜ | 71.11 | 77.19 | 83.27 | 89.36 | 95.44 | 107.60 | 119.77 |
| 5.40 | 70.87 | 76.94 | 83.01 | 89.07 | 95.14 | 107.28 | 119.42 |
| 5½ | 69.90 | 75.92 | 81.94 | 87.96 | 93.98 | 106.02 | 118.06 |
| 5⅝ | 68.72 | 74.68 | 80.64 | 86.59 | 92.55 | 104.47 | 116.38 |
| 5¾ | 67.57 | 73.46 | 79.36 | 85.26 | 91.15 | 102.95 | 114.74 |
| 5⅞ | 66.43 | 72.27 | 78.11 | 83.95 | 89.78 | 101.46 | 113.13 |
| 6. | 65.33 | 71.11 | 76.89 | 82.66 | 88.44 | 100.00 | 111.56 |
| 6⅛ | 64.25 | 69.97 | 75.69 | 81.41 | 87.13 | 98.57 | 110.01 |
| 6¼ | 63.19 | 68.85 | 74.51 | 80.18 | 85.84 | 97.17 | 108.50 |
| 6⅜ | 62.15 | 67.76 | 73.36 | 78.07 | 84.58 | 95.79 | 107.01 |
| 6½ | 61.14 | 66.69 | 72.24 | 77.79 | 83.34 | 94.45 | 105.55 |
| 6⅝ | 60.14 | 65.64 | 71.14 | 76.64 | 82.13 | 93.13 | 104.12 |
| 6¾ | 59.17 | 64.62 | 70.06 | 75.50 | 80.95 | 91.83 | 102.72 |
| 6⅞ | 58.22 | 63.61 | 69.00 | 74.39 | 79.78 | 90.57 | 101.35 |
| 7. | 57.29 | 62.63 | 67.97 | 73.31 | 78.64 | 89.32 | 100.00 |

## BUYING AND SELLING BONDS

**299.** Bonds are generally bought and sold through investment bankers or private bankers.

The commission for buying and selling bonds is the same as for buying and selling stocks.

**300.** The following table is an abbreviated form of the sales, and the opening, highest, lowest, and closing prices of bonds traded in on the New York Exchange on a recent date.

TABLE OF SALES AND RANGE OF PRICES

| SALES | BONDS | OPEN. | HIGH. | LOW. | CLOS. | NET CHANGE |
|---|---|---|---|---|---|---|
| 5,000 | Am. Hide & Leather 6s | 103½ | 103½ | 103 | 104 | + ¾ |
| 8,000 | Brooklyn Rapid Transit con. 5s | 103 | 103 | 102⅝ | 103 | |
| 6,000 | Chesapeake & Ohio 5s | 106⅞ | 107¼ | 106¾ | 107¼ | + ⅜ |
| 81,000 | Chicago, Burlington & Quincy 4s | 93⅞ | 93⅞ | 92¾ | 92¾ | − 1 |
| 15,000 | Erie 1st con. 4s | 85⅜ | 85¾ | 85 | 85 | − ⅜ |
| 1,000 | Illinois Central 4s | 93½ | 93½ | 93½ | 93½ | |
| 11,000 | Lehigh Valley con. 4½s | 99⅝ | 99⅛ | 99⅛ | 99⅛ | − ⅝ |
| 1,000 | Louisville & Nashville gold 5s | 108 | 110 | 110 | 110 | + 2 |
| 2,000 | Manhattan Ry. con. 4s | 92 | 92½ | 91⅜ | 91½ | − ½ |
| 8,000 | Missouri Pacific 4s | 59 | 57 | 56 | 56 | − 2 |
| 24,000 | N.Y. Central & Hudson River 4s 1934 | 91⅞ | 91¾ | 89⅛ | 90 | − 1⅞ |
| 35,000 | Reading general 4s | 94¾ | 95 | 94½ | 94½ | − ¼ |
| 1,000 | Standard Gas 6s | 89⅝ | 89⅝ | 89⅝ | 89 | − ⅝ |
| 4,000 | Texas Pacific 1st 5s | 102 | 102⅛ | 102 | 102⅛ | − ⅜ |
| 62,000 | Union Pacific 1st 4s | 97⅛ | 97⅞ | 97⅛ | 96⅜ | − ¾ |
| 196,000 | United Steel 5s | 102¼ | 102⅝ | 102¼ | 101⅞ | − ⅜ |
| 18,000 | Wabash 1st 5s | 103½ | 104 | 103½ | 102¾ | − ⅝ |
| 8,000 | West Shore 4s | 93¾ | 94 | 93¾ | 93½ | − ¼ |

In the first column is shown the par value of the bonds sold; in the second, the name of the bonds and the interest they bear; in the third, fourth, fifth, and sixth, respectively, the opening, highest, lowest, and closing prices of the day. In the last column, *Net Change*, the net changes between the closing prices of the given day and the closing prices of the day preceding. Thus, on the day given, $8000 worth of Brooklyn Rapid Transit bonds bearing 5% interest were sold. The opening price was $103 per $100 of par value; the highest price, $103; the lowest price, $102.62½; the closing price, $103; there was no change between the closing price of the day given and the day preceding.

**301. Example.** What is the cost of \$50,000 (par value) Chicago, Burlington & Quincy 4% bonds at the highest price quoted in the table (page 270)?

SOLUTION. \$100 of par value cost \$$93\frac{7}{8}$ + \$$0.12\frac{1}{2}$ brokerage, or \$94.
∴ \$50,000 of par value will cost 500, *i.e.*, (\$50,000 ÷ \$100) times \$94, or \$47,000.

### WRITTEN EXERCISE

(Omit the interest in solving these problems)

1. What is the cost of \$25,000 American Hide and Leather bonds at the opening price in the table?

2. I gave my broker orders to sell \$10,000 Chesapeake & Ohio 5% bonds and buy \$10,000 Texas Pacific 1st 5% bonds. If he sold at the highest price in the table and bought at the lowest price, what balance should he place to my credit?

3. Find the proceeds from these sales: \$1000 United Steel 5% bonds at the opening price in the table; \$5000 Illinois Central 4% bonds at the opening price in the table; \$75,000 Chicago, Burlington & Quincy 4% bonds at the closing price in the table; \$10,000 Erie 4% bonds at the lowest price in the table.

4. June 1, 1915, a certain city borrowed \$250,000 with which to build a new high school, and issued $4\frac{1}{2}$% 10-yr. coupon bonds as security. If these bonds sold (through a broker) at $101\frac{5}{8}$, how much was received by the city? If A bought five \$1000 bonds, how much did they cost him? If interest is payable semiannually, what date (of maturity) should the last interest note of each bond bear? What will be the amount of each interest note?

5. Find the total cost of the following purchases: \$20,000 Erie 4% bonds at the closing price in the table; \$2000 Illinois Central 4% bonds at the lowest price in the table; \$5000 Louisville & Nashville 5% bonds at the lowest price in the table; \$15,000 Missouri Pacific 4% bonds at the opening price in the table; \$10,000 Manhattan Railway 4% bonds at the lowest price in the table; \$3000 West Shore 4% bonds at the opening price in the table.

# APPENDIX A

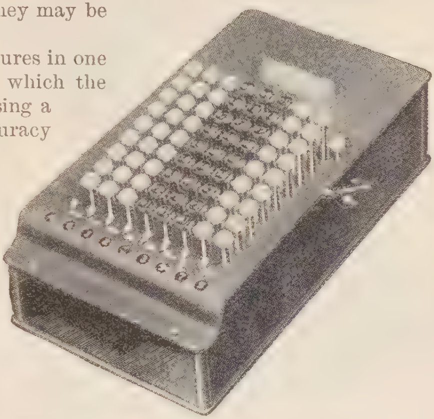
## ADDING MACHINES

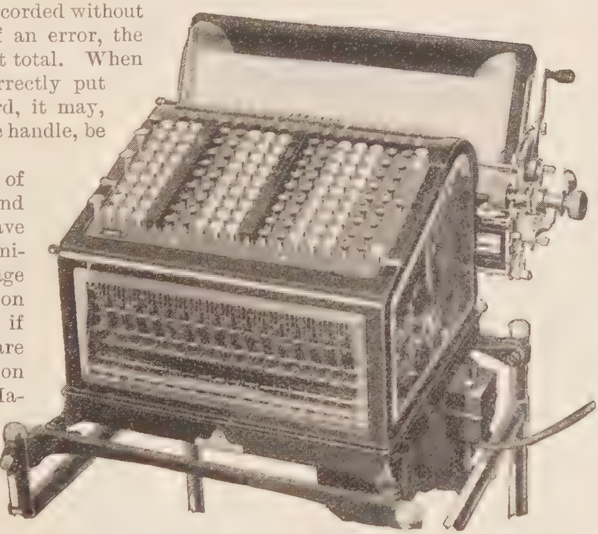
Machines or mechanical devices for performing arithmetical calculations are now commonly used in business offices; in banks, factories, insurance offices, and wholesale and retail houses they may be regarded as indispensable.

A machine will list and add figures in one fifth or one sixth of the time in which the work can be done by a person using a pen or a pencil, and with an accuracy that a person cannot equal. The operations of subtraction, multiplication, division, and trade discount may be as readily performed as those in addition.

The machine writes figures as rapidly as a typewriter, and as legibly; figures are recorded by simply touching the keys. The figures written down are added automatically, and at any time, by the mere operation of a handle, will be recorded without the possibility of an error, the absolutely correct total. When an item is incorrectly put into the keyboard, it may, before pulling the handle, be corrected.

Machines are of different sizes, and some machines have paper carriages similar to the carriage on a typewriter; on these carriages, if desired, results are printed and carbon copies made. Machines may be furnished with an electric drive, thus avoiding the handle pull.





Machines can be equipped for adding dollars and cents; feet and inches; dozens and gross; hours and minutes; tons and hundred weights; pounds and bushels; grains and pennyweights; English pounds, shillings, and pence, or any other kind of foreign money; dates and amounts; or any kind of figures.

Machines may be equipped with the *unlimited split* device for dividing the keyboard into two or more sections, for listing and adding two or more sets of figures at one operation. Thus they may also be equipped with devices for automatically listing and adding across the sheet or form in two or more columns. There is a *duplex* adding machine with two sets of wheels, to accumulate two separate totals at the same time. With a machine of this type, totals of groups of items may be secured and a grand total of the group totals accumulated at the same time.

Adding-subtracting machines add debits, subtract credits, and automatically compute the difference and print it. There are special adding machines for handling monthly statements and for ledger posting and cost accounting; a pay-roll machine that, with one operation, prints the employees' numbers and the amount of pay on the pay-roll sheet and pay envelopes.

The following are some of the uses of these machines in offices: proving daily postings; daily ledger balance; daily cash balance; daily recapitulation of sales (as cash, credit, C.O.D., etc.); checking invoices and freight bills; figuring discounts; computing commissions; summary of day's receipts and disbursements; figuring estimates; making out pay envelopes; analysis of outstanding accounts; analysis of accounts payable; balancing petty cash account; footing ledger accounts before taking the trial balance; taking off the trial-balance figures (debits and credits); reconciling cashbook balance with bank balance, listing the number and the amount of each outstanding check; making monthly statements giving month, date, total of debits, total of credits, balance and special terms; compiling statements of cost of production; footing inventories and calculating extensions; posting customers' ledger.

The cuts show various types of calculating machines.

# APPENDIX B

## TABLES OF MEASURES

### MEASURES OF CAPACITY

*Liquid Measure*

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
= 231 cubic inches

*Dry Measure*

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
= 2150.42 cubic inches

**Barrels** and **hogsheads** vary in size; but in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl., or 63 gal., a hogshead.

A **heaped bushel**, used for measuring apples, corn in the ear, etc., equals 2747.71 cu. in. A **dry quart** equals 67.2 cu. in., and a **liquid quart** 57.75 cu. in.

### MEASURES OF WEIGHT

*Avoirdupois Weight*

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton

*Troy Weight*

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

*Apothecaries' Weight*

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound

*Comparative Weights*

1 lb. troy or apothecaries' = 5760 gr.
1 oz. troy or apothecaries' = 480 gr.
1 lb. avoirdupois = 7000 gr.
1 oz. avoirdupois = $437\frac{1}{2}$ gr.

The ton of 2000 lb. is sometimes called a **short ton**. There is a ton of 2240 lb., called a **long ton**, used in all customhouse business and in some wholesale transactions in mining products.

In weighing diamonds, pearls, and other jewels, the unit generally employed is the **carat**, equal to 3.2 troy grains. The term "carat" is also used to express the number of parts in 24 that are pure gold. Thus, gold that is 14 carats fine is $\frac{14}{24}$ pure gold and $\frac{10}{24}$ alloy.

*Miscellaneous Weights*

1 keg of nails = 100 pounds
1 cental of grain = 100 pounds
1 quintal of fish = 100 pounds

1 barrel of salt = 280 pounds
1 barrel of flour = 196 pounds
1 barrel of pork or beef = 200 pounds

A cubic foot of water contains about $7\frac{1}{2}$ gal. and weighs $62\frac{1}{2}$ lb., avoirdupois.

## MEASURES OF EXTENSION

### *Long Measure*

12 inches = 1 foot
3 feet = 1 yard
5½ yards, or 16½ feet = 1 rod
320 rods, or 5280 feet = 1 mile

### *Surveyors' Long Measure*

7.92 inches = 1 link
25 links = 1 rod
4 rods, or 100 links = 1 chain
80 chains = 1 mile

City lots are usually measured by feet and decimal fractions of a foot; farms, by rods or chains.

### *Miscellaneous Long Measures*

4 inches = 1 hand
6 feet = 1 fathom
120 fathoms = 1 cable length
1.15 miles, nearly, = 1 knot, or
1 nautical or geographical mile

### *Square Measure*

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

The **hand** is used in measuring the height of horses at the shoulder. The **fathom** and **cable length** are used by sailors for measuring depths at sea. The **knot** is used by sailors in measuring distances at sea. Three knots are frequently called a **league**.

### *Surveyors' Square Measure*

625 square links = 1 square rod
16 square rods = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile
36 square miles = 1 township

### *Cubic Measure*

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord
1 cubic yard = 1 load (of earth, etc.)
24¾ cubic feet = 1 perch

The square rod is sometimes called a **perch**. The word **rood** is sometimes used to mean 40 sq. rd. or ¼ A. In the government surveys, 1 sq. mi. is called a **section**.

The perch of stone or masonry varies in different parts of the country; but it is usually considered as 1 rd. long, 1 ft. high, and 1½ ft. thick, or 24¾ cu. ft.

### *Angular Measure*

60 seconds = 1 minute
60 minutes = 1 degree

90 degrees = 1 right angle
360 degrees = 1 circumference

Angular (also called circular) measure is used principally in surveying, navigation, and geography for measuring arcs of angles, for reckoning latitude and longitude, for determining locations of places and vessels, and for computing difference of time.

A minute of the earth's circumference is equal to a geographical mile. A degree of the earth's circumference at the equator is therefore equal to about 69 statute miles.

### MEASURES OF TIME

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
30 days = 1 commercial month

12 months = 1 year
360 days = 1 commercial year
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

September, April, June, and November have 30 da. each; all of the other months have 31 da. each, except February, which has 28 da. in a common year and 29 da. in a leap year.

Centennial years that are divisible by 400 and other years that are divisible by 4 are leap years.

In running trains across such a broad stretch of country as the United States, it is highly important to have a uniform time over considerable territory. Recognizing this, in 1883, the railroad companies of the United States and Canada adopted for their own convenience a system of **standard time.** This system divides the United States into **four time belts,** each covering approximately 15° of longitude, $7\frac{1}{2}^\circ$ of which are east and $7\frac{1}{2}^\circ$ west of the governing meridian. The region of **eastern time** lies approximately $7\frac{1}{2}^\circ$ each side of the 75th meridian, and the time throughout this belt is the same as the local time of the 75th meridian. Similarly, the regions of **central, mountain,** and **Pacific time** lie approximately $7\frac{1}{2}^\circ$ each side of the 90th, 105th, and 120th meridians, respectively, and the time throughout each belt is determined by the local time of the governing meridian of that belt. There is just one hour's difference between adjacent time belts. Thus, when it is 11 o'clock A.M. by eastern time, it is 10 o'clock A.M. by central time, 9 o'clock A.M. by mountain time, and 8 o'clock A.M. by Pacific time. Since railroad companies change the time at important stations and termini, regardless of the longitude of such stations and termini, the boundaries of the time belts are quite irregular.

### MEASURES OF VALUE

*United States Money*

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

*English Money*

4 farthings = 1 penny
12 pence = 1 shilling
20 shillings = 1 pound sterling
= $4.8665

The term "eagle" is seldom used in business. The mill is not a coin, but the name is frequently used in some calculations. In Canada the units of money are the same as in the United States. 1 far. = $\frac{50}{99}$¢; 1*d.* = $2\frac{2}{99}$¢; 1*s.* = $24\frac{1}{3}$¢.

*French Money*

100 centimes = 1 franc = $0.193

*German Money*

100 pfennigs = 1 mark = $0.238

### MISCELLANEOUS MEASURES

*Counting by 12*

12 things = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross

*Counting Sheets of Paper*

24 sheets = 1 quire
20 quires = 1 ream
= 480 sheets

## BUSINESS ABBREVIATIONS

| | |
|---|---|
| A | acre |
| Apr. | April |
| Aug. | August |
| bbl. | barrel; barrels |
| bdl. | bundle; bundles |
| bg. | bag; bags |
| bkt. | basket; baskets |
| bl. | bale; bales |
| bu. | bushel; bushels |
| bx. | box; boxes |
| cd. | cord; cords |
| ch. | chain; chains |
| c.i.f. | carriage and insurance free |
| Co. | company; county |
| C.O.D. | collect on delivery |
| Coll. | collection |
| Cr. | creditor; credit |
| cs. | case; cases |
| ct. | cent; cents; centime |
| cu. ft. | cubic foot; cubic feet |
| cu. in. | cubic inch; cubic inches |
| cu. yd. | cubic yard; cubic yards |
| cwt. | hundredweight |
| *d.* | pence |
| da. | day; days |
| Dec. | December |
| doz. | dozen; dozens |
| Dr. | debtor; debit; doctor |
| E. | east |
| ea. | each |
| e.g. | *exempli gratia,* for example |
| etc. | *et cætera,* and so forth |
| far. | farthing; farthings |
| Feb. | February |
| f.o.b. | free on board |
| fr. | franc; francs |
| ft. | foot; feet |
| gal. | gallon; gallons |
| gi. | gill; gills |
| gr. | grain; grains |
| gro. | gross |
| hhd. | hogshead; hogsheads |
| hf. cht. | half chest; half chests |
| hr. | hour; hours |
| i.e. | *id est,* that is |
| in. | inch; inches |
| Jan. | January |
| kg. | keg; kegs |
| l. | link; links |
| lb. | pound; pounds |
| Mar. | March |
| Mdse. | merchandise |
| Messrs. | *Messieurs,* Gentlemen; Sirs |
| mi. | mile; miles |
| min. | minute; minutes |
| mo. | month; months |
| Mr. | Mister |
| Mrs. | Mistress |
| N. | north |
| No. | number |
| Nov. | November |
| Oct. | October |
| oz. | ounce; ounces |
| p. | page |
| pc. | piece; pieces |
| per. | by the; by |
| per cent. | *per centum,* by the hundred |
| pk. | peck; pecks |
| pkg. | package; packages |
| pp. | pages |
| pr. | pair; pairs |
| pt. | pint; pints |
| pwt. | pennyweight; pennyweights |
| qr. | quire; quires |
| qt. | quart; quarts |
| rd. | rod; rods |
| rm. | ream; reams |
| Rm.(or M.) | *Reichsmark,* Mark |
| *s.* | shilling; shillings |
| S. | South |
| sec. | second; seconds |
| sq. ch. | square chain; square chains |
| sq. ft. | square foot; square feet |
| sq. mi. | square mile; square miles |
| sq. rd. | square rod; square rods |
| sq. yd. | square yard; square yards |
| T. | ton |
| tb. | tub; tubs |
| Tp. | township; townships |
| viz. | *videlicet,* namely; to wit |
| via | by way of |
| wk. | week; weeks |
| wt. | weight; weigh |
| yd. | yard; yards |
| yr. | year; years. |

## BUSINESS SYMBOLS

| | |
|---|---|
| $^a/_c$ | account |
| $^a/_s$ | account sales |
| + | addition |
| ( ), ‾ | aggregation |
| & | and |
| . . . . . | and so on |
| @ | at; to |
| $^c/_o$ | care of |
| ¢ | cent; cents |
| √ | check mark |
| ° | degree |
| ÷ | division |
| $ | dollar; dollars |
| = | equal; equals |
| | foot; feet; minutes |
| C | hundred |
| ″ | inch; inches; seconds |
| × | multiplication |
| # | number, if written before a figure; pounds, if written after a figure |
| $1^1$ | one and one fourth |
| $1^2$ | one and two fourths; one and one half |
| $1^3$ | one and three fourths |
| ⅌ | per; by |
| % | per cent; hundredth; hundredths |
| £ | pounds sterling |
| ∵ | since |
| − | subtraction |
| ∴ | therefore |
| M | thousand |
| $^5/_6$ | 5 shillings 6 pence; five sixths |

# INDEX

Abbreviations, 278
Above par, 258
Abstract number, 36
Adding machines, 273
Addition, 2, 78, 107, 172
Aliquot parts, 138
Amount, 176, 232
Angular measure, 276
Apothecaries' weight, 275
Assessment, 257, 260
At a discount, 254, 258
At par, 254
At a premium, 254, 258
Average clause, 211
Avoirdupois weight, 275

Bank discount, 230, 231
Bank drafts, 246, 248
Bank loans, 238
Bank money order, 243
Banker's sixty-day method of interest, 219
Banking, 216
Base, 176, 180
Below par, 258
Bill of lading, 254
Bills, 31, 32, 45, 49, 116, 145, 146, 150, 151, 152, 153, 154, 155, 156, 161, 165, 190, 208
Bills and accounts, 150
Blanket policy, 210
Bond table, 269
Bonds, 265, 266
Buying bonds, 270
Buying by the hundred, 89
Buying by the thousand, 89
Buying by the ton, 90
Buying stocks, 262

Cable length, 276
Cancellation, 97
Capital stock, 256
Carat, 275
Cashier's check, 249
Certificate of deposit, 249
Change memorandum, 159
Charter, 256
Checking results, 12, 24, 38, 43, 44, 55, 71, 72, 73
Checks, 246, 250, 260
Clearing house, 246, 247
Code, 244
Co-insurance, 211
Collateral note, 240
Collateral trust bond, 265
Collection and exchange, 236, 252
Commercial bank, 230
Commercial discounts, 188
Commercial drafts, 231, 252
Common denominator, 106
Common divisor, 98
Common fractions, 101
Common stock, 258
Comparative weights, 275
Complement, 27
Composite number, 95
Concrete number, 36
Consecutive numbers, 10
Conversion of fractions, 127
Corporation, 256
Counting by 12, 277
Counting sheets of paper, 277
Coupon bond, 266
Cubic measure, 276

Day method of interest, 217
Days of grace, 231
Debenture bonds, 265
Decimal, 75
Decimal fractions, 75
Decimal units, 75
Demand note, 239
Denominate quantities, 169
Denominator, 101
Deposit slip, 251
Depositors' ledger, 30, 34, 35, 50
Difference, 176
Discount series, 188, 191
Dividend, 52, 257, 260
Division, 52, 57, 85, 88, 121, 173
Divisor, 52
Divisors, 98
Domestic exchange, 242
Drafts, 230, 231, 238, 252

Drawee, 231
Drawer, 231
Dry measure, 275

English money, 277
Even number, 95
Exchange, 242, 249
Exponent, 36
Express money order, 243
Expressage, 164

Factor, 36, 95
Factoring, 96
Fathom, 276
Final results, 105
Finding the base, 180
Finding the cost, 199
Finding the gain or loss, 197
Finding the per cent of gain or loss, 198
Finding the percentage, 176
Finding the rate, 178
Fire insurance, 209
Fire losses, 133, 134
Firm note, 236
Fixed policy, 210
Floating policy, 210
Fluctuation of rates of exchange, 254
Fractional relations, 124
Fractions, 75
Freight bill, 165
Freightage, 164
French money, 277

Gain and loss, 196
Gain or loss on selling price, 202
German money, 277
Government bonds, 265
Graphic representations, 126, 133, 185
Greatest common divisor, 98
Gross price, 189
Gross weight, 30
Grouping, 3, 6

Hand, 276
Heaped bushel, 275
Holder, 230
Horizontal addition, 16

Important per cents, 176
Improper fraction, 102
Income bond, 265
Insurance, 209
Insurance rates, 213
Insurer, 210
Interest, 216
Invoice, 150

Joint and several note, 236
Joint note, 236

Key, 204
Knot, 276

League, 276
Least common denominator, 106
Least common multiple, 100
Letter ordering goods, 162
Liquid measure, 275
Listing goods for catalogues, 207
Long measure, 276
Long ton, 275

Making change, 25
Market value, 258
Marking goods, 204
Maturity, 230
Maturity table, 232
Measures of capacity, 275
Measures of extension, 276
Measures of time, 277
Measures of value, 277
Measures of weight, 275
Miscellaneous measures, 277
Miscellaneous weights, 275
Mixed numbers, 102
Model figures, 11, 13, 14, 15
Money orders, 242
Mortgage bonds, 265
Multiple, 36
Multiplication, 36, 41, 43, 45, 46, 47 82, 114, 120, 173
Municipal bonds, 265
Mutual insurance company, 210

Net price, 189
Net weight, 31
Notation, 76
Notes, 239, 240
Numeration, 76
Numeration table, 76
Numerator, 101

Odd number, 95
Open policy, 210
Ordinary policy, 210

Par value, 258
Parcel post, 63
Parenthesis, 23
Pay rolls, 159, 160, 163

Payee, 231
Pay-roll memorandum, 160
Per cent, 76, 175
Per cents of decrease, 182
Per cents of increase, 181
Percentage, 175
Perch, 276
Policy, 210
Postal money order, 242
Postal service, 63
Power, 37
Preferred stock, 257
Premium, 210
Prime number, 95
Principal, 216
Proceeds, 232
Promissory notes, 239, 240
Properties of 9, 71
Properties of 11, 72
Property insurance, 209

Quotient, 52

Rate, 176, 211, 216
Rate of exchange, 243, 248, 254
Reading decimals, 76
Reduction, 103, 104, 105, 106, 170, 171
Registered bond, 267
Remainder, 52
Repeaters, 204
Review of the common tables, 169
Review tests, 22, 35, 51, 94, 137, 187, 195, 241
Rood, 276

Scrip, 259
Section, 276
Selling by the hundred, 89
Selling by the thousand, 89
Selling by the ton, 90
Share, 256
Short methods, 41, 57, 108, 118
Short ton, 275
Sight draft, 252
Similar fractions, 106
Simple interest, 217
Six per cent method of interest, 227
Sixteen to one, 118
Solution of problems, 128
Specific policy, 210
Square measure, 276
Standard time, 277
Stock broker, 258
Stock certificates, 256, 257, 258
Stock company, 256
Stock insurance company, 210
Stockholder, 256
Stocks and bonds, 256
Subtraction, 23, 80, 112, 172
Surveyors' long measure, 276
Surveyors' square measure, 276

Table of aliquot parts, 140
Table of bond quotations, 270
Table of common measures, 275
Table of important per cents, 176
Table of insurance rates, 213
Table of stock quotations, 262
Table of time, 234
Table of twelfths, 206
Tare, 30
Tariff, or rate, book, 213
Telegram, 244
Telegraphic money order, 244
Telegraphic rates, 245
Term of discount, 232
Terms of a fraction, 102
Tests of divisibility, 96
Time sheets, 159, 160, 163
Time slip, 163
Trade discount, 188
Troy weight, 275

Underwriter, 210
Unit fraction, 102
United States money, 277

Value, 232
Valued policy, 210
Vinculum, 23

Weigh tickets, 90